



جامعة المستقبل  
AL MUSTAQBAL UNIVERSITY

**كلية العلوم**  
**قسم الادلة الجنائية**

**Lecture (1)**

**Basic concept of Complex Analysis**

**المادة : Complex Analysis**

**المرحلة : الثانية**

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## **Content**

- The general Aim
- The Behavioral objectives
- Complex analysis importance
- What is a Complex Number?
- The Algebra of Complex Numbers

**Addition of Complex number**

**Subtraction of Complex number**

**Multiplication of Complex number**

**Division of Complex number**

- Complex Conjugate
- Graphical Representation of Complex Number
- The Polar Representation





## **The General Aim**

The general aim of studying complex analysis is to explore the properties of functions of a complex variable and apply their powerful theoretical and practical tools to solve problems in mathematics, physics, and engineering.

## **The Behavioral objectives**

By the end of the lecture, the student will be able to:

- ✓ Define a complex number and state its general form.
- ✓ Explain the meaning of the real part and imaginary part of a complex number.
- ✓ Perform addition and subtraction of complex numbers correctly.
- ✓ Determine the conjugate of a given complex number.
- ✓ Divide two complex numbers using the conjugate method.
- ✓ Distinguish between the different operations on complex numbers and analyze their results.





**Al-Mustaqbal University**  
**College of Science**  
**Forensic Evidence Department**  
**Second Stage**



تخصيص

Complex Analysis  
الأدلة الجنائية (المرحلة الثانية)

إعلان للمف

رمز الفصل الدراسي  
vsp2rq1b



**vsp2rq1b**

<https://classroom.google.com/c/ODE1MDM4MzAxMzU0?cjc=vsp2rq1b>



## I. Complex analysis importance

Complex analysis has not only transformed the world of mathematics, but surprisingly, we find its **application** in many areas of **physics** and **engineering**.

For example, we can use complex numbers to describe the behavior of the electromagnetic field.

In atomic systems, which are described by quantum mechanics, complex numbers and complex functions play a central role.

## II. What is a Complex Number?

It's a solution of the equation:

$$x^2 + 1 = 0$$

$$x = \pm\sqrt{-1}$$

So we see that  $i^2 = -1$ , and we can solve equations like  $x^2 + 1 = 0$ .

## III. The Algebra of Complex Numbers

More general complex numbers can be written down. In fact, using real numbers **X** and **Y** we can form a complex number:

$$Z = x + iy$$

We call **X** the real part of the complex number **Z** and refer to **Y** as the imaginary part of **Z**.



## A. Addition of Complex number

If you have two complex numbers:

$$Z_1 = x + iy \quad \text{and} \quad Z_2 = x + iy$$

where **x** is the real part, and **y** is the imaginary part.

then their sum is:

$$Z_1 + Z_2 = (x_1 + x_2) + (y_1 + y_2)i$$

### Example:

If  $Z_1 = 3 + 2i$  ,  $Z_2 = 5 + 7i$  Find the addition.

Sol//

$$Z_1 + Z_2 = (3 + 5) + (2 + 7)i$$

$$Z_1 + Z_2 = 8 + 9i$$

### Activity//

Find the Sum of:  $Z_1 = 4(2 + 3i)$ ,  $Z_2 = 5(6 + 3i)$



## B. Subtraction of Complex number

If you have two complex numbers:

$$Z_1 = x_1 + iy_1 \quad \text{and} \quad Z_2 = x_2 + iy_2$$

where **x** is the real part, and **y** is the imaginary part.

then their sub is:

$$Z_1 - Z_2 = (x_1 + x_2) + (y_1 + y_2)i$$

### Example:

If  $Z_1 = 3 - 2i$  ,  $Z_2 = 5 - 7i$  Find the Sub.

Sol//

$$Z_1 - Z_2 = (3 - 5) + (2 - 7)i$$

$$Z_1 - Z_2 = 8 - 9i$$

## C. Multiplication of Complex number

If you have two complex numbers:

$$Z_1 = x_1 + iy_1 \quad \text{and} \quad Z_2 = x_2 + iy_2$$

where **x** is the real part, and **y** is the imaginary part.

then their multiplication is:

$$Z_1 * Z_2 = (x_1 + iy_1) + (x_2 + iy_2)$$

Expanding using distributive law:

$$= x_1x_2 + x_1y_2i + x_2y_1i - y_1y_2$$

$$= (x_1x_2 - y_1y_2) + (x_1y_2 + x_2y_1)i$$



### Example:

If  $Z_1 = 2 + 3i$  ,  $Z_2 = 4 + 5i$  Find the Multiplication.

Sol//

$$\begin{aligned} Z_1 * Z_2 &= (2 + 3i) + (4 + 5)i \\ &= (2 * 4 - 3 * 5) + (2 * 5 + 3 * 4)i \\ &= (8 - 15) + (10 + 12)i \\ &= -7 + 22i \end{aligned}$$

### D. Division of Complex number

If you have two complex numbers:

$$Z_1 = a + ib \quad \text{and} \quad Z_2 = m + in$$

where **x** is the real part, and **y** is the imaginary part.

then their division is:

$$\begin{aligned} \frac{c}{k} &= \frac{a + ib}{m + in} \\ &= \frac{a + ib}{m + in} \frac{(m - in)}{(m - in)} \\ &= \frac{am + ibm - ian + bn}{m^2 + n^2} \\ &= \frac{am + bn}{m^2 + n^2} + i \frac{bm - an}{m^2 + n^2} \end{aligned}$$





### Example:

If  $Z_1 = 1 + 3i$  ,  $Z_2 = 2 - 6i$  Find the Division.

