Ministry of Higher Education and Scientific Research – Iraq
AL-Mustaqbal University

Department of Electrical Engineering techniques

الرياضيات التكاملية

المحاضرة 8

Theory of matrices and determinants. Properties of matrix operations

اعداد

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Solving a 2D Determinant

For any 2d square matrix or a square matrix of order 2×2, we can use the determinant formula to calculate its determinant:

$$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Its 2D determinant can be calculated as:

$$|C| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$|C| = (a \times d) - (b \times c)$

For example: $C = \begin{bmatrix} 8 & 6 \\ 3 & 4 \end{bmatrix}$

$$|C| = \begin{vmatrix} 8 & 6 \\ 3 & 4 \end{vmatrix}$$

$|C| = (8 \times 4) - (6 \times 3) = 32 - 18 = 14$

Solving A 3D Determinants

For any 3d square matrix or a square matrix of order 3×3, this is the procedure to calculate its determinant.

$$C = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

Its determinant can be calculated as:

$$C = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

Its determinant can be calculated as:

$$|C| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

number and by its sign.

- $|C| = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$
- Finally sum them up.

$$|C| = a_1 \cdot \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \cdot \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \cdot \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$|C| = a_1 (b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1 (a_2b_3 - a_3b_2)$$

Example 2: Find the determinant of the matrix A where

$$A = \begin{bmatrix} 1 & 3 & 2 \\ -3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix}.$$

Solution:

$$|C| = 1 \cdot \begin{vmatrix} -1 & -3 \\ 3 & 1 \end{vmatrix} - 3 \cdot \begin{vmatrix} -3 & -3 \\ 2 & 1 \end{vmatrix} + 2 \cdot \begin{vmatrix} -3 & -1 \\ 2 & 3 \end{vmatrix}$$

Using the determinants rule,

=|C| = 1. (-1 -(-9) - 3. (-3 -(-6) + 2.(-9 -(-2))

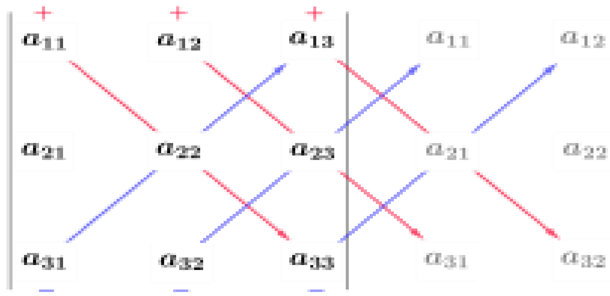
= 1. (-1 +9) - 3. (-3 +6) + 2 .(-9 +2)

= 8 - 9 -14

=|C| = -15

Answer: The determinant of the given matrix is -15.

من الممكن حل المحدد الثلاثي بالطريقة التالية



خواص المحددات

Addition of 2 x 2 Matrices

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

Subtraction of 2 x 2 Matrices

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$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \times \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1a_2 + b_1c_2 & a_1b_2 + b_1d_2 \\ c_1a_2 + d_1c_2 & c_1b_2 + d_1d_2 \end{bmatrix}$$