



Ministry of Higher Education and
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الرياضيات التكاملية

7 محاضرة

Solved Problems in Definite Integrals

حل امثلة على التكامل المحدد مع اجوبتها

اعداد

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Evaluate the following definite integrals.

1. $\int_1^5 (3x^2 + 4x + 1) dx$

$$\begin{aligned}\int_1^5 (3x^2 + 4x + 1) dx &= (x^3 + 2x^2 + x) \Big|_1^5 \\ &= [5^3 + 2(5^2) + 5] - [1^3 + 2(1^2) + 1] \\ &= 180 - 4 \\ &= 176\end{aligned}$$

2. $\int_4^9 (2x + 3\sqrt{x}) dx$

$$\begin{aligned}\int_4^9 (2x + 3\sqrt{x}) dx &= (x^2 + 2x^{3/2}) \Big|_4^9 \\ &= [9^2 + 2(9^{3/2})] - [4^2 + 2(4^{3/2})] \\ &= 135 - 32 \\ &= 103\end{aligned}$$

4. $\int_{-1}^0 x^3(1 - 2x^4)^3 dx$

$$\begin{aligned}\int_{-1}^0 x^3(1 - 2x^4)^3 dx &= -\frac{(1 - 2x^4)^4}{32} \Big|_{-1}^0 \\ &= -\left[\left(\frac{1}{32}\right) - \left(\frac{1}{32}\right)\right] \\ &= 0\end{aligned}$$

**Example**

Find $\int_{-2}^3 x^3 dx$.

Solution

$$\begin{aligned}\int_{-2}^3 x^3 dx &= \left[\frac{x^4}{4} \right]_{-2}^3 \\ &= \left(\frac{(3)^4}{4} \right) - \left(\frac{(-2)^4}{4} \right) \\ &= \frac{81}{4} - \frac{16}{4} \\ &= \frac{65}{4} \\ &= 16.25\end{aligned}$$

Example

Find $\int_0^{\pi/2} \cos x \, dx$.

Solution

$$\begin{aligned}\int_0^{\pi/2} \cos x \, dx &= [\sin x]_0^{\pi/2} \\ &= \sin \left(\frac{\pi}{2} \right) - \sin 0 \\ &= 1 - 0 \\ &= 1\end{aligned}$$



Compute each of the following definite integrals.

$$1. \int_0^6 x^2 dx$$

$$12. \int_{-1}^2 \frac{1}{x} dx$$

$$23. \int_0^2 e^{-3x} dx$$

$$2. \int_1^7 x^2 dx$$

$$13. \int_0^4 \frac{1}{2} \sqrt{x} dx$$

$$24. \int_0^{\ln 8} (e^x - e^{-x}) dx$$

$$3. \int_1^3 2x - 5 dx$$

$$14. \int_7^{12} \sqrt{x-3} dx$$

$$25. \int_0^3 2^x dx$$

$$4. \int_{-7}^7 x dx$$

$$15. \int_{-\pi}^{\pi} \sin x dx$$

$$26. 2 \int_{-1}^1 \sqrt{1-x^2} dx$$

$$5. \int_0^5 x^2 - 2x dx$$

$$16. \int_0^{\pi} \sin x dx$$

(Hint: Don't try (yet) to solve this algebraically. Instead, draw a picture!)

