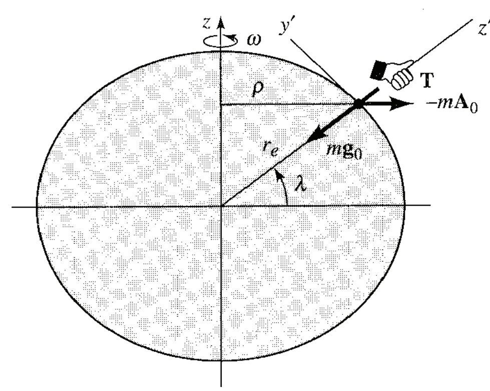
5.4. Effects of Earth's Rotation

❑ Consider a coordinate system that is moving with the Earth. Because the angular speed of Earth's rotation is 2π radians per day, *the effects of such rotation is relatively small*.

❑ Nevertheless, it is the spin of the Earth that makes the equatorial radius is some 13 miles greater than the polar radius, i.e. *equatorial bulge*.

5.4.1. Static Effects: The Plumb line

Let us describe the motion of the plumb bob in a local frame of reference whose origin is at the position of the bob. Our frame of reference is attached to the surface of the Earth, so it is undergoing translation as well as rotation.



❑ The **translation** of the frame takes place along a circle whose radius is $\rho = r_e \cos\lambda$, where r_e is the radius of the Earth and λ is the geocentric latitude of the plumb bob. Hence;

$$A_0 = \omega^2 \rho = \omega^2 r_e \cos\lambda \qquad (126)$$

❑ Its rate of **rotation** is ω, the same as that of the Earth about its axis. Let us now examine the terms of Eq. (124):

- The *force* $-m\mathbf{a}'$ is **zero**, because the bob is at rest in the local frame of reference, i.e. $\mathbf{a}' = 0$.

- The *Coriolis force* $-2m\boldsymbol{\omega} \times \mathbf{v}'$ is **zero**, because $\mathbf{v}' = 0$.

- The *transverse force* $-m\dot{\boldsymbol{\omega}} \times \mathbf{r}'$ is **zero**, because ω is **constant**.

- The *centrifugal force* $-m\,\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}')$ is **zero**, because the origin of the local coordinate system is centered on the bob. I.e, $\mathbf{r}' = 0$.

The only surviving terms in Eq. (124) are the *real forces* $\mathbf{F}$ and the *inertial term* $-m\mathbf{A}_0$, which arises because the local frame of reference is accelerating. Thus,

$$\mathbf{F} - m\mathbf{A}_0 = 0 \qquad (127)$$

In other words, the rotation of the Earth causes the acceleration $\mathbf{A}_0$ of the local frame. The bob does not hang on a line pointing toward the center of the Earth because the *inertial* force $-m\mathbf{A}_0$ throws it outward, away from Earth's axis of rotation. The magnitude of this force is;

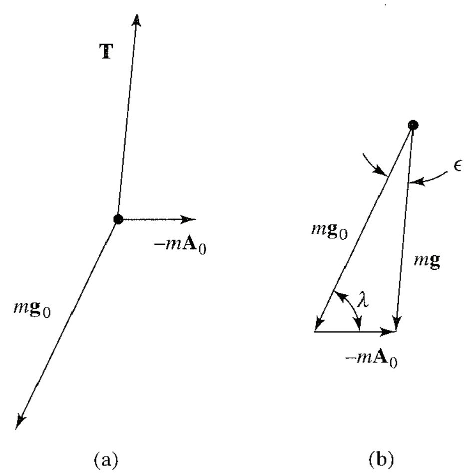
$$m\omega^2 r_e \cos\lambda$$

It is a **maximum** when $\lambda = 0$ at the *Earth's equator* and a **minimum** at either *pole*.

The *tension* $\mathbf{T}$ in the string balances out the real gravitational force $m\mathbf{g}_0$ and the inertial force $-m\mathbf{A}_0$, i.e;

$$(\mathbf{T} + m\mathbf{g}_0) - m\mathbf{A}_0 = 0 \qquad (128)$$

Now, when we hang a plumb bob, we normally think that the tension $\mathbf{T}$ balances out the local force of gravity, which we call $m\mathbf{g}$. We can see from the Figure that:



$$\text{mg} = \text{mg}_0 - m\text{A}_0$$

or

$$\mathbf{g} = \mathbf{g}_0 - \mathbf{A}_0 \qquad (129)$$

As can be seen the *inertial* reaction $-m\mathbf{A}_0$, directed away from Earth's axis, causes the direction of the plumb line to deviate by a small angle ε away from the direction toward Earth's center.

We can easily calculate the value of the angle ε. From the Figure we have;

$$\frac{\sin\varepsilon}{m\omega^2 r_e \cos\lambda} = \frac{\sin\lambda}{mg}$$

or, because ε is small

$$\sin\varepsilon \approx \varepsilon = \frac{\omega^2 r_e}{2g}\sin 2\lambda \qquad (130)$$

Thus, ε vanishes at the equator ($\lambda = 0$) and the poles ($\lambda = \pm 90$). The maximum deviation of the direction of the plumb line from the center of the Earth occurs at $\lambda = 45°$

where;
$$\varepsilon_{\max} = \frac{\omega^2 r_e}{2g} \approx 0.1^o$$

5.4.2. Dynamic Effects:

1- Falling Body

A body that is dropped from a height h above the ground, as it falls, it will drift to the east. The eastward drift is given by:

$$x' = \tfrac{1}{3}\omega\left(\frac{8h^3}{g}\right) cos\,\lambda \qquad (131)$$

For a height of 100 m at latitude of 45°, the drift is 1.55 cm

2- Deflection of a Rifle Bullet

If we fire a projectile to east with high initial speed v_0 , the projectile will bend to the south. If H is the horizontal range of the projectile, the transverse deflection is then;

$$\Delta \approx \frac{\omega H^2}{v_0}\left|sin\,\lambda\right| \qquad (132)$$

This is the same for any direction in which the projectile is initially aimed, provided the trajectory is flat.