Ministry of Higher Education and
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الرياضيات التكاملية

محاضرة 1

التكامل المحدد Definite integral

اعداد

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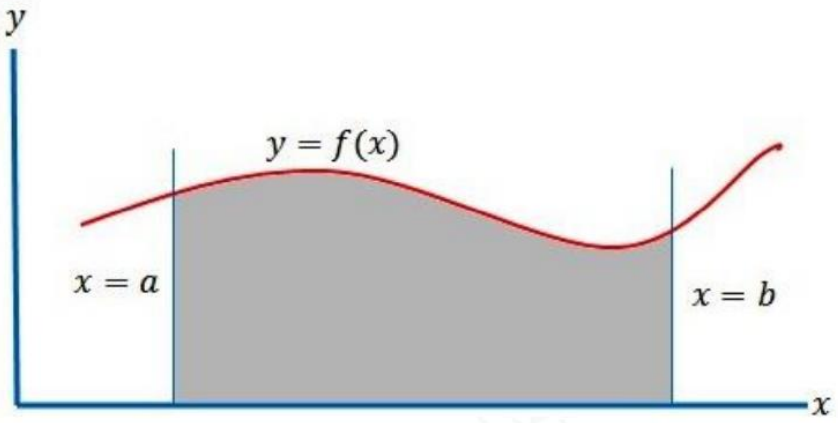
التكامل المحدد *Definite integral*

لتكن الدالة $f(x)$ دالة مستمرة على الفترة $[a,b]$ فان التكامل المحدد يُعرّف بأنه $\int_a^b f(x)dx$ المساحة الواقعة تحت المنحني $f(x)$ وفوق المحور السيني والمحددة بالمستقيمين المتوازيين $x=a$ و $x=b$

كما موضح في الشكل أدناه :



بعض خواص التكامل المحدد **Some propereties of definite integral**

لتكن الدالة $f(x)$ دالة مستمرة على فترة التكامل $a \le x \le b$ فان :

1. *Order of Integration:*	$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$	A definition
2. *Zero Width Interval:*	$\int_a^a f(x)\,dx = 0$	A definition when $f(a)$ exists
3. *Constant Multiple:*	$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$	Any constant k
4. *Sum and Difference:*	$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$	
5. *Additivity:*	$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$	

Definite Integrals

1. $\int_0^1 (x^2 + 3x + 2)dx$
2. $\int_{-\pi/2}^{\pi/2} \sin(x)dx$
3. $\int_0^{\pi} \cos^2(x)dx$ *(Hint: Use the identity* $\cos^2(x) = \frac{1+\cos(2x)}{2}$*)*
4. $\int_1^2 \frac{1}{x}dx$

(a) $\int_0^1 (x^2 + 3x + 2)dx$

Find the antiderivative:

$$\int (x^2 + 3x + 2)dx = \frac{x^3}{3} + \frac{3x^2}{2} + 2x$$

Evaluate at the limits:

$$\int_0^1 (x^2 + 3x + 2)dx = \left[\frac{1^3}{3} + \frac{3 \cdot 1^2}{2} + 2 \cdot 1\right] - \left[\frac{0^3}{3} + \frac{3 \cdot 0^2}{2} + 2 \cdot 0\right]$$
$$= \left(\frac{1}{3} + \frac{3}{2} + 2\right) - 0 = \frac{1}{3} + \frac{9}{6} + \frac{12}{6} = \frac{1}{3} + \frac{21}{6} = \frac{23}{6}$$

(b) $\int_{-\pi/2}^{\pi/2} \sin(x)dx$

The antiderivative of $\sin(x)$ is $-\cos(x)$:

$$\int_{-\pi/2}^{\pi/2} \sin(x)dx = [-\cos(x)]_{-\pi/2}^{\pi/2}$$
$$= -\cos\left(\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right)$$
$$= -0 + 0 = 0$$

(c) $\int_0^{\pi} \cos^2(x)dx$

Using the identity $\cos^2(x) = \frac{1+\cos(2x)}{2}$:

$$\int_0^{\pi} \cos^2(x)dx = \int_0^{\pi} \frac{1+\cos(2x)}{2}dx$$

Split the integral:

$$= \frac{1}{2}\int_0^{\pi} 1dx + \frac{1}{2}\int_0^{\pi} \cos(2x)dx$$

Evaluate each term:

1. $\int_0^{\pi} 1dx = [x]_0^{\pi} = \pi - 0 = \pi$
2. $\int_0^{\pi} \cos(2x)dx = \left[\frac{\sin(2x)}{2}\right]_0^{\pi} = \frac{\sin(2\pi)}{2} - \frac{\sin(0)}{2} = 0$

So:

$$\int_0^{\pi} \cos^2(x)dx = \frac{1}{2}(\pi + 0) = \frac{\pi}{2}$$

1;Evaluate the following <u>definite</u> integrals:

1-

$$\int_1^2 (3x^2+2x-2)dx$$

2-

$$\int_0^3 \frac{2x}{\sqrt{x^2+16}}dx$$

3-

$$\int_1^4 (\frac{1}{\sqrt{x}}+\sqrt{x})dx$$

SOLUSION

1-

$$\int_1^2 (3x^2+2x-2)dx$$

$$= [\frac{3x^3}{3}+\frac{2x^2}{2}-2x]_1^2 = [x^3+x^2-2x]_1^2$$

$$= (8+4-4)-(1+1-2) = 8$$

2-

$$\int_0^3 \frac{2x}{(x^2+16)^{\frac{1}{2}}}dx \Rightarrow \int_0^3 (x^2+16)^{-\frac{1}{2}}2xdx$$

$$= [\frac{(x^2+16)^{\frac{1}{2}}}{\frac{1}{2}}]_0^3 = 2[\sqrt{x^2+16}]_0^3$$

$$= [\sqrt{9+16}-\sqrt{0+16}]$$

$$= 2[5-4] = 2(1) = 2$$

3-

$$\int_1^4 (\frac{1}{\sqrt{x}}+\sqrt{x})dx$$

$$= \int_1^4 (x^{\frac{-1}{2}}+\frac{1}{x^2})dx = [\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\frac{x^{\frac{3}{2}}}{\frac{3}{2}}]_1^4$$

$$= [2x^{\frac{1}{2}}+\frac{2}{3}x^{\frac{3}{2}}]_1^4 = [2\sqrt{x}+\frac{2}{3}\sqrt{x^3}]_1^4$$

$$= (2\sqrt{4}+\frac{2}{3}\sqrt{(4)^3})-(2\sqrt{1}+\frac{2}{3}\sqrt{(1)^3}$$

$$= (4+\frac{2}{3}(8))-(2+\frac{2}{3})$$

$$= (4+\frac{16}{3})-(\frac{6+12}{3})$$

$$= (\frac{12+16}{3})-(\frac{8}{3}) = \frac{28}{3}-\frac{8}{3} = \frac{20}{3}$$

2-Evaluate the following definite integrals

1. $\int_1^4 (5x^2 - 8x + 5)dx$

2. $\int_1^9 (x^{\frac{3}{2}} + 2x + 3)dx =$

3. $\int_4^9 (\sqrt{x} + \frac{1}{3\sqrt{x}})dx =$

4. $\int_1^4 \frac{5}{x^3} dx$

5. $\int_{-1}^2 (1+3t)t^2\, dt$:

6. $\int_{-2}^1 (2t^2 - 1)^2\, dt$:

الاجوبة

1. **60**
2. **200.2**
3. **13.333**
4. **2.344**
5. **14.25**
6. **17.4**

Thanks for lessening

Any questions