

The effectiveness factor η is given by equation 10.215 as:

$$\eta = \frac{1}{\phi_L} \left(\coth 3\phi_L - \frac{1}{3\phi_L} \right)$$

$\phi_L > 3$, and hence:

$$\eta \approx \phi_L^{-1} \quad (\text{ii})$$

It is assumed that the reactor is operating in this regime and the assumption is then checked.

Substituting numerical values in equation 10.217:

$$8.2 \times 10^{-2} = \eta k (0.011) \quad (\text{iii})$$

From equations (i) and (ii):

$$\eta = \phi_L^{-1} = 0.822 \frac{1}{\sqrt{k}}$$

From equation (iii):

$$8.2 \times 10^{-2} = 0.822 \frac{1}{\sqrt{k}} k (0.011)$$

and:

$$k = 82.2 \text{ s}^{-1}$$

From equation (i):

$$\phi_L = 11.04 \text{ and } \underline{\underline{\eta = 0.0906}}$$

This result may be checked by using equation 10.215:

$$\begin{aligned} \eta &= \frac{1}{11.04} \left(\coth 33.18 - \frac{1}{3 \times 11.04} \right) \\ &= \underline{\underline{0.0878}} \end{aligned}$$

This value is sufficiently close for practical purposes to the value of 0.0906, calculated previously. If necessary, a second iteration may be carried out.

10.8 PRACTICAL STUDIES OF MASS TRANSFER

The principal applications of mass transfer are in the fields of distillation, gas absorption and the other separation processes involving mass transfer which are discussed in Volume 2, In particular, mass transfer coefficients and heights of transfer units in distillation, and in gas absorption are discussed in Volume 2, . In this section an account is given of some of the experimental studies of mass transfer in equipment of simple geometry, in order to provide a historical perspective.

10.8.1. The j -factor of Chilton and Colburn for flow in tubes

Heat transfer

Because the mechanisms governing mass transfer are similar to those involved in both heat transfer by conduction and convection and in momentum transfer (fluid flow), quantitative relations exist between the three processes, and these are discussed in Chapter 12. There is generally more published information available on heat transfer than on mass transfer, and these relationships often therefore provide a useful means of estimating mass transfer coefficients.

Results of experimental studies of heat transfer may be conveniently represented by means of the *j*-factor method developed by COLBURN⁽³⁴⁾ and by CHILTON and COLBURN⁽³⁵⁾ for representing data on heat transfer between a turbulent fluid and the wall of a pipe. From equation 9.64:

$$Nu = 0.023Re^{0.8}Pr^{0.33} \tag{equation 9.64}$$

where the viscosity is measured at the mean film temperature, and *Nu*, *Re* and *Pr* denote the Nusselt, Reynolds and Prandtl numbers, respectively.

If both sides of the equation are divided by the product *Re Pr*:

$$St = 0.023Re^{-0.2}Pr^{-0.67} \tag{10.223}$$

where *St*(= *h/C_pρu*) is the Stanton number.

Equation 10.223 may be rearranged to give:

$$j_h = St Pr^{0.67} = 0.023Re^{-0.2} \tag{10.224}$$

The left-hand side of equation 10.224 is referred to as the *j*-factor for heat transfer (*j_h*). Chilton and Colburn found that a plot of *j_h* against *Re* gave approximately the same curve as the friction chart (*φ* versus *Re*) for turbulent flow of a fluid in a pipe.

The right-hand side of equation 10.224 gives numerical values which are very close to those obtained from the Blasius equation for the friction factor *φ* for the turbulent flow of a fluid through a smooth pipe at Reynolds numbers up to about 10⁶.

$$\phi = 0.0396Re^{-0.25} \tag{equation 3.11}$$

<i>Re</i>	3 × 10 ³	10 ⁴	3 × 10 ⁴	10 ⁵	3 × 10 ⁵	10 ⁶
<i>φ</i> (equation 3.11)	0.0054	0.0040	0.0030	0.0022	0.0017	0.0013
<i>φ</i> (equation 10.224)	0.0046	0.0037	0.0029	0.0023	0.0019	0.0015

Mass transfer

Several workers have measured the rate of transfer from a liquid flowing down the inside wall of a tube to a gas passing countercurrently upwards. GILLILAND and SHERWOOD⁽³⁶⁾ vaporised a number of liquids including water, toluene, aniline and propyl, amyl and butyl alcohols into an air stream flowing up the tube. A small tube, diameter *d* = 25 mm and length = 450 mm, was used fitted with calming sections at the top and bottom, and the pressure was varied from 14 to about 300 kN/m².

