2.1 Introduction

The error is the failure of instrument in exactly specifying the value of the quantity to be measured or can be defined as the departure of the measured value from the true value.

No measurement can be made with perfect accuracy, but it is important in any measurement system to qualify the maximum error in order to reducing it from the instrument output reading.

By proper analysis of the measurement data, the error can be predicted and avoided or eliminated from the instrument output reading.

2.2 Classification of Errors

Errors may come from different sources and are usually classified under three main categories:

1. Gross Errors

The gross errors are mistakes or blunders include:

- Misreading of instrument.
- Incorrect adjustment of instrument.
- Improper application of instrument.
- Computational mistakes.

2. Systematic Errors

Systematic errors in the output of any instrument are due to factors inherent in the manufacture of the instrument. This type of errors can be reduced or corrected. Systematic errors may be subdivided into:

a. Instrumental Errors

These are defects or shortcoming of instruments that may arise due to:

- Tolerance in the components of the instrument.
- Wear in instrument components over the period of time.
- Error in calibration.
- Using unsuitable or defective elements in the structure of instrument.

b. Environmental Errors

The environmental errors are introduce by physical effects (temperature, pressure, humidity, …..) that influence the instrument, the quantity to be measured and the experimentalist.

C. Observational Errors

The observational errors pertain to habits of the observer, such as:

- Imperfect techniques.
- Poor judgment.
- Peculiarities in making observation.

d. System disturbance due to measurement

In general, the process of measurement always disturbs the system being measured. The magnitude of disturbance varies from system to another and is affected by the type of the instrument used for measurement.

Measurements in electrical circuits are prone to errors induced through the loading effect on the circuit when instruments are applied to make voltage and current measurements.

To illustrate the loading effect, we shall consider the simple electric circuit shown in Figure 2.1. In this circuit, the voltage across the resistance R_2 is to be measured by a voltmeter whose internal resistance R_s.

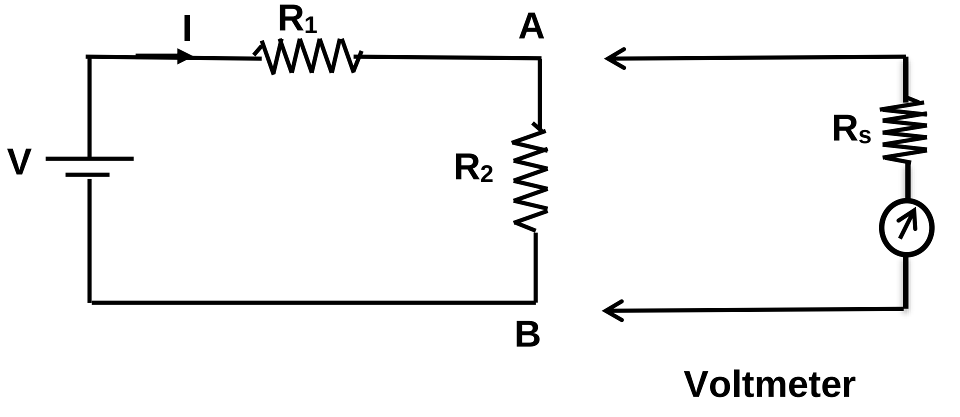

Figure 2.1 The Loading effect on circuit by voltmeter

If we denote the voltage that exist prior to measurement by E_o, i.e.

$$E_0 = I R_2$$

or

$$E_o = \frac{V}{R_1 + R_2} R_2$$

When the voltmeter is added to the circuit at measurement, the resistance between the points A and B will be R_{AB}, where

$$R_{AB} = \frac{R_2 R_s}{R_2 + R_s}$$

Therefor; the voltage E_m measured by voltmeter will be:

$$E_m = I\, R_{AB}$$

or

$$E_m = \frac{V}{R_1 + R_{AB}}\, R_{AB}$$

Therefore,

$$E_m = \frac{V}{R_1 + \frac{R_2 R_s}{R_2 + R_s}} \; \frac{R_2 R_s}{R_2 + R_s}$$

By rearranging the above equation we get:

$$E_m = \frac{V\, R_1 R_2}{R_{1\,(R_2+R_s)} + R_2 R_s}$$

Dividing E_m by E_o, we get:

$$\frac{E_m}{E_0} = \frac{R_s(\,R_1 + R_2)}{R_s(\,R_1 + R_2) + R_1 R_2}$$

The percentage error in measurement due to the loading effect of voltmeter is given by:

$$Error = \left(1 - \frac{E_m}{E_0}\right) \times 100\%$$

It is obvious that as R_s gets larger, the ratio E_m/E_o gets closer to unity and show that the error will be minimized. Therefore; in the design strategy, it should be to make R_s as large as possible in order to minimize the disturbance of the measurement system.

e. Modifying inputs in measurement systems

The variation of the environmental conditions away from the calibration conditions cause the characteristics of the measuring instruments to vary to some extent. The environmental condition variation is considered as input to the measuring system because their effect is the same as the value of the measured quantity had changed.

In general, it is very difficult to avoid the modifying input because it is impossible to control the environmental conditions surrounding the measurement system. But, the effect of the modifying input on the instrument output can be reduced with:

1. Proper analysis.
2. Careful instrument design.
3. Using some techniques such as the method of opposing inputs, high gain feedbacks, or using signal filtering, …………………, etc.

