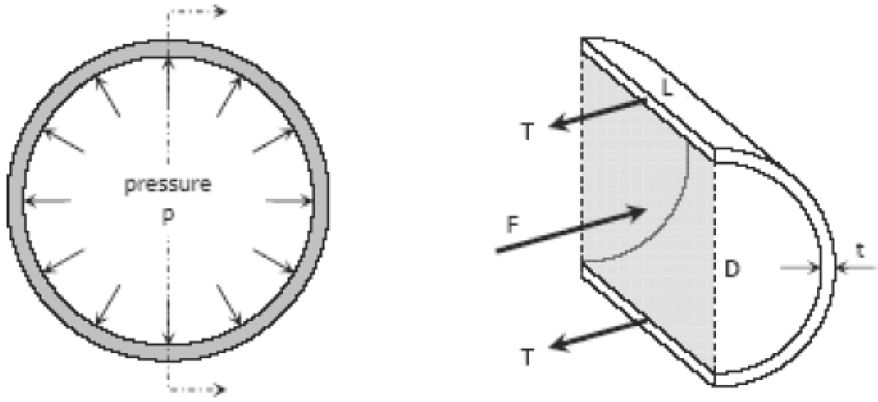
Thin-Walled Cylindrical Pressure Vessels

A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

Tangential Stress (Circumferential Stress):

Consider the tank shown being subjected to an internal pressure p. The length of the tank is L and the wall thickness is t. Isolating the right half of the tank:



$$F = P_o * A = P_o * D * L$$

$$T = \sigma_t * A_{wall} = \sigma_t * t * L$$

$$\sum F_H = 0$$

$$F = 2 * T$$

$$P_o * D * L = 2 * \sigma_t * t * L$$

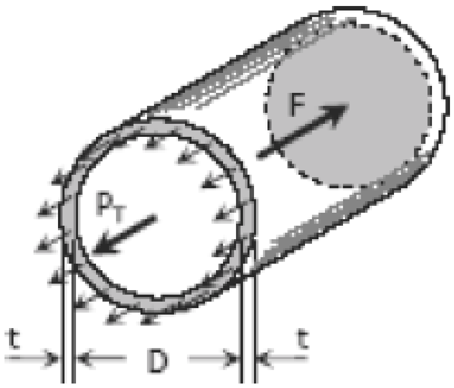
$$\sigma_t = \frac{P_o * D}{2 * t}$$

If there exist an external pressure p_o and an internal pressure p_i, the formula may be expressed as:

Longitudinal Stress:

Consider the free body diagram in the transverse section of the tank:

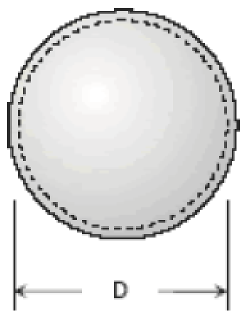
The total force acting at the rear of the tank (F) must equal to the total longitudinal stress on the wall ($P_T = \sigma_L * A_{wall}$) . Since (t) is so small compared to (D), the area of the wall is close to (π*D*t)



It can be observed that the tangential stress is twice that of the longitudinal stress.

<u>Spherical Shell:</u>

If a spherical tank of diameter (D) and thickness (t) contains gas under a pressure of (p), the stress at the wall can be expressed as:



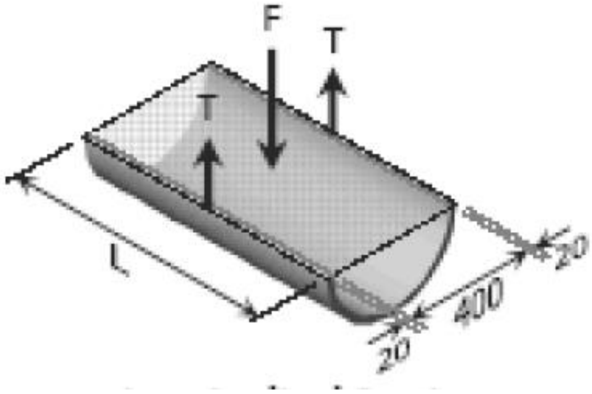
Problem (1): A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m^2.

(a) Calculate the tangential and longitudinal stresses in the steel.

(b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m^2?

(c) If the internal pressure were increased until the vessel burst, sketch the type of fracture that would occur.