The experimental results were correlated using an equation of the form:

$$\frac{h_D}{u} \frac{C_{Bm}}{C_T} \left(\frac{\mu}{\rho D} \right)^{0.56} = 0.023Re^{-0.17} \tag{10.225}$$

The index of the Schmidt group *Sc* is less than the value of 0.67 for the Prandtl group for heat transfer but the range of values of *Sc* used was very small.

There has for long been uncertainty concerning the appropriate value to be used for the exponent of the Schmidt number in equation 10.225. SHERWOOD, PIGFORD and WILKE⁽²⁷⁾

have analysed experimental results obtained by a number of workers for heat transfer to the walls of a tube for a wide range of gases and liquids (including water, organic liquids, oils and molten salts), and offered a logarithmic plot of the Stanton number ($St = h/C_p \rho u$) against Prandtl number (Pr) for a constant Reynolds number of 10,000. Superimposed on the graph are results for mass transfer obtained from experiments on the dissolution of solute from the walls of tubes composed of solid organics into liquids, and on the evaporation of liquid films from the walls of tubes to turbulent air streams using a wetted-wall column, again all at a Reynolds number of 10,000: these results were plotted as Stanton number for mass transfer (h_D/u) against Schmidt number (Sc). There is very close agreement between the results for heat transfer and for mass transfer, with a line slope of about -0.67 giving a satisfactory correlation of the results. The range of values of Prandtl and Schmidt numbers was from about 0.4 to 10,000. This established that the exponents for both the Prandtl and Schmidt numbers in the j -factors should be the same, namely 0.67. These conclusions are consistent with the experimental results of LINTON and SHERWOOD⁽³⁷⁾ who measured the rates of dissolution of cast tubes of benzoic acid, cinnamic acid and β -naphthol into water, giving Schmidt numbers in the range 1000 to 3000.

In defining a j -factor (j_d) for mass transfer there is therefore good experimental evidence for modifying the exponent of the Schmidt number in Gilliland and Sherwood's correlation (equation 10.225). Furthermore, there is no very strong case for maintaining the small differences in the exponent of Reynolds number. On this basis, the j -factor for mass transfer may be defined as follows:

$$j_d = \frac{h_D}{u} \frac{C_{Bm}}{C_T} Sc^{0.67} = 0.023 Re^{-0.2} \quad (10.226)$$

The term C_{Bm}/C_T (the ratio of the logarithmic mean concentration of the insoluble component to the total concentration) is introduced because $h_D(C_{Bm}/C_T)$ is less dependent than h_D on the concentrations of the components. This reflects the fact that the analogy between momentum, heat and mass transfer relates only to that part of the mass transfer which is *not* associated with the *bulk flow* mechanism; this is a fraction C_{Bm}/C_T of the total mass transfer. For equimolecular counterdiffusion, as in binary distillation when the molar latent heats of the components are equal, the term C_{Bm}/C_T is omitted as there is no bulk flow contributing to the mass transfer.

SHERWOOD and PIGFORD⁽⁷⁾ have shown that if the data of GILLILAND and SHERWOOD⁽³⁶⁾ and others^(35,38,39) are plotted with the Schmidt group raised to this power of 0.67, as shown in Figure 10.14, a reasonably good correlation is obtained. Although the points are rather more scattered than with heat transfer, it is reasonable to assume that both j_d and j_h are approximately equal to ϕ . Equations 10.224 and 10.226 apply in the absence of ripples which can be responsible for a very much increased rate of mass transfer. The constant of 0.023 in the equations will then have a higher value.