3. Random Errors

The random errors are accidental errors whose magnitude and sign fluctuate in a manner that cannot be predicted from the knowledge of the measuring system and the condition of measurement.

The random errors are also known as the residual errors. Generally, the random errors are minimized by employing the statistical analysis for a large number of measurement readings.

Examples – L2

Ex2.1 Classify the error types.

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1. Gross errors.
2. Systematic errors, include:
   a. Instrumental errors.
   b. Environmental errors.
   c. Observational errors.
   d. System disturbance due to measurement.
   e. Modifying input errors.
3. Random errors.

Ex2.2 Define the following terms:
(1)Error.
(2) Random errors.

Sol.

(1) The error is the failure of instrument in exactly specifying the value of the quantity to measured.

(2) The random errors are accidental errors whose magnitude and sign fluctuate in a manner that cannot be predicted from the knowledge of the measuring system and the condition of measurement.

Ex2.3 List the points according to which modifying input errors can be minimized.

Sol. The effect of the modifying input on the instrument output can be minimized with:

1. Proper analysis.
2. Careful instrument design.
3. Using some techniques such as the method of opposing inputs, high gain feedbacks, or using signal filtering, …………………, etc.

Ex2.4 In the circuit shown in Fig. 1, the voltage across terminals A and B is measured by a voltmeter has an internal resistance $R_S = 1000\ \Omega$.

(1) Derive an expression for the loading effect caused by the measurement process.

(2) Determine the error caused by the internal resistance of the measuring instrument if $R_1 = 100\ \Omega$, $R_2 = 200\ \Omega$ and $R_3 = 300\ \Omega$.

(3) Determine the error if the voltmeter is replaced by one whose internal resistance $R_S = 5000\ \Omega$. Compare between the two results.

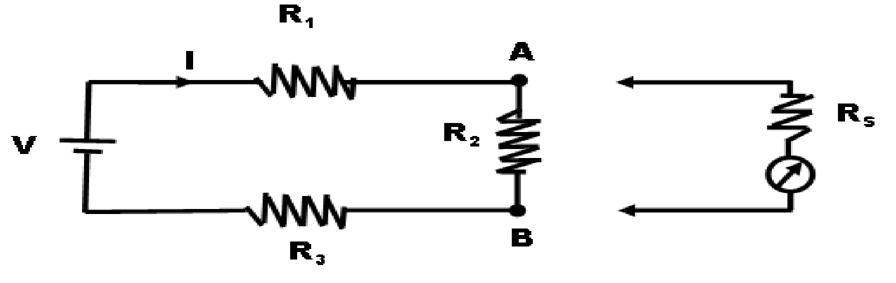

Fig.(1)

Sol. **(1)**

$$E_o = I\ R_2$$

Or,

$$E_o = \frac{V}{R_1 + R_2 + R_3}\ R_2$$

And

$$E_m = I\ R_{AB}$$

Or,

$$E_m = \frac{V}{R_1 + R_3 + R_{AB}}\ R_{AB}$$

But,

$$R_{AB} = \frac{R_2 R_S}{R_2 + R_S}$$

By substituting R_{AB} in E_m , we get:

$$E_m = \frac{V}{R_1 + R_3 + \frac{R_2 R_S}{R_2 + R_S}} \cdot \frac{R_2 R_S}{R_2 + R_S}$$

Or,

$$E_m = \frac{V R_2 R_S}{(R_1 + R_3)(R_2 + R_S) + R_2 R_S}$$

With some arrangements, we get:

$$E_m = \frac{V R_2 R_S}{R_S (R_1 + R_2 + R_3) + R_1 R_2 + R_2 R_3}$$

Now,

$$\frac{E_m}{E_o} = \frac{\frac{V R_2 R_S}{R_S (R_1 + R_2 + R_3) + R_1 R_2 + R_2 R_3}}{\frac{V R_2}{R_1 + R_2 + R_3}}$$

Or,

$$\frac{E_m}{E_o} = \frac{R_S (R_1 + R_2 + R_3)}{R_S (R_1 + R_2 + R_3) + R_1 R_2 + R_2 R_3}$$

(2)

$$Error = \left(1 - \frac{E_m}{E_o}\right) \times 100\,\%$$

Where,

$$\frac{E_m}{E_o} = \frac{1000(100 + 200 + 300)}{1000(100 + 200 + 300) + 100 \times 200 + 200 \times 300}$$

Or

$$\frac{E_m}{E_o} = \frac{600000}{600000 + 20000 + 60000} = \frac{600000}{680000} = 0.88$$

Therefore,

$$Error = (1 - 0.88) \times 100\ \%$$

Or,

$$Error = 12\ \%$$

(3)

When $R_S = 5000\ \Omega$

$$\frac{E_m}{E_o} = \frac{5000(100 + 200 + 300)}{5000(100 + 200 + 300) + 100 \times 200 + 200 \times 300}$$

Or,

$$\frac{E_m}{E_o} = \frac{3000000}{3000000 + 20000 + 60000} = \frac{3000000}{3080000} = 0.97$$

Therefore,

$$Error = (1 - 0.97) \times 100\ \%$$

Or,

$$Error = 3\ \%$$

It is obvious that as R_s gets larger, the ratio E_m/E_o gets closer to unity and show that the error will be minimized.