4.Linear second order homogenous equation with constant coefficient:-

$$a_n(x)\frac{d^n y}{dx^n}+a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}}+\ldots+a_1(x)\frac{dy}{dx}+a_0(x)y=F(x)$$

homogenous of f(x)=0∴

For n=2 (second order first degree)

$$a_o\frac{d^2y}{dx^2}+a1\frac{dy}{dx}+a2y=0(general\ form)$$

$$\bar{\bar{y}}+\bar{y}+y=0$$

$$(D^2+D+2)Y=0$$

$$(r^2+r+2)=0$$

Have there are three cases

1.real roots

$$r_1=r_2=r$$

$$y=(c_1e^{rx}+c2\ xe^{rx})(general\ solution)$$

2.if $r_1\neq r_2$

$$y=(c_1e^{r1x}+c2\ e^{r2x})(general\ solution)$$

2.if r_1&r_2 are complex number

$$y = e^{\alpha x}[c1\cos\beta x + c2\sin\beta x]$$

$\alpha = the\ real\ part$ & $\beta =$ **The imaginary part**

Example 1// Solve the equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

Solution/

$$(D^2 + D - 2)y = 0$$

$$(r^2 + r - 2) = 0 \rightarrow$$

$$(r+2)(r-1) = 0$$

$$r_1 = -2 \ \& \ r_2 = 1 (r_1 \neq r_2)$$

$$y = (c_1 e^{r1x} + c2\ e^{r2x}) = (c1 e^{-2x} + c2\ e^{1x})$$

Example 2// Solve $\bar{\bar{y}} + 4\bar{y} + 4y = 0$

Solution /

$$(D^2 + 4D + 4)y = 0$$

$$(r^2+4r+4)=0 \quad \rightarrow$$

$$(r+2)(r+2)=0$$

$$r_1=-2 \ \& \ r_2=-2 \qquad (r_1=r_2)$$

$$y=(c_1e^{rx}+c2\ xe^{rx})=(c1e^{-2x}+c2\ xe^{-2x})$$

Example 3// Solve $\frac{d^2y}{dx^2}+w^2y=0$

Solution /

$$(D^2+w^2)y=0$$

$$(r^2+w^2)=0 \quad \rightarrow r^2=-w^2 \rightarrow r=iw$$

$$\alpha=0 \quad \& \quad \beta=w$$

$$y=e^{\alpha x}[c1\cos\beta x+c2\sin\beta x]$$

$$y=e^{0x}[c1\cos wx+c2\sin wx]$$

Example 4// Solve $\bar{\bar{y}}+\bar{y}+y=0$

Solution /

$$(D^2+D+1)y=0$$

$$(r^2+r+1)=0 \quad \rightarrow$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4 * 1 * 1}}{2 * 1} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{\mathbf{-1 \pm} \boldsymbol{i}\sqrt{\mathbf{3}}}{\mathbf{2}}$$

$$\boldsymbol{\alpha} = \frac{\mathbf{-1}}{\mathbf{2}} \quad \& \quad \boldsymbol{\beta} = \frac{\sqrt{\mathbf{3}}}{\mathbf{2}}$$

$$\boldsymbol{y = e^{\alpha x}[c1\ cos\ \beta x + c2\ sin\ \beta x]}$$

$$\boldsymbol{y = e^{\frac{-1}{2}x}[c1\ cos\ \frac{\sqrt{3}}{2}x + c2\ sin\ \frac{\sqrt{3}}{2}x]}$$