Sol//

$$\frac{1 + 3i}{2 - 6i} = \frac{(1 + 3i)(2 + 6i)}{(2 - 6i)(2 + 6i)} = \frac{2 + 6i + 6i + 18i^2}{2^2 + 6^2} = \frac{-16 + 12i}{40} = \frac{-2}{5} + \frac{3}{10}i.$$

### IV. Complex Conjugate

For a complex number:

$$Z = x + iy$$

where  $x$  is the real part and  $y$  is the imaginary part, its **complex conjugate** is:

$$\bar{Z} = x - yi$$

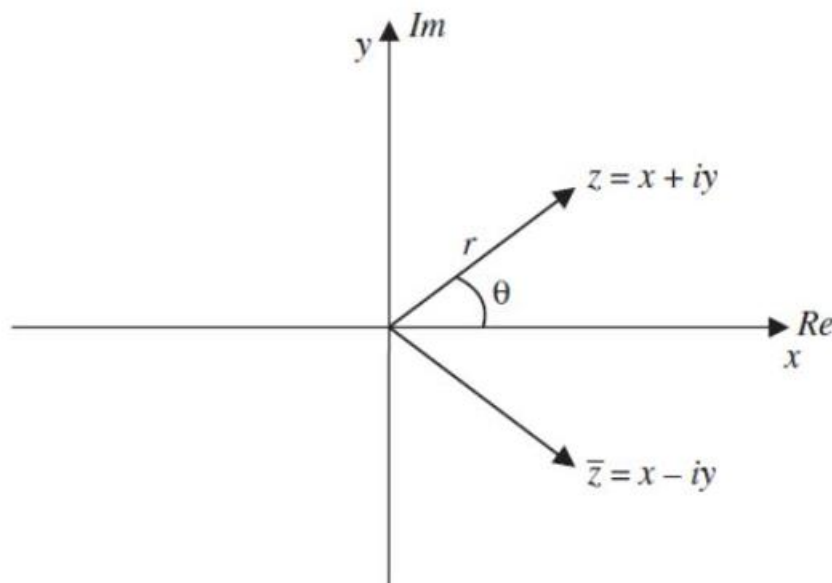
The conjugate keeps the real part the same but changes the sign of the imaginary part.

Note that:

$$\bar{\bar{Z}} = Z = \overline{x - iy} = x + iy$$



## V. Graphical Representation of Complex Number



The complex plane, showing  $z = x + iy$  and its complex conjugate as vectors.

## VI. The Polar Representation

Let  $z = x + iy$  is the Cartesian representation of a complex number.

To write down the polar representation, we begin with the definition of the polar coordinates  $(r, \theta)$ :

$$x = r \cos \theta \quad , \quad y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2}$$

We can write  $Z$  as:

$$\begin{aligned} Z &= x + iy = r \cos \theta + r \sin \theta \\ &= r(\cos \theta + i \sin \theta) \end{aligned}$$



Very important complex transformation:

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}, \quad \sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

The modulus of a complex variable  $z$  is given by:

$$|z|^2 = x^2 + y^2 \quad \Rightarrow \quad |z| = \sqrt{x^2 + y^2}$$

Note that  $r > 0$  and that we have

$$\tan = \frac{y}{x}$$

as a means to convert between polar and Cartesian representations.

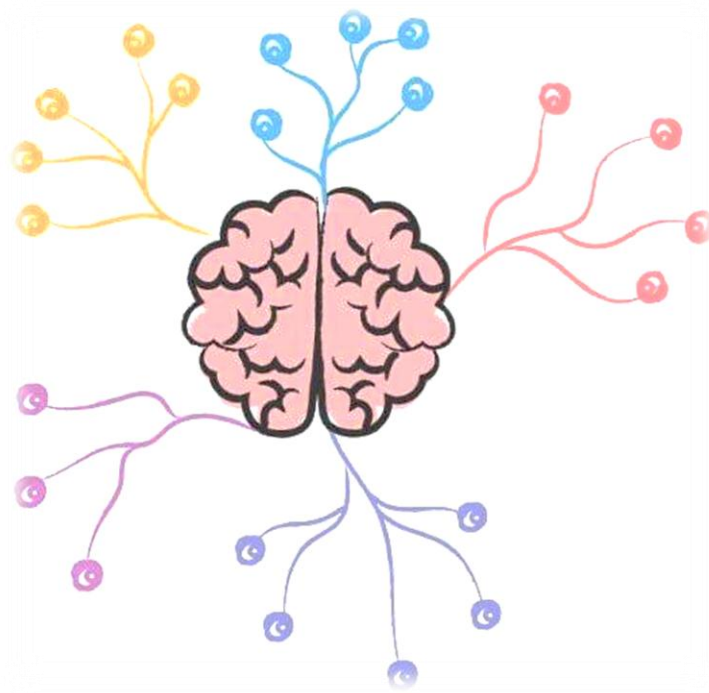
The value of  $\theta$  for a given complex number is called the **argument of  $z$**  or **arg  $z$** .



## **TASK:**

In **Group**, Calculate in the form  $x + iy$ , the following complex numbers:

1.  $(1 + 3i) + (2 - 6i)$
2.  $(1 + 3i) - (2 - 6i)$
3.  $(1 + 3i)(2 - 6i)$

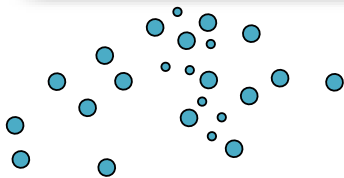


**Note: The Answer must be sent to the Google Classroom**





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التغذية الراجعة