



Laplace Transforms In The MATLAB

1. Basic Laplace Transforms

Here are the Laplace transforms of some fundamental functions:

- **Constant Function:**

$$\mathcal{L}\{1\} = \frac{1}{s}$$

- **Power Function t^n** (where n is a non-negative integer):

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

- **Exponential Function e^{at} :**

$$\mathcal{L}\{e^{at}\} = \frac{1}{s - a}$$

(where a is a constant).

- **Sine Function $\sin(at)$:**

$$\mathcal{L}\{\sin(at)\} = \frac{a}{s^2 + a^2}$$

- **Cosine Function $\cos(at)$:**

$$\mathcal{L}\{\cos(at)\} = \frac{s}{s^2 + a^2}$$



2. Properties of the Laplace Transform

Linearity (Superposition Property):

The Laplace transform is linear, which means

$$\mathcal{L}\{c_1 f(t) + c_2 g(t)\} = c_1 \mathcal{L}\{f(t)\} + c_2 \mathcal{L}\{g(t)\}$$

where c_1 and c_2 are constants.

Time-Shift Property:

If $f(t)$ has a Laplace transform $F(s)$, then:

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$$

where $u(t-a)$ is the Heaviside step function.

Differentiation in Time Domain:

If $f(t)$ has a Laplace transform $F(s)$, then

Frequency-Shift Property:

If $f(t)$ has a Laplace transform $F(s)$, then:

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

Scaling Property:

If $f(t)$ has a Laplace transform $F(s)$, then:

$$\mathcal{L}\{f(at)\} = \frac{1}{|a|}F\left(\frac{s}{a}\right)$$

where a is a constant.



$$\mathcal{L} \left\{ \frac{d^n f(t)}{dt^n} \right\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

where $f^{(n)}(0)$ denotes the n -th derivative of $f(t)$ evaluated at $t = 0$.

Integration in Time Domain:

If $f(t)$ has a Laplace transform $F(s)$, then:

$$\mathcal{L} \left\{ \int_0^t f(\tau) d\tau \right\} = \frac{1}{s} F(s)$$

Evaluating Laplace Transform by Using Matlab

1. Evaluate Laplace transform of the function $f(t) = 3t + 2e^{3t}$

Step-1

First declare t as a variable:

```
>> syms t
```

Step-2

Define the function f

Practice Problems:

- | | |
|-----------------------------|--------------------------------|
| (a) $5 - 3t$ | (b) $7t^3 - 2 \sin 3t$ |
| (c) $3 - 2t + 4 \cos 2t$ | (d) $\cosh 3t$ |
| (e) $\sinh 2t$ | (f) $5e^{-2t} + 3 - 2 \cos 2t$ |
| (g) $4t e^{-2t}$ | (h) $2e^{-3t} \sin 2t$ |
| (i) $t^2 e^{-4t}$ | (j) $6t^3 - 3t^2 + 4t - 2$ |
| (k) $2 \cos 3t + 5 \sin 3t$ | (l) $t \cos 2t$ |



```
>> syms t
>> f=3*t+2*exp(3*t)

f =

3*t + 2*exp(3*t)

>> laplace(f)

ans =

2/(s - 3) + 3/s^2

>> syms t
>> f=5-3*t

f =

5 - 3*t

>> laplace(f)

ans =

(3*((5*s)/3 - 1))/s^2
```