By equating j_h and j_d (equations 10.224 and 10.226), the mass transfer coefficient may be expressed in terms of the heat transfer coefficient, giving:

$$h_D = \left(\frac{h}{C_p \rho} \right) \left(\frac{C_T}{C_{Bm}} \right) \left(\frac{Pr}{Sc} \right)^{0.67} \quad (10.227)$$

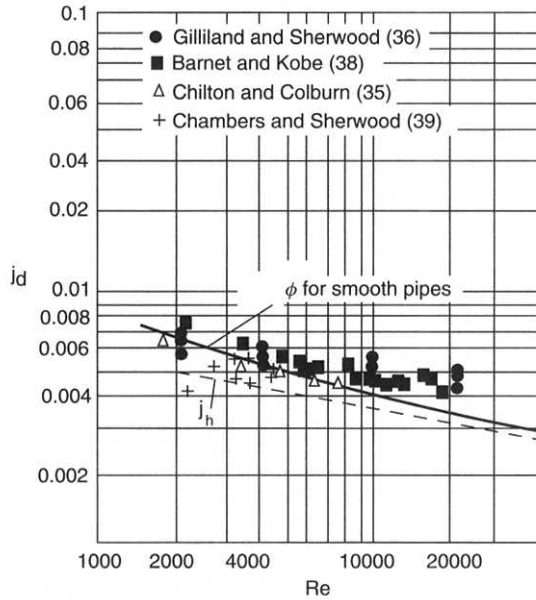


Figure 10.14. Mass transfer in wetted-wall columns

10.8.2. Mass transfer at plane surfaces

Many of the earlier studies of mass transfer involved measuring the rate of vaporisation of liquids by passing a turbulent air stream over a liquid surface. In addition, some investigations have been carried out in the absence of air flow, under what have been termed still air conditions. Most of these experiments have been carried out in some form of wind tunnel where the rate of flow of air and its temperature and humidity could be controlled and measured. In these experiments it was found to be important to keep the surface of the liquid level with the rim of the pan in order to avoid the generation of eddies at the leading edge.

HINCHLEY and HIMUS⁽⁴⁰⁾ measured the rate of evaporation from heated rectangular pans fitted flush with the floor of a wind tunnel (0.46m wide by 0.23m high), and showed that the rate of vaporisation was proportional to the difference between the saturation vapour pressure of the water P_s and the partial pressure of water in the air P_w . The results for the mass rate of evaporation W were represented by an empirical equation of the form:

$$W = \text{constant} (P_s - P_w) \tag{10.228}$$

where the constant varies with the geometry of the pan and the air velocity.

This early work showed that the driving force in the process was the pressure difference ($P_s - P_w$). Systematic work in more elaborate equipment by POWELL and GRIFFITHS⁽⁴¹⁾, WADE⁽⁴²⁾, and PASQUILL⁽⁴³⁾ then followed. Wade, who vaporised a variety of organic liquids, including acetone, benzene and tri-chloroethylene at atmospheric pressure, used a small pan 88 mm square in a wind tunnel. Powell and Griffiths stretched canvas sheeting over rectangular pans and, by keeping the canvas wet at all times, ensured that it behaved

as a free water surface. In all of these experiments the rate of vaporisation showed a similar form of dependence on the partial pressure difference and the rate of flow of the air stream. Powell and Griffiths found that the vaporisation rate per unit area decreased in the downwind direction. For rectangular pans, of length L , the vaporisation rate was proportional to $L^{0.77}$. This can be explained in terms of the thickening of the boundary layer (see Chapter 11) and the increase in the partial pressure of vapour in the air stream arising from the evaporation at upstream positions.

In these experiments, it might be anticipated that, with high concentrations of vapour in the air, the rate of evaporation would no longer be linearly related to the partial pressure difference because of the contribution of bulk flow to the mass transfer process (Section 10.2.3), although there is no evidence of this even at mole fractions of vapour at the surface as high as 0.5. Possibly the experimental measurements were not sufficiently sensitive to detect this effect.

SHERWOOD and PIGFORD⁽⁷⁾ have plotted the results of several workers^(41,42,43,44,45) in terms of the Reynolds number Re_L , using the length L of the pan as the characteristic linear dimension. Figure 10.15, taken from this work, shows j_d plotted against Re_L for a number of liquids evaporating into an air stream. Although the individual points show some scatter, j_d is seen to follow the same general trend as j_h in this work. The Schmidt number was varied over such a small range that the correlation was not significantly poorer if it was omitted from the correlation.

