2. Ordinary Differential Equation (ODE):-

Ordinary Differential Equations (ODE).

A differential equation is an equation that contains one or more derivatives of a differentiable function. An equation with partial derivatives is called a Partial Differential Equation. While, an equation with ordinary derivatives, which is derivatives of a function of a single variable, is called an Ordinary Differential Equation.

The order of a differential equation is the order of the equation's highest order derivative. A differential equation is linear if it can be put in the form

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_1(x)\frac{dy}{dx} + a_0(x)y = F(x)$$

The degree of a differential equation is the power (exponent) of the equation's highest order derivative.

Example

First order, first degree, linear $\quad \frac{dy}{dx} = 5y, \quad 3\frac{dy}{dx} - \sin x = 0$

Third order, second degree, nonlinear $\quad \left(\frac{d^3 y}{dx^3}\right)^2 + \left(\frac{d^2 y}{dx^2}\right)^5 - \frac{dy}{dx} = e^x$

If F(x) in the equation above equal to zero the differential equation is called homogeneous, otherwise it is called nonhomogeneous differential equation.

Example

First order, first degree ,linear, non-Homogenous $\frac{dy}{dx}+2xy=\sin x$

Second order, first degree,linear,Homogenous $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+6y=0$

3. Methods and techniques for solving differential equations and systems of differential equations, for first order:-

Solution of First Order First degree Ordinary Differential Equations.

$$a_0+a1\ \frac{dy}{dx}=Q(X)[where\ a0, a1\ are\ function\ of\ x]$$

To solve this type of O.D.E is y(x)we divide into four types

1.variable separable.

2.Homogenous.

3.Exat

4.linear.

1) variable separable Equations.

A first order differential equations is separable if it can be put in the form:

$$M(x)dx+N(y)dy=0$$

To solve these equations, integrate M with respect to x and N with respect to y to obtain an equation that relates y and x .

Example// Solve $\frac{dy}{dx} = 1 + y^2$

Solution /

$$\frac{dy}{1 + y^2} = dx \rightarrow \int \frac{dy}{1 + y^2} = \int dx$$

$$\tan^{-1} y = x + c$$

Example // Solve $(x + 1)dy - (y^2 + 1)dx = 0$

Solution/

$$(x + 1)dy - (y^2 + 1)dx = 0 \qquad \div (x + 1)(y^2 + 1)$$

$$\frac{dy}{(y^2 + 1)} = \frac{dx}{(x + 1)} \rightarrow \int \frac{dy}{y^2 + 1} = \int \frac{dx}{(x + 1)}$$

$$\tan^{-1} y = \ln|x + 1| + c$$

Example//Solve $\frac{dy}{dx} = (1 + y^2)e^x$

Solution/

$$\frac{dy}{(1 + y^2)} = e^x\, dx \rightarrow \int \frac{dy}{(1 + y^2)} = \int e^x\, dx$$

$$\tan^{-1} y = e^x + c$$

Example//Solve $e^{x+y}dx = \frac{dy}{x}$

Solution/

$$e^{x+y}dx = \frac{dy}{x} \rightarrow e^x e^y dx = \frac{dy}{x} \rightarrow xe^x dx = \frac{dy}{e^y}$$

$$\int \frac{dy}{e^y} = \int xe^x dx \rightarrow \int e^{-y} = \int xe^x dx$$

$$-e^{-y} = xe^x - \int e^x dx + c \rightarrow$$

$$-e^{-y} = xe^x - e^x + c$$