Solution



a) 1) Tangential stress

$$F = 2T$$

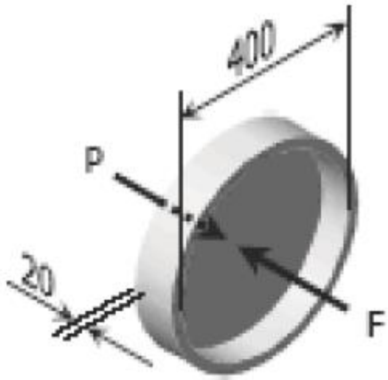
$$F = P_o * D * L = 2 * \sigma_t * t * L$$

$$\sigma_t = \frac{P_o * D}{2 * t}$$

$$\sigma_t = \frac{4.5 * 400}{2 * 20}$$

$$\sigma_t = 45\ MPa$$

2) Longitudinal stress



$$F = P$$

$$\frac{\pi}{4} * D^2 * P_o = \sigma_L * \pi * D * t$$

$$\sigma_L = \frac{P_o * D}{4 * t}$$

$$\sigma_L = 22.5\ MPa$$

b) From (a)

$$\sigma_t = \frac{P_o * D}{2 * t} \text{ and } \sigma_L = \frac{P_o * D}{4 * t}$$

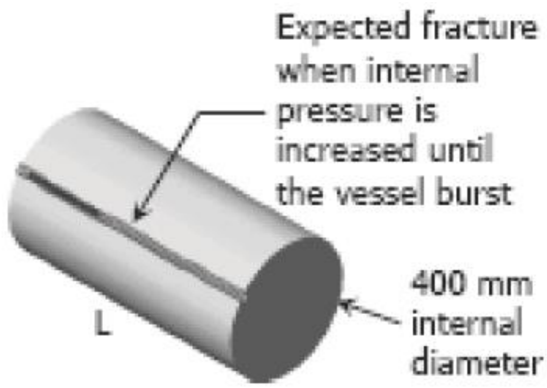
This shows that the tangential stress is the critical

$$\sigma_t = \frac{P_o * D}{2 * t}$$

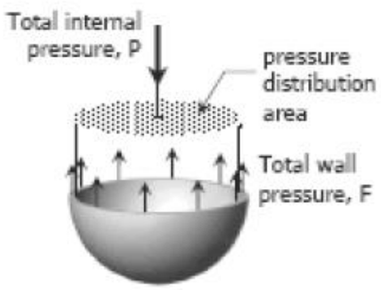
$$120 = \frac{P_o * 400}{2 * 20}$$

$$P_o = 12\ MPa$$

c) The bursting force will cause a stress on the longitudinal section that is twice to that of the transverse section thus fracture is expected as shown



Problem (2): The wall thickness of a 4-ft-diameter spherical tank is 5/16 in. Calculate the allowable internal pressure if the stress is limited to 8000 psi.



Total internal pressure:

$$P = p\left(\tfrac{1}{4}\pi D^2\right)$$

Resisting wall:

$$F = P$$

$$\sigma A = p\left(\tfrac{1}{4}\pi D^2\right)$$

$$\sigma(\pi D t) = p\left(\tfrac{1}{4}\pi D^2\right)$$

$$\sigma = \frac{pD}{4t}$$

$$8000 = \frac{p(4\times 12)}{4\left(\frac{5}{16}\right)}$$

$$\mathbf{p = 208.33\ psi}$$

Problem (3): Calculate the minimum wall thickness for a cylindrical vessel that is to carry a gas at a pressure of 1400 psi. The diameter of the vessel is 2 ft, and the stress is limited to 12 ksi.

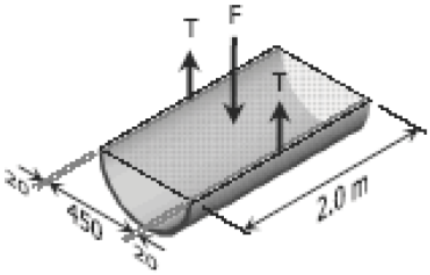
The critical stress is the tangential stress

$$\sigma_t = \frac{pD}{2t}$$

$$12\,000 = \frac{1400(2\times 12)}{2t}$$

$$\mathbf{t = 1.4\ in}$$

Problem (4): A cylindrical pressure vessel is fabricated from steel plating that has a thickness of 20 mm. The diameter of the pressure vessel is 450 mm and its length is 2.0 m. Determine the maximum internal pressure that can be applied if the longitudinal stress is limited to 140 MPa, and the circumferential stress is limited to 60 MPa.