$$6. \int_{-2}^2 6x^3 - 4x dx$$

$$17. \int_0^{\pi/2} \cos x dx$$

$$27. a) \int_1^N \frac{1}{x^2} dx$$

$$7. \int_1^3 \frac{1}{x^2} dx$$

$$18. \int_0^{\pi/2} \cos 2x dx$$

$$b) \lim_{N \rightarrow \infty} \int_1^N \frac{1}{x^2} dx$$

$$7. \int_1^3 \frac{1}{x^2} dx$$

$$18. \int_0^{\pi/2} \cos 2x dx$$

$$b) \lim_{N \rightarrow \infty} \int_1^N \frac{1}{x^2} dx$$

$$8. \int_{-4}^{-1} \frac{1}{x^2} dx$$

$$19. \int_0^1 \frac{1}{1+x^2} dx$$

$$28. a) \int_1^N e^{-2x} dx$$

$$9. \int_{-1}^3 \frac{1}{x^2} dx$$

$$20. \int_{1/2}^1 \frac{1}{\sqrt{1-x^2}} dx$$

$$b) \lim_{N \rightarrow \infty} \int_1^N e^{-2x} dx$$

$$10. \int_1^4 \frac{1}{x} dx$$

$$21. \int_0^1 e^{2x} dx$$

$$29. a) \int_0^N \frac{1}{x^2+1} dx$$



$$11. \int_{-4}^{-1} \frac{1}{x} dx$$

$$22. \int_0^5 e^{-x} dx$$

$$b) \lim_{N \rightarrow \infty} \int_0^N \frac{1}{x^2 + 1} dx$$

Answers

$$1. 72$$

$$9. \text{undefined}$$

$$17. 1$$

$$25. \frac{7}{\ln 2}$$

$$2. 114$$

$$10. \ln 4$$

$$18. 0$$

$$26. \pi$$

$$3. -2$$

$$11. -\ln 4$$

$$19. \frac{\pi}{4}$$

$$27. a) 1 - \frac{1}{N}$$

$$4. 0$$

$$12. \text{undefined}$$

$$20. \frac{\pi}{3}$$

$$b) 1$$

$$5. \frac{50}{3}$$

$$13. \frac{8}{3}$$

$$21. \frac{1}{2}e^2 - \frac{1}{2}$$

$$28. a) \frac{1}{2e^2} - \frac{1}{2e^{2N}}$$

$$6. 0$$

$$14. \frac{38}{3}$$

$$22. 1 - e^{-5}$$

$$b) \frac{1}{2e^2}$$

$$7. \frac{2}{3}$$

$$15. 0$$

$$23. \frac{1}{3} - \frac{1}{3}e^{-6}$$

$$29. a) \arctan N$$

$$8. \frac{3}{4}$$

$$16. 2$$

$$24. \frac{49}{8}$$

$$b) \frac{\pi}{2}$$

Solved problems in indefinite Integrals



2. Find:

$$(a) \int (4x^2 - 2x) dx$$

$$(e) \int (3x^2 - 2x + 1) dx$$

$$(i) \int x^2 \sqrt{x} dx$$

$$(b) \int (2x^4 - 3x^3) dx$$

$$(f) \int (x^5 - 4x^3 + x^2 - x + 1) dx$$

$$(j) \int \left(3x^2 + \frac{2}{x^3} \right) dx$$

$$(c) \int (x^2 - x^{-2}) dx$$

$$(g) \int (x - 1)(x + 1) dx$$

$$(k) \int (x^{3/2} + x^{-3/2}) dx$$

$$(d) \int (2x + 1)^2 dx$$

$$(h) \int \frac{(x^3 - 2x^2 + x)}{x} dx$$

$$(l) \int \left(\frac{1}{x\sqrt{x}} + x^{-1.3} \right) dx$$

3. Find the following indefinite integrals.

$$(a) \int (12x^2 - 4) dx$$

$$(i) \int 2 du$$

$$(e) \int 2u^2(4u^3 + 3u^2 - 1) du$$

$$(b) \int \frac{2x^2 - 3x}{x} dx$$

$$(f) \int \left(2t + \frac{1}{t} \right) dt$$

$$(h) \int \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right) dx$$

$$(c) \int (3t^2 - 2t + 5) dt$$

$$(g) \int (x^2 - \sqrt{x})^2 dx$$

$$(k) \int \left(\frac{3x^4 + 1}{x^2} \right) dx$$

$$(d) \int \left(\frac{1}{t^2} + t \right) dt$$

$$(j) \int \frac{v^2 - 4}{v - 2} dv$$

$$(l) \int (1 - \sqrt{u})(3 + 2u) du$$

Answers:

$$2. (a) \frac{4x^3}{3} - x^2 + C$$

$$(e) x^3 - x^2 + x + C$$

$$(i) \frac{2}{7} x^{7/2} + C$$

$$(b) \frac{2x^5}{5} + \frac{3x^4}{4} + C$$

$$(f) \frac{x^6}{6} - x^4 + \frac{x^3}{3} + \frac{x^2}{2} + x + C$$

$$(j) x^3 - \frac{1}{x^2} + C$$

$$(c) \frac{x^3}{3} + x^{-1} + C$$

$$(g) \frac{x^3}{3} - x + C$$

$$(k) \frac{2}{5} x^{5/2} - 2x^{-1/2} + C$$

$$(d) \frac{4x^3}{3} + 2x^2 + x + C$$

$$(h) \frac{x^3}{3} + x^2 + x + C$$

$$(l) -\frac{2}{\sqrt{x}} - \frac{x^{-0.3}}{0.3} + C$$



3. (a) $4x(x^2 - 1) + C$	(i) $2u + C$	(e) $\frac{4u^6}{3} + \frac{6u^5}{5} - \frac{2u^5}{2} + C$
(b) $x^2 - 3x + C$	(f) $\frac{4t^3}{3} + 4t - \frac{1}{t} + C$	(h) $\frac{x^3}{3} + \frac{1}{x} + C$
(c) $t^3 - t^2 + 5t + C$	(g) $\frac{x^5}{5} - \frac{4}{7}x^{7/2} + \frac{x^2}{2} + C$	(k) $x^3 - \frac{1}{x} + C$
(d) $\frac{t^2}{2} - \frac{1}{t} + C$	(j) $\frac{v^2}{2} + 2v + C$	(l) $u^2 + 3u - 2u^{3/2} - \frac{4}{5}u^{5/2} + C$

Thanks for lessening

Any questions?