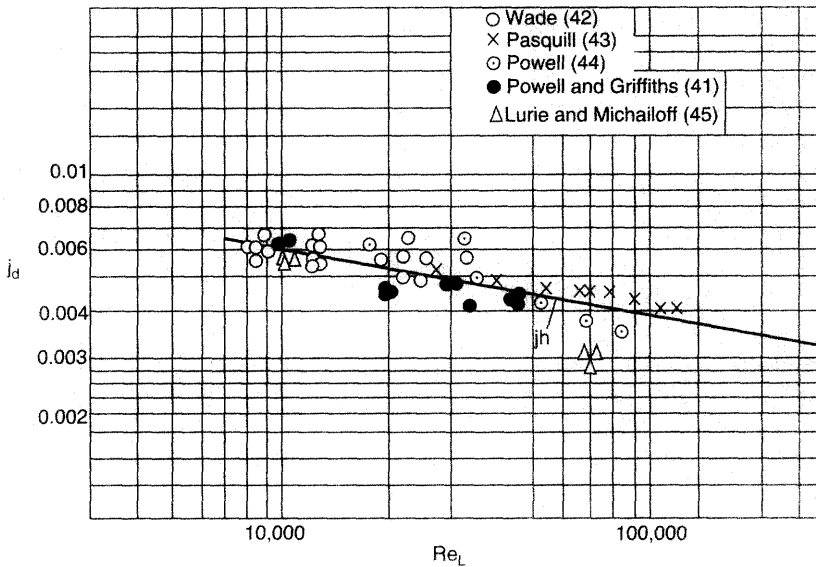


Figure 10.15. Evaporation from plane surfaces

MAISEL and SHERWOOD⁽⁴⁶⁾ also carried out experiments in a wind tunnel in which water was evaporated from a wet porous surface preceded by a dry surface of length L_0 . Thus, a velocity boundary layer had become established in the air before it came into

contact with the evaporating surface. The results were correlated by:

$$j_d = 0.0415 Re_L^{-0.2} \left[1 - \left(\frac{L_o}{L} \right)^{0.8} \right]^{-0.11} \quad (10.229)$$

where L is the total length of the surface (dry + wet).

10.8.3. Effect of surface roughness and form drag

The results discussed in Section 10.8.2 give reasonably good support to the treatment of heat, mass, and momentum transfer by the j -factor method, although it is important to remember that, in all the cases considered, the drag is almost entirely in the form of skin friction (that is, viscous drag at the surface). As soon as an attempt is made to apply the relation to cases where form drag (that is, additional drag caused by the eddies set up as a result of the fluid impinging on an obstruction) is important, such as beds of granular solids or evaporation from cylinders or spheres, the j -factor and the friction factor are found no longer to be equal. This problem receives further consideration in Volume 2. SHERWOOD^(47,48) carried out experiments where the form drag was large compared with the skin friction, as calculated approximately by subtracting the form drag from the total drag force. In this way, reasonable agreement between the corresponding value of the friction factor ϕ and j_h and j_d was obtained.

GAMSON *et al.*⁽⁴⁹⁾ have successfully used the j -factor method to correlate their experimental results for heat and mass transfer between a bed of granular solids and a gas stream.

PRATT⁽⁵⁰⁾ has examined the effect of using artificially roughened surfaces and of introducing "turbulence promoters", which increase the amount of form drag. It was found that the values of ϕ and the heat and mass transfer coefficients were a minimum for smooth surfaces and all three quantities increased as the surface roughness was increased. ϕ increased far more rapidly than either of the other two quantities however, and the heat and mass transfer coefficients were found to reach a limiting value whereas ϕ could be increased almost indefinitely. Pratt has suggested that these limiting values are reached when the velocity gradient at the surface corresponds with that in the turbulent part of the fluid; that is, at a condition where the buffer layer ceases to exist (Chapter 11).

