جامعة المستقبل
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Lecture: (4)

ENTROPY

Subject: Coding Techniques
First Stage
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Entropy

Self-information is defined in terms of the individual messages or symbols a source may produce. A communication system is not designed around a particular message but rather all possible messages.

In practice, a source may not produce symbols with equal probabilities. Then each symbol will carry different amount of information.
Need to know on average how much information is transmitted per symbol in a M - symbols message ? Or, the average information content of symbols in a long message.

Consider a source emits one of M possible symbols in a statistically independent manner with probabilities , respectively. (i.e. occurrence of a symbol at any given time does not influence the symbol emitted any other time).

Entropy of a discrete source is average amount of information in its messages (i.e bit per symbol):

$$H(X) = \overline{I(X)} = -\sum_{x\in X} p(x)\log_2 p(x) = \sum_{x\in X} p(x)\log_2 \frac{1}{p(x)}.$$

The entropy is simply a weighted average of the information of each message, and therefore the average number of bits of information in the set of messages. Larger entropies represent more information.

Entropy effectively provides the average content and this may be used to estimate the bandwidth of a channel required to transmit a particular code, or the size of memory to store a certain number of symbols within a code.

Entropy properties:

1. Nonnegativity:

$$H(X) \geq 0,$$

with zero value only for the deterministic (noninformative) source.

2. Entropy is limited from above as

$$H(X) \leq \log_2 M$$

where equality takes place for only source of M equiprobable messages pi=1/M.

3. Additivity for independent sources:

$$H(XY) = H(X) + H(Y).$$

Entropy

Entropy	Number of possible messages	Amount of Uncertainty	Amount of Information in message
Low Entropy	One message out of 10	Small uncertainty	Small amount of information
High Entropy	One message out of 1,000,000	Large uncertainty	Large amount of information

Here are some examples of entropies for different probability distributions over five messages.

$$p(S) = \{0.25, 0.25, 0.25, 0.125, 0.125\}$$
$$H = 3 \times 0.25 \times \log_2 4 + 2 \times 0.125 \times \log_2 8$$
$$= 1.5 + 0.75$$
$$= 2.25$$

$$p(s) = \{0.75, 0.625, 0.625, 0.625, 0.625\}$$
$$H = 0.75 \times \log_2 \frac{4}{3} + 4 \times 0.625 \times \log_2 16$$
$$= 0.3 + 1$$
$$= 1.3$$

Q/ Assuming the symbols are independent:

a a a b b b b c c c c d d

P(a) = 3/13

P(b) = 4/13

P(c) = 4/13

P(d) = 2/13

Simple example:

Let's consider a source S, of uniform law, that sends messages from the 26-character French (a,b,c, ..., z). To this alphabet we add the "space" character as a word separator.

The alphabet is made up of 27 characters: $H(S) = -\sum_{i=1}^{27} \frac{1}{27} \log_2 \frac{1}{27} = \log_2(27) = 4.75$ bits of information per character. Actually, the entropy is close to 4 bits of information per character on a very large amount of French text.

Example : A weather information source transmits visibility information with the probabilities given in the below Table. Evaluate the entropy of the source code.

Visibility	Probability
Very poor	1/4
Poor	1/8
Moderate	1/8
Good	1/2

Solution:

Now, with n = 4:

P(x1) = 1/4

P(x2) = 1/8

P(x3) = 1/8

P(x4) = ½ the entropy can be found:

$$H(X) = \sum_{i=1}^{i=n} P(x_i)\log_2(1/P(x_i))$$

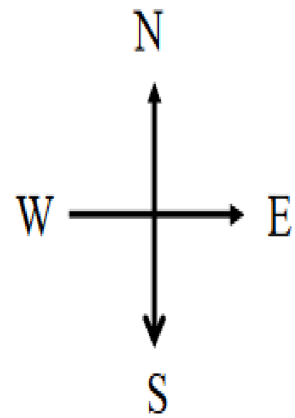
$$= P(x_1)\log_2(1/P(x_1)) + P(x_2)\log_2(1/P(x_2)) + P(x_3)\log_2(1/P(x_3)) + P(x_4)\log_2(1/P(x_4))$$

$$= (1/4) \times 2 + (1/8) \times 3 + (1/8) \times 3 + (1/2) \times 1$$

$$= 1.75 \text{ bits/symbol}$$

Robot Example:

- 4-way random walk



$$prob(x=S)=\frac{1}{2},\ prob(x=N)=\frac{1}{4}$$

$$prob(x=E)=prob(x=W)=\frac{1}{8}$$

$$H(X)=-(\frac{1}{2}\log_2\frac{1}{2}+\frac{1}{4}\log_2\frac{1}{4}+\frac{1}{8}\log_2\frac{1}{8}+\frac{1}{8}\log_2\frac{1}{8})=1.75bps$$