Ministry of Higher Education and Scientific Research – Iraq
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Department of Electrical Engineering techniques

الرياضيات التكاملية

المحاضرة 9-10

Matrix

Determinant of 2×2 Matrix

Example 1: Find the determinant of the matrix below.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = (1)(4) - (2)(3)$$

$$= 4 - 6$$

$$= -2 \checkmark$$

Example 2: Calculate the determinant of the matrix below.

$$B = \begin{bmatrix} -5 & -4 \\ -2 & -3 \end{bmatrix}$$

$$\det \begin{bmatrix} -5 & -4 \\ -2 & -3 \end{bmatrix} = (-5)(-3) - (-4)(-2)$$

$$= 15 - 8$$

$$= 7 \checkmark$$

Example 3,4,5: Find the determinant of the matrix below.

$$|A| = \begin{vmatrix} 4 & 3 \\ 5 & -1 \end{vmatrix} = (4 \times -1) \quad - \quad (3 \times 5) = -4 - 15 = -19$$

Example

Suppose $B = \begin{pmatrix} 6 & 2 \\ 3 & 5 \end{pmatrix}$.

$$|B| = \begin{vmatrix} 6 & 2 \\ 3 & 5 \end{vmatrix} = (6 \times 5) \quad - \quad (2 \times 3) = 30 - 6 = 24$$

Example

Suppose $D = \begin{pmatrix} 6 & 4 \\ 3 & 2 \end{pmatrix}$.

$$|D| = \begin{vmatrix} 6 & 4 \\ 3 & 2 \end{vmatrix} = (6 \times 2) \quad - \quad (4 \times 3) = 12 - 12 = 0$$

Ex:

The determinant of a 2×2 matrix is -8. Find the value of K.

$$\det \begin{bmatrix} k & 3 \\ -9 & 7 \end{bmatrix} = -8$$

$$\det \begin{bmatrix} k & 3 \\ -9 & 7 \end{bmatrix} = -8$$

$$(7)(k) - (3)(-9) = -8$$

$$7k - (-27) = -8$$

$$7k + 27 = -8$$

$$7k = -35$$

$$k = -5$$

Example 1: Solve for the determinant of the 3×3 matrix below.

$$\det \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix} = 2 \cdot \det \begin{bmatrix} 0 & -1 \\ 4 & 5 \end{bmatrix} - (-3) \cdot \det \begin{bmatrix} 2 & -1 \\ 1 & 5 \end{bmatrix} + 1 \cdot \det \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= 2\left[0 - (-4)\right] + 3\left[10 - (-1)\right] + 1\left[8 - 0\right]$$

$$= 2(0+4) + 3(10+1) + 1(8)$$

$$= 2(4) + 3(11) + 8$$

$$= 8 + 33 + 8$$

$$= 49 \checkmark$$

Example 2: Solve for the determinant of the 3×3 matrix below.

$$\det\begin{bmatrix} 1 & 3 & 2 \\ -3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix} = 1 \cdot \det\begin{bmatrix} -1 & -3 \\ 3 & 1 \end{bmatrix} - (3) \cdot \det\begin{bmatrix} -3 & -3 \\ 2 & 1 \end{bmatrix}$$

$$= 1\left[-1-(-9)\right] - 3\left[-3-(-6)\right] + 2\left[-9-(\right.$$
$$= 1(-1+9) - 3(-3+6) + 2(-9+2)$$
$$= 1(8) - 3(3) + 2(-7)$$
$$= 8 - 9 - 14$$
$$= -15 \checkmark$$

Example 3: Solve for the determinant of the 3×3 matrix below.

$$\det\begin{bmatrix} -5 & 0 & -1 \\ 1 & 2 & -1 \\ -3 & 4 & 1 \end{bmatrix} = -5 \cdot \det\begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix} - (0) \cdot \det\begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix} + (-1) \cdot \det\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$$

$$= -5\left[2-(-4)\right] - 0\left[1-(3)\right] - 1\left[4-(-6)\right]$$
$$= -5(2+4) - 0 - 1(4+6)$$
$$= -5(6) - 1(10)$$
$$= -30 - 10$$
$$= -40 \checkmark$$

Example 4: Solve for the determinant of the 3×3 matrix below.

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix} = \begin{bmatrix} \boxed{1} & -2 & 3 \\ 2 & \boxed{0} & \boxed{3} \\ 1 & \boxed{5} & \boxed{4} \end{bmatrix} - \begin{bmatrix} 1 & \boxed{-2} & 3 \\ \boxed{2} & 0 & \boxed{3} \\ \boxed{1} & 5 & \boxed{4} \end{bmatrix} -$$

$$= \boxed{1} \times \begin{vmatrix} 0 & 3 \\ 5 & 4 \end{vmatrix} - \boxed{-2} \times \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + \boxed{3}$$

$$= 1 \times (0-15) + 2 \times (8-3) + 3 \times ($$

$$= 1(-15) + 2(5) + 3(10)$$

$$= -15 + 10 + 30$$

$$= 25$$

Example 5: Calculate the determinant of the three-by-three matrix below.

$$\begin{bmatrix} -5 & -5 & -5 \\ 3 & -1 & -2 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{-5} & -5 & -5 \\ 3 & \boxed{-1} & \boxed{-2} \\ 4 & \boxed{2} & \boxed{1} \end{bmatrix} - \begin{bmatrix} -5 & \boxed{-5} & -5 \\ \boxed{3} & -1 & \boxed{-2} \\ \boxed{4} & 2 & \boxed{1} \end{bmatrix} + \begin{bmatrix} -5 & -5 & \boxed{-5} \\ \boxed{3} & \boxed{-1} & -2 \\ \boxed{4} & \boxed{2} & 1 \end{bmatrix}$$

$$= \boxed{-5} \times \begin{vmatrix} -1 & -2 \\ 2 & 1 \end{vmatrix} - \boxed{-5} \times \begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix} + \boxed{-5} \times \begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix}$$

$$= -5(3) + 5(11) - 5(10)$$

$$= -10$$