Based on circumferential stress (tangential):

$$\Sigma F_V = 0$$

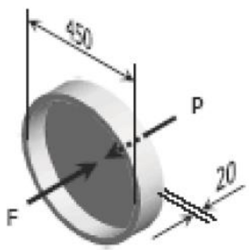
$$F = 2T$$

$$p(DL) = 2(\sigma_t Lt)$$

$$\sigma_t = \frac{pD}{2t}$$

$$60 = \frac{p(450)}{2(20)}$$

$$p = 5.33 \text{ MPa}$$



Based on longitudinal stress:

$$\Sigma F_H = 0$$

$$F = P$$

$$p\left(\tfrac{1}{4}\pi D^2\right) = \sigma_l(\pi D t)$$

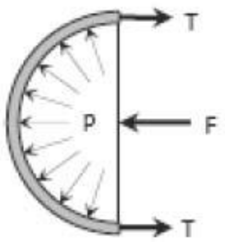
$$\sigma_l = \frac{pD}{4t}$$

$$140 = \frac{p(450)}{4(20)}$$

$$p = 24.89 \text{ MPa}$$

Use p = **5.33 MPa**

Problem (5): A water tank, 22 ft in diameter, is made from steel plates that are ½ in. thick. Find the maximum height to which the tank may be filled if the circumferential stress is limited to 6000 psi. The specific weight of water is 62.4 lb/ft3.



$\sigma_t = 6000$ psi

$$\sigma_t = \frac{6000\ \text{lb}}{\text{in}^2}\left(\frac{12\ \text{in}}{\text{ft}}\right)^2$$

$\sigma_t = 864\ 000\ \text{lb/ft}^2$

Assuming pressure distribution to be uniform:

$p = \gamma h = 62.4h$

$F = pA = 62.4h(Dh)$

$F = 62.4(22)h^2$

$F = 1372.8h^2$

$T = \sigma_t A_t = 864\ 000(th)$

$T = 864\ 000\left(\frac{1}{2} \times \frac{1}{12}\right) h$

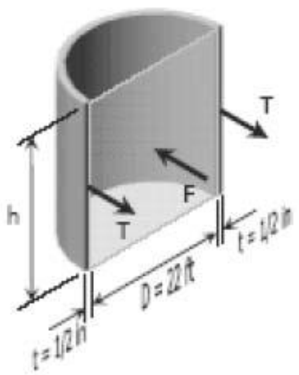
$T = 36\ 000h$

$\Sigma F = 0$

$F = 2T$

$1372.8h^2 = 2(36\ 000h)$

$h = \mathbf{52.45\ ft}$



Problem (7): The tank shown in the figure is fabricated from 1/8-in steel plate. Calculate the maximum longitudinal and circumferential stress caused by an internal pressure of 125 psi.

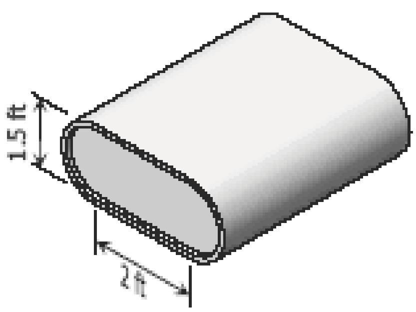
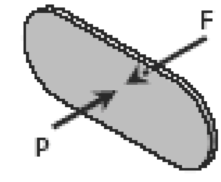


Figure P-141

solution



Longitudinal Stress:

$F = pA = 125[1.5(2) + \frac{1}{4}\pi(1.5)^2](12)^2$

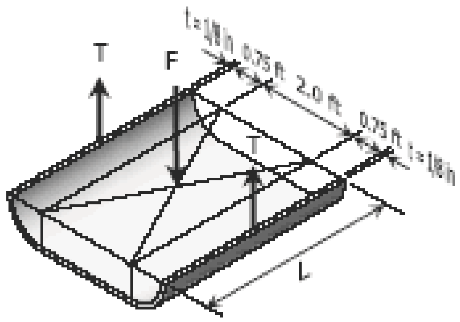
$F = 85\,808.62$ lbs

$P = F$

$\sigma_l\,[2(2 \times 12)\,(\frac{1}{8}) + \pi(1.5 \times 12)\,(\frac{1}{8})\,] = 85\,808.62$

$\sigma_l = 6\,566.02$ psi

σ_l **= 6.57 ksi**



Circumferential Stress:

$F = pA = 125[(2 \times 12)L + 2(0.75 \times 12)L]$

$F = 5250L$ lbs

$2T = F$

$2[\sigma_t\,(\frac{1}{8})\,L] = 5250L$

$\sigma_t = 21\,000$ psi

σ_t **= 21 ksi**