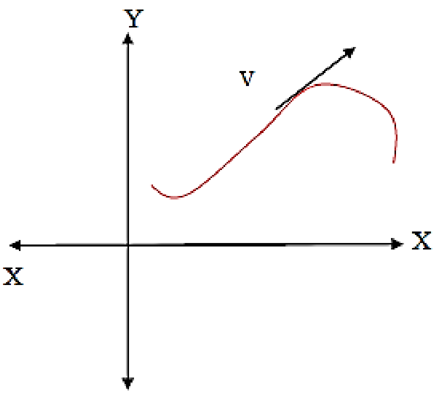
Mathematic I

Revision:

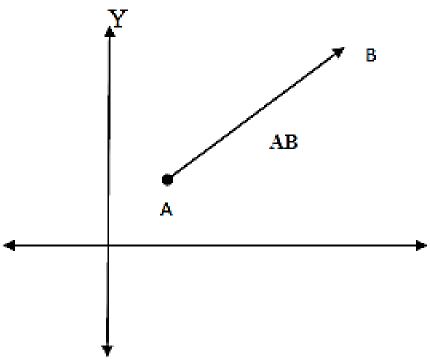
1. Vector.
2. Vector /Cross Product.
3.Complex Number
4. Matrix.
5. Determinates.
6. Grammer rule.
7. Integration methods/ powers of trigonometric function $\sin\theta$ & $\cos\theta$.
8. Integration methods/trigonometric substation.
9. Integration methods/partial fraction.
10. Integration by long division.
11. Integration methods/by part.
12. Tubular Integration
13. Integration application/area under the curve.
14. Integration application/volume.
15. Integration application/length of curve.
16. Integration application/area of surface.

1.<u>Vector</u>

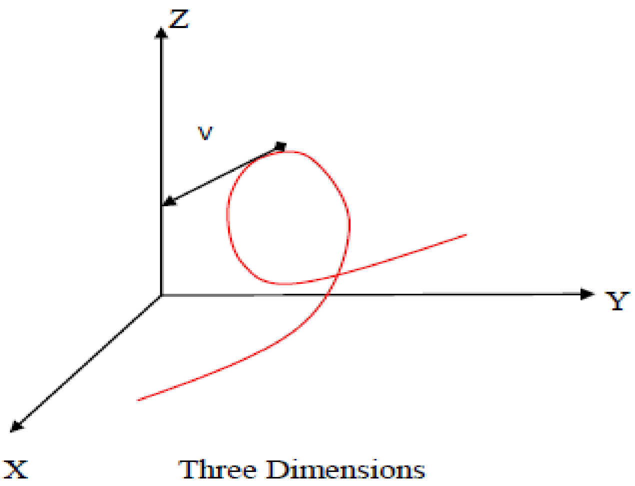
A quality such as force, displacement, or velocity is called a vector and represented by a direct line segment.



Two Dimensions



initial and terminal points.



Three Dimensions

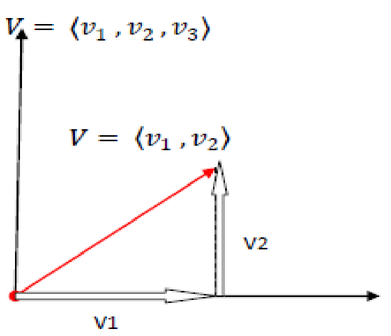
Definition of Vector:

A vector is a directed line segment $\overrightarrow{AB}$ has initial point (A) and terminal point (B), it's length denoted by $|\overrightarrow{AB}|$.

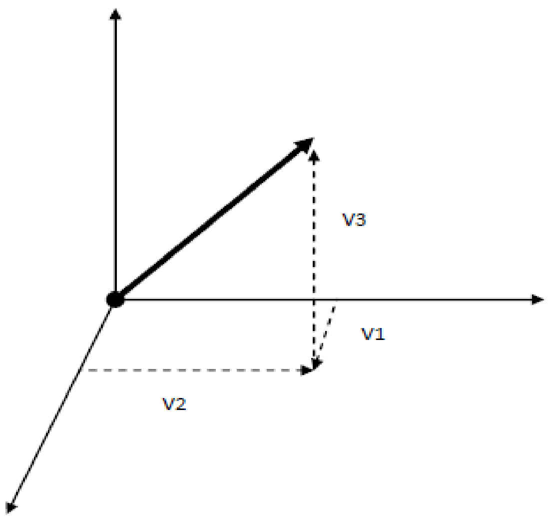
If $\vec{V}$ is a two dimensional vector in plane equal to the vector with initial points at the origin and terminal point (v_1, v_2), then the component of (V):

$$V = \langle v_1 , v_2 \rangle$$

If $\vec{V}$ is a three-dimensional vector with initial point at the origin at the terminal point(v_1, v_2, v_3), then the component of (V):