10.8.4 Mass transfer from a fluid to the surface of particles

It is necessary to calculate mass transfer coefficients between a fluid and the surface of a particle in a number of important cases, including:

- (i) gas absorption in a spray tower,
- (ii) evaporation of moisture from the surface of droplets in a spray tower,
- (iii) reactions between a fluid and dispersed liquid droplets or solid particles as, for instance, in a combustion process where the oxygen in the air must gain access to the external surfaces, and
- (iv) catalytic reactions involving porous particles where the reactant must be transferred to the outer surface of the particle before it can diffuse into the pores and make contact with the active sites on the catalyst.

There have been comparatively few experimental studies in this area and the results of different workers do not always show a high degree of consistency. Frequently, estimates of mass transfer coefficients have been made by applying the analogy between heat transfer and mass transfer, and thereby utilising the larger body of information which is available on heat transfer.

Interest extends from transfer to single particles to systems in which the particles are in the form of fixed or fluidised beds. The only case for which there is a rigorous analytical solution is that for heat by conduction and mass transfer by diffusion to a sphere.

Mass transfer to single particles

Mass transfer from a single spherical drop to still air is controlled by molecular diffusion and, at low concentrations when bulk flow is negligible, the problem is analogous to that of heat transfer by conduction from a sphere, which is considered in Chapter 9, Section 9.3.4. Thus, for steady-state radial diffusion into a large expanse of stationary fluid in which the partial pressure falls off to zero over an infinite distance, the equation for mass transfer will take the same form as that for heat transfer (equation 9.26):

$$Sh' = \frac{h_D d'}{D} = 2 \quad (10.230)$$

where Sh' is the Sherwood number which, for mass transfer, is the counterpart of the Nusselt number $Nu' (= hd/k)$ for heat transfer to a sphere. This value of 2 for the Sherwood number is the theoretical minimum in any continuous medium and is increased if the concentration difference occurs over a finite, as opposed to an infinite, distance and if there is turbulence in the fluid.

For conditions of forced convection, FRÖSSLING⁽⁵¹⁾ studied the evaporation of drops of nitrobenzene, aniline and water, and of spheres of naphthalene, into an air stream. The drops were mainly small and of the order of 1 mm diameter. POWELL⁽⁴⁴⁾ measured the evaporation of water from the surfaces of wet spheres up to 150 mm diameter and from spheres of ice.

The experimental results of Frössling may be represented by the equation:

$$Sh' = \frac{h_D d'}{D} = 2.0(1 + 0.276Re^{0.5}Sc^{0.33}) \quad (10.231)$$

SHERWOOD and PIGFORD⁽⁷⁾ found that the effect of the Schmidt group was also influenced by the Reynolds group and that the available data were fairly well correlated as shown in Figure 10.16, in which $(h_D d')/D$ is plotted against $Re'Sc^{0.67}$.

GARNER and KEYE^(52,53) dissolved pelleted spheres of organic acids in water in a low-speed water tunnel at particle Reynolds numbers between 2.3 and 255 and compared their results with other data available at Reynolds numbers up to 900. Natural convection was found to exert some influence at Reynolds numbers up to 750. At Reynolds numbers greater than 250, the results are correlated by equation 10.230:

$$Sh' = 0.94Re^{0.5}Sc^{0.33} \quad (10.232)$$

Mass transfer under conditions of natural convection was also investigated.

RANZ and MARSHALL⁽⁵⁴⁾ have carried out a comprehensive study of the evaporation of liquid drops and confirm that equation 10.231 correlates the results of a number of

