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الرياضيات التكاملية

Completing The Square Formula

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Completing The Square Formula

$x^2 + bx + c = (x + b/2)^2 + (c – b^2/4)$

This method is known as completing the square method. We have achieved it geometrically.

We know that,

$x^2 + bx + c = 0$

This can be written as:

$(x + b/2)^2 + (c – b^2/4) = 0$

$\Rightarrow (x + b/2)^2 = -(c – b^2/4)$

This formula can be used to solve the quadratic equations **by completing the square** technique.

Completing the Square Examples

Example 1: Find the roots of the quadratic equation $x^2 + 4x – 5 = 0$ by the method of completing the square.

Solution:

Given quadratic equation is:

$x^2 + 4x – 5 = 0$

Comparing the equation with the standard form,

$b = 4, c = -5$

$(x + b/2)^2 = -(c – b^2/4)$

So,

$[x + (4/2)]^2 = -[-5 – (4^2/4)]$

$(x + 2)^2 = 5 + 4$

$\Rightarrow (x + 2)^2 = 9$

$\Rightarrow (x + 2) = \pm\sqrt{9}$

$\Rightarrow (x + 2) = \pm 3$

$\Rightarrow x + 2 = 3, x + 2 = -3$

$\Rightarrow x = 1 , -5$

Therefore, the roots of the given equation are 1 and -5.

Example 2: Find the roots of the quadratic equation $3x^2 – 5x + 2 = 0$ by completing the square.

Solution:

Given quadratic equation is:

$3x^2 – 5x + 2 = 0$

The given equation is not in the form to which we apply the method of completing squares, i.e. the coefficient of x^2 is not 1. To make it 1, we need to divide the whole equation with 3.

$x^2 – 5/3\ x + 2/3 = 0$

Comparing with the standard form,

b = -5/3; c = ⅔

$c – b^2/4 = ⅔ – [(-5/3)^2/4] = (2/3) – (25/36) = (24 – 25)/36 = -1/36$

Substituting these values in the equation $(x + b/2)^2 = -(c – b^2/4)$ we get,

$\Rightarrow (x – 5/6)^2 = 1/36$

$\Rightarrow (x – 5/6) = \pm \sqrt{(1/36)}$

⇒ x – 5/6 = ±1/6

⇒ x = 1, -2/3

Therefore, the roots of the given equation are 1 and -⅔.

Example 3: Find the roots of the quadratic equation by completing the square

$$x^2 + 6x = 40$$

Solution:

$$(x + 3)^2 = 49$$

X=-3+7=4

X=-3-7=-10

Example 4:

$$x^2 + 10x + 4 = 15$$

Solution:

$$x = 1, \qquad x = -11$$

Check:

$$x^2 + 10x + 4 = 15$$
$$(1)^2 + 10(1) + 4 \stackrel{?}{=} 15$$
$$1 + 10 + 4 \stackrel{?}{=} 15$$
$$15 = 15 \checkmark$$

$$x^2 + 10x + 4 = 15$$
$$(-11)^2 + 10(-11) + 4 \stackrel{?}{=} 15$$
$$121 + 110 + 4 \stackrel{?}{=} 15$$
$$15 = 15 \checkmark$$

Example 5:

$$x^2 + 2x - 15 = 9$$

Solution:

$$x = 4, \qquad x = -6$$

Example 6:

$$3x^2 - 12x - 15 = 0$$

Solution:

$$x = 5, \qquad x = -1$$

Check:

$$\begin{aligned} x &= 5 \\ 3x^2 - 12x - 15 &= 0 \\ 3(5)^2 - 12(5) - 15 &\stackrel{?}{=} 0 \\ 75 - 60 - 15 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$$\begin{aligned} x &= -1 \\ 3x^2 - 12x - 15 &= 0 \\ 3(-1)^2 - 12(-1) - 15 &\stackrel{?}{=} 0 \\ 3 + 12 - 15 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

Example 7:

$$2x^2 - 3x = 20$$

Solution:

$$x = 4, \qquad x = -\frac{5}{2}$$