Component of two dimensional Vector



Component of three dimensional vectors

The magnitude of length of the vector $v = \overrightarrow{PQ}$ is the non-negative number.

$$|V| = \sqrt{v_1{}^2 + v_2{}^2 + v_3{}^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Vectors Operation:

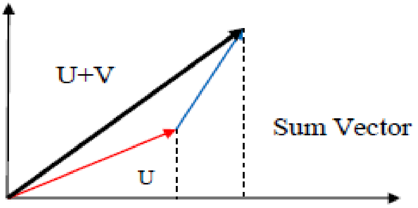
Vector Addition multiplication of a vector by scalar:

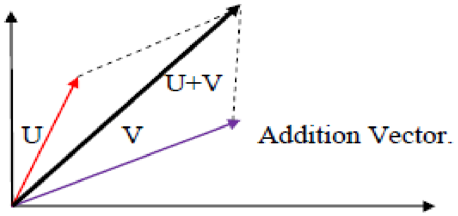
Let $U = (u_1, u_2, u_3), and\ \ V = (v_1, v_2, v_3)$

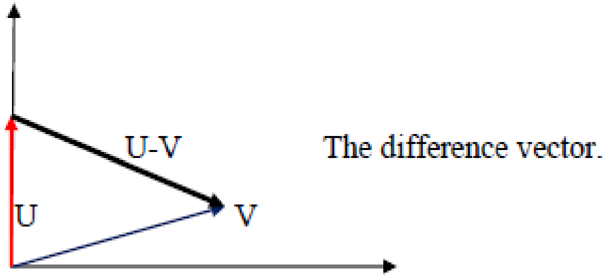
K is scalar.

Addition: $U + V = (u_1 + v_1 , u_2 + v_2, u_3 + v_3)$

Scalar multiplication: $K.U = (K.u_1, K.u_2,\ K.u_3$







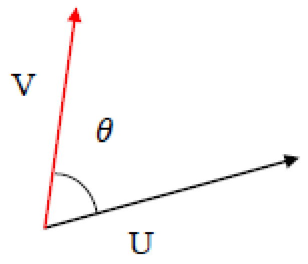
<u>Properties of Vector.</u>

1. U±V=V±U

2. a(U+V)=aU+aV

3. U.V= V.U(Dot product)

4. angle between two vector

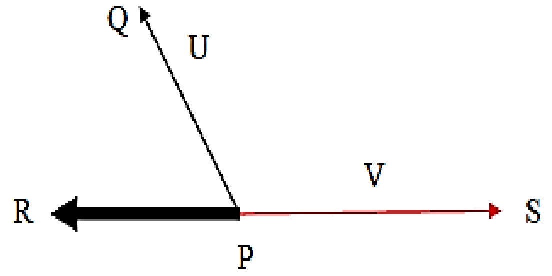


$$\theta = \cos^{-1}\left(\frac{U.V}{|u|.|v|}\right)$$

5.Vector Projection.

$U = \overrightarrow{PQ}$

$V = \overrightarrow{PS}$



The vector projection of $U = \overrightarrow{PQ}$ onto non-zero vector $V = \overrightarrow{PS}$ is the vector $\overrightarrow{PR}$.

$Proj_v\, U$ (*the vector Projection U onto V*)

$$Proj_v\, U = (\,|U|.\cos\theta)\,.\frac{V}{|V|}$$

$$\because\ |U|.\cos\theta = \frac{U.V}{|V|}$$

$$Proj_v\, U = \left(\frac{U.V}{|V|}\right).\frac{V}{|V|} = \left(\frac{U.V}{|V|^2}\right).V \qquad (vector\ value).$$

Example 1: Find the component form and length of vector with initial point P (-3, 4, 1) and terminal point Q (-5, 2, 2):

Solution:

a) The standard position vector V representing $\overrightarrow{PQ}$ has components:

$v_1 = x_2 - x_1 = -5 - 3 = -2$

$v_2 = y_2 - y_1 = 2 - 4 = -2$

$v_3 = z_2 - z_1 = 2 - 1 = 1$

The component form of $\overrightarrow{PQ}$ is:

V= (-2, -2, 1)

b) The magnitude of vector length $|V|$ *or* $|\overrightarrow{PQ}|$ is:

$$|V| = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$|V| = \sqrt{(-2)^2 + (-2)^2 + (1)^2} = \sqrt{9} = 3$$

Example 1: Let U = (-1, 3 , 1) , V = (4, 7, 0), Find:

1. 2U + 3V:

$2U + 3V = 2(-1,3,1) + 3(4,7,0) = (-2,6,2) + (12,21,0) =$

(10,27,2)

2. $U - V =$

$$U - V = (-1,3,1) - (4,7,0) = (-5,-4,1)$$

Example: Find the dot product of U.V, where $U = (1, -2. -1), and\ V = (-6, 2, -3)$.

