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Lecture: (7)

Shannon-Fano code

Subject: Coding Techniques

First Stage

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Shannon - Fano code

To encode a message using Shannon-Fano method, you can follow the below steps :

1. Sort the symbols in descending order according to their probabilities.
2. Divide the list of symbols into two parts : upper and lower, so that the summation of the probabilities of the upper part is equal ***as possible*** to the summation of the lower part symbols.
3. Assign "0" code to each of the upper part symbols, and "1" code to each of the lower part symbols.
4. Divide each of the upper and lower part into upper and lower subdivision as in step (2) above, and assign the code "0" and "1" as in step (3) above.
5. Continue in step(4) until each subdivision contains only one symbols.

Example 1:

A source produce 5 independent symbols (x1, x2, x3, x4, x5) with its corresponding probabilities 0.1, 0.3, 0.15, 0.25, 0.2 . design a binary code for the above source symbol using Shannon – fanon method .

Sol

symbols	P_i	code	l_i
x_2	0.3	0 0	2
x_4	0.25	0 1	2
x_5	0.2	1 0	2
x_3	0.15	1 1 0	3
x_1	0.1	1 1 1	3

$$L = \sum P_i l_i = 2*0.3 + 2*0.25 + 2*0.2 + 3*0.15 + 3*0.1$$
$$= 2.25 \text{ Bits\symbol}$$

The entropy (the smallest number of bits needed, on average, to represent each symbol) is

$$H = -\sum P_i \log P_i$$
$$= -(0.3 \log 0.3 + 0.25 \log 0.25 + 0.2 \log 0.2 + 0.15 \log 0.15 + 0.1 \log 0.1)$$
$$= 2.228 \text{ Bits/symbol}$$
$$R = L - H = 2.25 - 2.228 = 0.022$$
$$\xi_{code} = \frac{H(x)}{L} * 100\% = \frac{2.228}{2.25} * 100\% = 99\%$$

Note

If we use fixed length coding

$$L = \lceil \log_2 5 \rceil = \lceil 2.3219 \rceil = 3$$

$$R = L - H = 3 - 2.228 = 0.772$$

$$\xi\ code = \frac{H(x)}{L} * 100\ \% = \frac{2.228}{3} * 100\ \% = 74\ \%$$

.: coding in Shannon –fanon is more efficient than coding in fixed length coding .

Example 2:

A source produce 5 independent symbols (x1, x2, x3, x4, x5) with its corresponding probabilities 0.1, 0.05, 0.25, 0.5, 0.1. design a binary code for the above source symbol using Shannon – fanon method .

Sol

symbols	P_i	code	l_i
x_4	0.5	0	1
x_3	0.25	1 0	2
x_1	0.1	1 1 0	3
x_5	0.1	1 1 1 0	4
x_2	0.05	1 1 1 1	4

$$L = \sum P_i\, l_i = 1{*}0.5 + 2{*}0.25 + 3{*}\ 0.1 + 4{*}\ 0.1 + 4{*}\ 0.05$$
$$= 1.9 \text{ Bits / symbol}$$

The entropy (the smallest number of bits needed, on average, to represent each symbol) is

$H = -\sum P_i \log P_i$

$= -(0.5 \log 0.5 + 0.25 \log 0.25 + 0.1 \log 0.1 + 0.1 \log 0.1 + 0.05 \log 0.05)$

$= 1.88$ Bits /symbol

$R = L - H = 1.9 - 1.88 = 0.02$

$\xi\ code = \frac{H(x)}{L} * 100\ \%$.

$= \frac{1.88}{1.9} * 100\ \% = 99\ \%$

Example 3:

A source produce 5 independent symbols (x1, x2, x3, x4, x5) with its corresponding probabilities 0.1, 0.35, 0.3, 0.05, 0.2. design a binary code for the above source symbol using Shannon – fanon method .

