Introduction

A resonant circuit consists of R, L, and C elements and whose frequency response characteristic changes with changes in frequency. In this tutorial we will look at the frequency response of a series resonance circuit and see how to calculate its resonant and cut-off frequencies.

Thus far we have analyzed the behavior of a series RLC circuit whose source voltage is a fixed frequency steady state sinusoidal supply. We have also seen in our tutorial about series RLC circuits that two or more sinusoidal signals can be combined using phasors providing that they have the same frequency supply.

But what would happen to the characteristics of the circuit if a supply voltage of fixed amplitude but of different frequencies was applied to the circuit. Also what would the circuits “frequency response” behaviour be upon the two reactive components due to this varying frequency.

In a series RLC circuit there becomes a frequency point were the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor. In other words, XL = XC. The point at which this occurs is called the Resonant Frequency point, (fr) of the circuit, and as we are analysing a series RLC circuit this resonance frequency produces a Series Resonance.

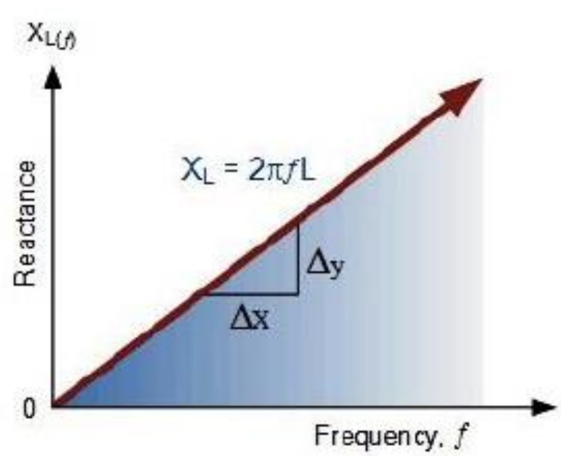
This Phase Difference, Φ depends upon the reactive value of the components being used and hopefully by now we know that reactance, (X) is zero if the circuit element is resistive, positive if the circuit element is inductive and negative if it is capacitive thus giving their resulting impedances as:

Series Resonance circuits are one of the most important circuits used electrical and electronic circuits. They can be found in various forms such as in AC mains filters, noise filters and also in radio and television tuning circuits producing a very selective tuning circuit for the receiving of the different frequency channels. Consider the simple series RLC circuit below.

Inductive Reactance against Frequency The graph of inductive reactance against frequency is a straight line linear curve. The inductive reactance value of an inductor increases linearly as the frequency across it increases. Therefore, inductive reactance is positive and is directly proportional to frequency ($X_L \propto f$)

The same is also true for the capacitive reactance formula above but in reverse. If either the Frequency or the Capacitance is increased the overall capacitive reactance would decrease. As the frequency approaches infinity the capacitors reactance would reduce to practically zero causing the circuit element to act like a perfect conductor of 0Ω.

But as the frequency approaches zero or DC level, the capacitors reactance would rapidly increase up to infinity causing it to act like a very large resistance, becoming more like an open circuit condition. This means then that capacitive reactance is “Inversely proportional” to frequency for any given value of capacitance and this shown below:

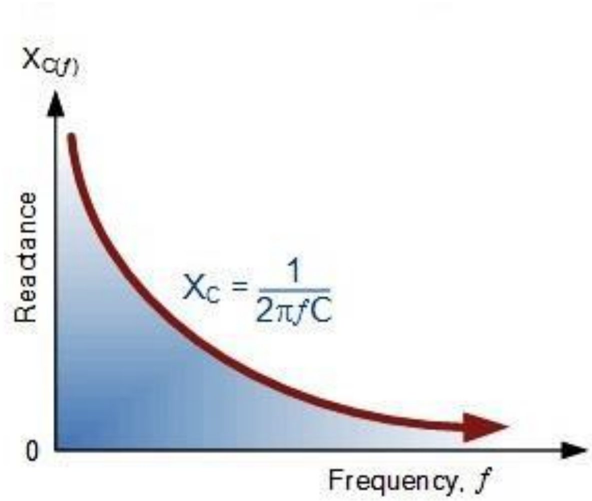


Capacitive Reactance against Frequency

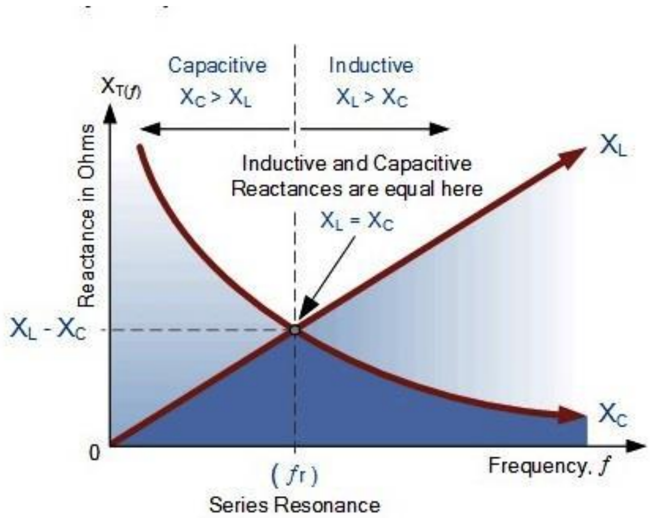
The graph of capacitive reactance against frequency is a hyperbolic curve. The Reactance value of a capacitor has a very high value at low frequencies but quickly decreases as the frequency across it increases. Therefore, capacitive reactance is negative and is inversely proportional to frequency ($X_C \propto f^{-1}$)

We can see that the values of these resistances depends upon the frequency of the supply. At a higher frequency XL is high and at a low frequency XC is high.