Solution:

$$U.V = (1 \times -6) + (-2 \times 2) + (-1 \times -3)$$

$$= -6 - 4 + 3 = -7$$

Example : Find the angle θ between $U = i - 2j - 2k, and\ V = 6i + 3j + 2k$.

Solution:

$U.V = (1 \times 6) + (-2 \times 3) + (-2 \times 2)$

$= 6 - 6 - 4 = -4$

$|U| = \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{9} = 3$

$|V| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = 7$

$\theta = \cos^{-1} \frac{U.V}{|U|.|V|}$

$\theta = \cos^{-1} \frac{(-4)}{3\times7} \cong 1.76\ rad.$

Example☐ Find the vector projection of $U = 6i + 3j + 2k$ *onto* $V = i - 2j - 2k$. and the scalar component of U in the direction of V.

Solution:

1) Find Proj_v u:

$$Proj_V u = \frac{U.V}{|V|.|V|}.V$$

$$Proj_V u = \frac{(6\times1)+(3\times-2)+(2\times-2)}{(1^2+(-2)^2+(-2)^2)}.(i - 2j - 2k)$$

$$Proj_V u = \frac{-4}{9}.(i - 2j - 2k) = \frac{-4}{9}\, i + \frac{8}{9}\, j + \frac{8}{9}\, k$$

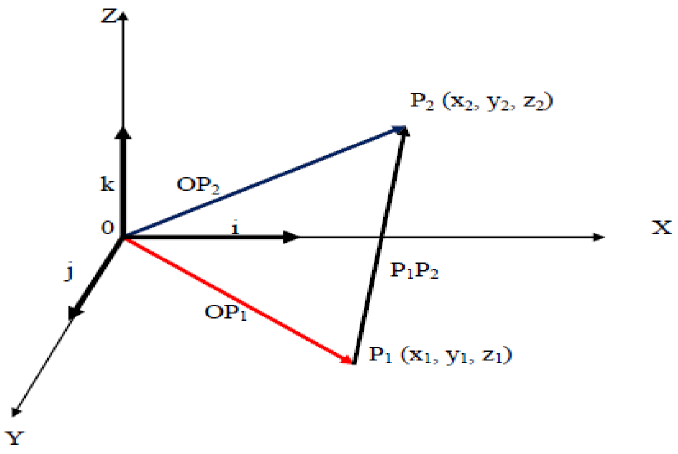
2) The scalar component of U in the direction of V.

$$|U|.\cos\theta = \frac{U.V}{|V|} = \frac{(6\times6)+(3\times-2)+(2\times-2)}{\sqrt{(1)^2+(-2)^2+(-2)^2}} = \frac{6-6+4}{\sqrt{9}} = \frac{4}{3}$$

Unit Vector:-

The length of vector is called unit vector. The standard of units vectors are:

$i = (1, 0, 0)$, $j = (0, 1, 0)$, $k = (0, 0, 1)$



$V = v_1\, i + v_2\, j + v_3\, k$

$i, j, and\ k\ \ as\ scalar\ vector.$

$i = component\ of\ vector\ v_1$, $j = component\ of\ vector\ v_2$

$k = component\ of\ vector\ v_3$

$V_1=(X_2-X_1)$, $V_2=(Y_2-Y_1)$,$V_3=(Z_2-Z_1)$

$$\textbf{Unit vector}=\frac{v}{|v|} = \frac{\overline{P1P2}}{|\overline{P1P2}|}$$

Example ?: Find a unit vector in the direction of the vector $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.

Solution:

$$1.\ \overrightarrow{P1P2} = V1\ i + v2\ j + v3\ k$$
$$= (x2 - x1)i + (y2 - y1)j + (z2 - z1)k$$
$$=(3-1)i+(2-0)j+(0-1)k=2i+2j-k$$

$$2.\ |\overrightarrow{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(3-1)^2 + (2-0)^2 + (0-1)^2}$$
$$= \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

3. Unit vector:

$$U = \frac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|} = \frac{2\,i + 2j - k}{3}$$

$$U = \frac{2}{3}\,i + \frac{2}{3}\,j - \frac{1}{3}\,k$$