Sol

symbols	P_i	code	l_i
x_2	0.35	0	1
x_3	0.3	1 0	2
x_5	0.2	1 1 0	3
x_1	0.1	1 1 1 0	4
x_4	0.05	1 1 1 1	4

$L = \sum P_i l_i = 1*0.35 + 2*0.3 + 3*0.2 + 4*0.1 + 4*0.05$

$= 2.15$ Bit / symbol

The entropy (the smallest number of bits needed, on average, to represent each symbol) is

$$H = -\sum P_i \log P_i$$
$$= -(0.35 \log 0.35 + 0.3 \log 0.3 + 0.2 \log 0.2 + 0.1 \log 0.1 + 0.05 \log 0.05)$$
$$= 2.062 \text{ Bit/symbol}$$

$$R = L - H = 2.15 - 2.062 = 0.088$$
$$\xi_{code} = \frac{H(x)}{L} * 100\ \% .$$
$$= \frac{2.062}{2.15} * 100\ \% = 96\ \%$$

Sol 2

symbols	P_i	code	l_i
x_2	0.35	0 0	2
x_3	0.3	0 1	2
x_5	0.2	1 0	2
x_1	0.1	1 1 0	3
x_4	0.05	1 1 1	3

$$L = \sum P_i l_i = 1*0.35 + 2*0.3 + 2*0.2 + 3*0.1 + 3*0.05$$
$$= 2.15 \text{ Bit / symbol}$$

Example 4:

A source produce 7 independent symbols (x1, x2, …. , x7) with its corresponding probabilities 0.10, 0.15, 0.10, 0.05, 0.25, 0.20, 015. design a binary code for the above source symbol using Shannon – fanon method .

Sol

Symbols	Prob.	Code	li
X5	0.25	00	2
X6	0.20	01	2
X2	0.15	100	3
X7	0.15	101	3
X1	0.10	110	3
X3	0.10	1110	4
X4	0.05	1111	4

The average size of this code is

$$L = 0.25 \times 2 + 0.20 \times 2 + 0.15 \times 3 + 0.15 \times 3 + 0.10 \times 3 + 0.10 \times 4 + 0.05 \times 4$$
$$= 2.7 \text{ bits/symbol.}$$

The entropy (the smallest number of bits needed, on average, to represent each symbol) is

$$H = -(0.25\log_2 0.25 + 0.20\log_2 0.20 + 0.15\log_2 0.15 + 0.15\log_2 0.15 + 0.10\log_2 0.10 + 0.10\log_2 0.10 + 0.05\log_2 0.05)$$
$$= 2.67 \text{ bits/ symbols.}$$

$$R = L - H = 2.7 - 2.67 = 0.03$$

$$\xi_{code} = \frac{H(x)}{L} * 100\ \% .$$
$$= \frac{2.67}{2.7} * 100\ \% = 99\ \%$$

Example 5 :

A source produce 5 independent symbols (A, B, C, D, Z) with the below probabilities:

1. For the letters (A, B, C, D) , the probability of each letter is twice as

its successor

2. The probability of the letter Z is equal to the summation of the probabilities of B and D.

design a binary code for the above source symbol using Shannon – fanon method .

Sol

symbols	Pi	code	Ii
A	0.4	0	1
Z	0.25	1 0	2
B	0.2	1 1 0	3
C	0.1	1 1 1 0	4
D	0.05	1 1 1 1	4

$L = \sum I_i \, p(x_i)$

$= 1*0.4 + 2*0.25 + 3*0.2 + 4*0.1 + 4*0.05 = 2.1$ Bit / symbol

$H = -\sum P(xi) \log P(xi)$

$= -(0.4 \log 0.4 + 0.25 \log 0.25 + 0.2 \log 0.2 + 0.1 \log 0.1 + 0.05 \log 0.05) = 2.025$ Bit/symbol

ξ code $= H(x)/L * 100\,\% . = 2.025/2.1 * 100\,\% = 96\,\%$