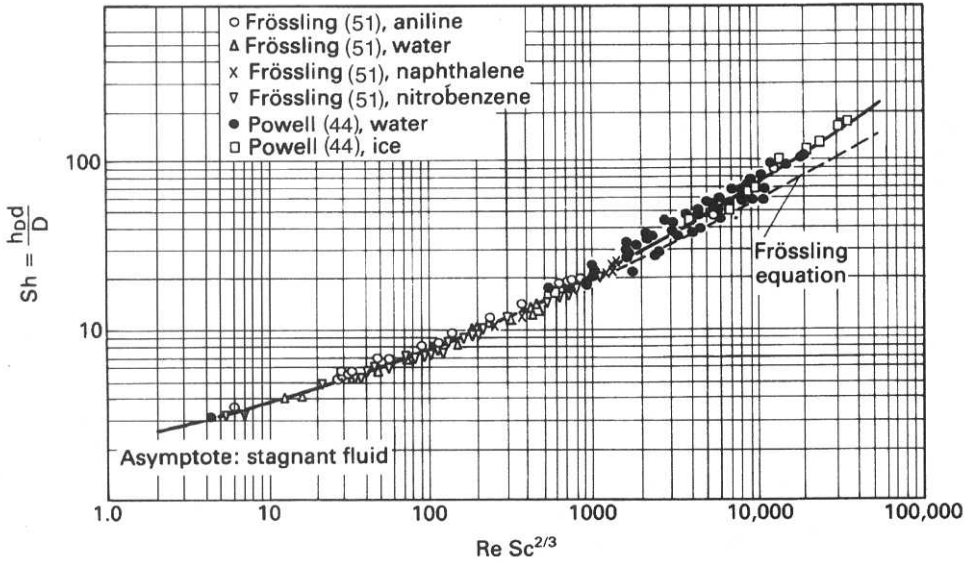


Figure 10.16. Mass transfer to single spheres

workers. A value of 0.3 in place of 0.276 for the coefficient is suggested, although the spread of the experimental results is such that the difference is not statistically significant.

ROWE *et al.*⁽⁵⁵⁾ have reviewed the literature on heat and mass transfer between spherical particles and a fluid. For heat transfer, their results which are discussed in Chapter 9, Section 9.4.6., are generally well represented by equation 9.100:

$$Nu' = 2 + \beta'' Re'^{0.5} Pr^{0.33} \quad (0.4 < \beta'' < 0.8) \quad (10.233)$$

For mass transfer:

$$Sh' = \alpha' + \beta' Re'^{0.5} Sc^{0.33} \quad (0.3 < \beta' < 1.0) \quad (10.234)$$

The constant α appears to be a function of the Grashof number, but approaches a value of about 2 as the Grashof number approaches zero.

In an experimental investigation⁽⁵⁵⁾ they confirmed that equations 10.233 and 10.234 can be used to represent the results obtained for transfer from both air and water to spheres. The constants β' , β'' varied from 0.68 to 0.79.

There is therefore broad agreement between the results of FRÖSLING⁽⁵¹⁾, RANZ and MARSHALL⁽⁵⁴⁾ and ROWE *et al.*⁽⁵⁵⁾. The variations in the values of the coefficient are an indication of the degree of reproducibility of the experimental results. However, BRIAN and HAYES⁽⁵⁶⁾ who carried a numerical solution of the equations for heat and mass transfer suggest that, at high values of $Re' Sc^{0.33}$, these equations tend to underestimate the value of the transfer coefficient, and an equation which can be expressed in the following form is proposed:

$$Sh' = [4.0 + 1.21(Re' Sc)^{0.67}]^{0.5} \quad (10.235)$$

Mass transfer to particles in a fixed or fluidised bed

Experimental results for fixed packed beds are very sensitive to the structure of the bed which may be strongly influenced by its method of formation. GUPTA and THODOS⁽⁵⁷⁾ have studied both heat transfer and mass transfer in fixed beds and have shown that the results for both processes may be correlated by similar equations based on j -factors (see Section 10.8.1). Re-arrangement of the terms in the mass transfer equation, permits the results for the Sherwood number (Sh') to be expressed as a function of the Reynolds (Re_c) and Schmidt numbers (Sc):

$$Sh' = 2.06 \frac{1}{e} Re_c^{0.425} Sc^{0.33} \quad (10.236)$$

where e is the voidage of the bed, and Re'_c is the particle Reynolds number incorporating the superficial velocity of the fluid (u_c).

KRAMERS⁽⁵⁸⁾ carried out experiments on heat transfer to particles in a fixed bed and has expressed his results in the form of a relation between the Nusselt, Prandtl and Reynolds numbers. This equation may be rewritten to apply to mass transfer, by using the analogy between the two processes, giving:

$$Sh' = 2.0 + 1.3Sc^{0.15} + 0.66Sc^{0.31} Re_c^{0.5} \quad (10.198)$$

In selecting the most appropriate equation for any particular operation, it is recommended that the original references be checked to ascertain in which study the experimental conditions were closest.

