3) Linear Equation (first order first degree)

Alinear of ordinary differential Equation (ODE)can always be put in to the standared from

$$\frac{dy}{dx} + \rho(x)y = Q(x)$$

$\rho = (integration\ factor) \rightarrow \rho = e^{\int \rho(x)dx}$

eqation$\left[y = e^{-\int \rho(x)dx} * (\int \rho * Q(x) + C)\right]$

Example// Solve $\frac{dy}{dx} = e^X - Y$

Solution/

$$\frac{dy}{dx} + Y = e^X \leftrightarrow \frac{dy}{dx} + \rho(x)y = Q(x)$$

$$\rho(x) = 1 \quad \& \ Q(x) = e^X$$

$$\rho = e^{\int \rho(x)dx} = e^{\int 1dx} = e^X$$

$$y = e^{-\int \rho(x)dx} * \left(\int \rho * Q(x) + C\right) = e^{-X} * \left(\int e^X * e^X dx + c\right)$$
$$=$$
$$e^{-X} * \left(\int e^{2X}\, dx + c\right) = e^{-X}\left(\frac{1}{2}e^{2X} + c\right) = \frac{1}{2}e^x + e^{-X}c$$

H.W. // Solve $\frac{dy}{dx} - 3y = x^2$

Bernoullis Equation:-

using when not linear equation

The general form

$$\frac{dy}{dx}+\rho(x)y=Q(x)y^n \qquad (n\neq 1)$$

Assume $u=y^{1-n}\rightarrow \frac{du}{dx}=(1-n)y^{-n}\frac{dy}{dx}$

$=\frac{dy}{dx}=\frac{1}{1-n}y^n\frac{du}{dx}\ sub\ in\ (1)$

$$\frac{dy}{dx}+\rho(x)y=Q(x)y^n \ \div\ y^n$$

$$\rightarrow\left[y^{-n}\ \frac{dy}{dx}+\rho(x)y^{1-n}=Q(x)\right]\ldots\ldots(1)$$

$$\rightarrow\left[y^{-n}\ \frac{1}{1-n}y^n\frac{du}{dx}+\rho(x)u=Q(x)\right]*(1-n)$$

$$=\left[\frac{du}{dx}+(1-n)\rho(x)u=(1-n)Q(x)\right]$$

Example// Solve $\frac{dy}{dx}+\frac{3}{x}y=x^2y^2$

Solution /

$$\frac{dy}{dx}+\frac{3}{x}y=x^2y^2 \qquad \div y^2\ (n=2)$$

$$\frac{dy}{dx}y^{-2}+\frac{3}{xy}=x^2$$

Assume $u = y^{1-n} \rightarrow u = y^{-1} \rightarrow \frac{du}{dx} = \frac{-1}{y^2}\frac{dy}{dx}$

$$\frac{dy}{dx} = -y^2\frac{du}{dx} \quad sub\ in\ equation$$

$$\rightarrow -y^2 * y^{-2}\frac{du}{dx} + \frac{3}{xy} = x^2 \rightarrow \frac{du}{dx} - \frac{3u}{x} = -x^2 \quad \boldsymbol{is\ linear}$$

$$\rho(x) = \frac{-3}{x} \quad \& \ Q(x) = -x^2$$

$$\rho = e^{\int \frac{-3}{x}dx} = e^{-3lnx} = x^{-3}$$

$$y = e^{-\int \rho(x)dx} * \left(\int \rho * Q(x) + C\right)$$

$$= x^3 * \left(\int x^{-3} * (-x^2)dx + c\right) =$$

$$x^3 * \left(\int \frac{-1}{x}dx + c\right) = x^3(-\ln x + c) = -x^3 lnx + x^3 c$$

H.W. // Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$