Then there must be a frequency point were the value of XL is the same as the value of XC and there is.



Series Resonance Frequency



where: fr is in Hertz, L is in Henries and C is in Farads.

Electrical resonance occurs in an AC circuit when the effects of the two reactances, which are opposite and equal, cancel each other out as XL = XC. The point on the above graph at which this happens is were the two reactance curves cross each other.

In a series resonant circuit, the resonant frequency, fr point can be calculated as follows.

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

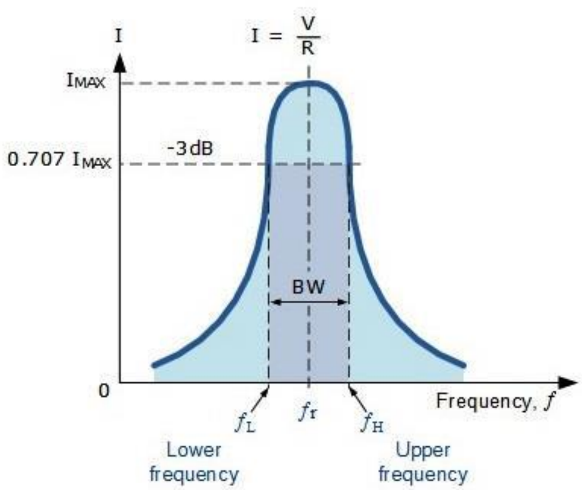
$$\therefore \ f_r = \frac{1}{2\pi\sqrt{LC}}\,(\text{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}\,(\text{rads})$$

We can see then that at resonance, mathematically the two reactances cancel each other out as XL – XC = 0. This makes the series LC combination act as a short circuit with the only opposition to current flow in a series resonance circuit being the resistance, R.

In complex form, the resonant frequency is the frequency at which the total impedance of a series RLC circuit becomes purely "real", that is no imaginary impedance's exist. This is because at resonance they are cancelled out. So the total impedance of the series circuit becomes just the value of the resistance and therefore: Z = R.

Then at resonance the impedance of the series circuit is at its minimum value and equal only to the resistance, R of the circuit. The circuit impedance at resonance is called the "dynamic impedance" of the circuit and depending upon the frequency, XC (typically at high frequencies) or XL (typically at low frequencies) will dominate either side of resonance as shown below.

Bandwidth of a Series Resonance Circuit



The frequency response of the circuits current magnitude above, relates to the "sharpness" of the resonance in a series resonance circuit. The sharpness of the peak is measured quantitatively and is called the Quality factor, Q of the circuit.

The quality factor relates the maximum or peak energy stored in the circuit (the reactance) to the energy dissipated (the resistance) during each cycle of oscillation meaning that it is a ratio of resonant frequency to bandwidth and the higher the circuit Q, the smaller the bandwidth, Q = fr /BW.

As the bandwidth is taken between the two -3dB points, the selectivity of the circuit is a measure of its ability to reject any frequencies either side of these points. A more selective circuit will have a narrower bandwidth whereas a less selective circuit will have a wider bandwidth.

The selectivity of a series resonance circuit can be controlled by adjusting the value of the resistance only, keeping all the other components the same, since Q = (XL or XC)/R.

1). Resonant Frequency, (f_r)

$$X_L = X_C \quad \Rightarrow \quad \omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r^2 = \frac{1}{LC} \quad \therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

2). Current, (I)

$$\text{at } \omega_r \quad Z_T = \text{min}, \quad I_S = \text{max}$$

$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{V_{max}}{\sqrt{R^2 + \left(\omega_r L - \frac{1}{\omega_r C}\right)^2}}$$

3). Lower cut-off frequency, (f_L)

$$\text{At half power, } \frac{P_m}{2},\ I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$Z = \sqrt{2}R,\ X = -R\ (\text{capacitive})$$

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

4). Upper cut-off frequency, (f_H)

$$\text{At half power, } \frac{P_m}{2},\ I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$Z = \sqrt{2}R,\ X = +R\ (\text{inductive})$$

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

5). Bandwidth, (BW)

$$BW = \frac{f_r}{Q},\quad f_H - f_L,\quad \frac{R}{L}\ (\text{rads})\quad \text{or}\quad \frac{R}{2\pi L}\ (\text{Hz})$$

6). Quality Factor, (Q)

$$Q = \frac{\omega_r L}{R} = \frac{X_L}{R} = \frac{1}{\omega_r CR} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Series Resonance Example

A series resonance network consisting of a resistor of 30Ω, a capacitor of 2uF and an inductor of 20mH is connected across a sinusoidal supply voltage which has a constant output of 9 volts at all frequencies.

Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Also sketch the corresponding current waveform for all frequencies.