Both heat transfer and mass transfer between a fluid and particles in a fluidised bed are discussed in Volume 2. The results are sensitive to the quality of fluidisation, and particularly to the uniformity of distribution of the particles in the fluid. In most cases, it is found that the same correlations for both heat transfer and mass transfer are applicable to fixed and fluidised beds.

10.9. FURTHER READING

- BENNETT, C.O. and MYERS, J.E.: *Momentum, Heat, and Mass Transfer*, 3rd edn (McGraw-Hill, New York, 1983).
- BIRD, R.B., STEWART, W.E. and LIGHTFOOT, E.N.: *Transport Phenomena* (Wiley, New York, 1960).
- CUSSLER, E.L.: *Diffusion. Mass transfer in fluid systems*. 2nd edn (Cambridge University Press, Cambridge, 1997).
- DANCKWERTS, P.V.: *Gas-liquid Reactions* (McGraw-Hill, New York, 1970).
- ECKERT, E.R.G. and DRAKE, R.M.: *Analysis of Heat and Mass Transfer* (McGraw-Hill, New York, 1972).
- EDWARDS, D. A., BRENNER, H. and WASAN, D. T. *Interface Transport Processes and Rheology* (Butterworth-Heinemann, Oxford, 1991).
- HEWITT, G.F., SHIRES, G.L. and POLEZHAEV, Y.V.: *Encyclopedia of Heat and Mass Transfer* (CRC Press, Boca Raton, New York, 1997)
- SHERWOOD, T.K.: *Chemical Engineering Education* (Fall, 1974), 204. A review of the development of mass transfer theory.
- SHERWOOD, T.K., PIGFORD, R.L. and WILKE, C.R.: *Mass Transfer* (McGraw-Hill, New York, 1975).
- TAYLOR, R., and KRISHNA, R.: *Multicomponent Mass Transfer* (Wiley, New York, 1993).
- THOMAS, J.M. and THOMAS, W. J.: *Practice and Principles of Homogeneous Catalysis* (VCH, 1977).
- TREYBAL, R.E.: *Mass-Transfer Operations*, 3rd edn (McGraw-Hill, New York, 1980).

10.10. REFERENCES

1. FICK, A.: *Ann. Phys.* **94** (1855) 59. Ueber Diffusion.
2. PRESENT, R.D.: *Kinetic Theory of Gases* (McGraw-Hill, New York, 1958).
3. STEFAN, J.: *Wiener Akad. Wissensch.* **68** (1873) 385; **79** (1879) 169; **98** (1889) 1418; *Ann. Physik* **41** (1890) 723. Versuche über die Verdampfung.
4. PERRY, R.H. and GREEN, D.W. (eds): *Perry's Chemical Engineers' Handbook*. 6th edn (McGraw-Hill, New York, 1984).
5. WINKELMANN, A.: *Ann. Physik* **22** (1884) 1, 152. Ueber die Diffusion von Gasen und Dämpfen.
6. GILLILAND, E.R.: *Ind. Eng. Chem.* **26** (1934) 681. Diffusion coefficients in gaseous systems.
7. SHERWOOD, T.K. and PIGFORD, R.L.: *Absorption and Extraction* (McGraw-Hill, New York, 1952).
8. FULLER, E.N., SCHEFFLER, P.D. and GIDDINGS, J.C.: *Ind. Eng. Chem.* **58** (1966) 19. A new method for prediction of binary gas-phase diffusion coefficients.
9. GREW, K.E. and IBBS, T.L.: *Thermal Diffusion in Gases* (Cambridge University Press, Cambridge, 1952).
10. HIRSCHFELDER, J.O., CURTISS, C.F. and BIRD, R.B.: *Molecular Theory of Gases and Liquids*, pp. 584–585 (Wiley, New York, 1954).
11. ARNOLD, J.H.: *Trans. Am. Inst. Chem. Eng.* **40** (1944) 361. Studies in diffusion: III, Steady-state vaporization and absorption.
12. MAXWELL, J.C.: *Phil. Trans. Roy. Soc.* **157** (1867) 49. The dynamical theory of gases.
13. TAYLOR, R. and KRISHNA, R.: *Multicomponent Mass Transfer* (Wiley, New York, 1993).
14. CUSSLER, E.L.: *Multicomponent Diffusion* (Elsevier, Amsterdam, 1976).
15. ZIELINSKI, J.M. and HANLEY, B.F.: *A.I.Ch.E.Jl.* **45** (1999) 1. Practical friction-based approach to modeling multicomponent diffusion.
16. WILKE C.R. and CHANG, P.: *A.I.Ch.E.Jl.* **1** (1955) 264. Correlation of diffusion coefficients in dilute solutions.
17. REID, R.C., PRAUSNITZ, J.M. and SHERWOOD, T.K.: *The Properties of Gases and Liquids* (McGraw-Hill, New York, 1977).
18. GOODRIDGE, F. and BRICKNELL, D.J.: *Trans. Inst. Chem. Eng.* **40** (1962) 54. Interfacial resistance in the carbon dioxide-water system.
19. WHITMAN, W.G.: *Chem. and Met. Eng.* **29** (1923) 147. The two-film theory of absorption.
20. HIGBIE, R.: *Trans. Am. Inst. Chem. Eng.* **31** (1935) 365. The rate of absorption of pure gas into a still liquid during short periods of exposure.
21. DANCKWERTS, P.V.: *Ind. Eng. Chem.* **43** (1951) 1460. Significance of liquid film coefficients in gas absorption.
22. TOOR, H.L. and MARCHELLO, J.M.: *A.I.Ch.E.Jl.* **4** (1958) 97. Film-penetration model for mass and heat transfer.
23. KISHINEVSKII, M.K.: *Zhur. Priklad. Khim.* **24** (1951) 542; *J. Appl. Chem. U.S.S.R.* **24** 593. The kinetics of absorption under intense mixing.
24. DANCKWERTS, P.V.: *Gas-Liquid Reactions* (McGraw-Hill, New York, 1970).
25. HARRIOTT, P.: *Chem. Eng. Sci.* **17** (1962) 149. A random eddy modification of the penetration theory.
26. DWIGHT, H.B.: *Tables of Integrals and Other Mathematical Data* (Macmillan, New York, 1957).
27. SHERWOOD, T.K., PIGFORD, R.L. and WILKE, C.R.: *Mass Transfer* (McGraw-Hill, New York, 1975).
28. DANCKWERTS, P. V.: *Trans. Faraday Soc.* **46** (1950) 300. Absorption by simultaneous diffusion and chemical reaction.
29. SCOTT, D.S. and DULLIEN, F.A.L.: *A.I.Ch.E.Jl.* **8** (1962) 113. Diffusion of ideal gases in capillaries and porous solids.
30. ARIS, R.: *The Mathematical Theory of Diffusion and Reaction in Permeable Catalysts*. (Oxford University Press, London, 1975).
31. THIELE, E.W.: *Ind. Eng. Chem.* **24** (1939) 916. Relation between catalyst activity and size of particle.
32. MICKLEY, H.S., SHERWOOD, T.K. and REED, C.E.: *Applied Mathematics in Chemical Engineering*, 2nd edn. (McGraw-Hill, New York, 1957).
33. RESTER, S. and ARIS, R.: *Chem. Eng. Sci.* **24** (1969) 793. Communications on the theory of diffusion and reaction—II The effect of shape on the effectiveness factor.
34. COLBURN, A.P.: *Trans. Am. Inst. Chem. Eng.* **29** (1933) 174. A method of correlating forced convection heat transfer data and a comparison with fluid friction.
35. CHILTON, T.H. and COLBURN, A.P.: *Ind. Eng. Chem.* **26** (1934) 1183. Mass transfer (absorption) coefficients —production from data on heat transfer and fluid friction.
36. GILLILAND, E.R. and SHERWOOD, T.K.: *Ind. Eng. Chem.* **26** (1934) 516. Diffusion of vapours into air streams.
37. LINTON, W.H. and SHERWOOD, T.K.: *Chem. Eng. Prog.* **46** (1950) 258. Mass transfer from solid shapes to water in streamline and turbulent flow.

38. BARNET, W.I. and KOBE, K.A.: *Ind. Eng. Chem.* **33** (1941) 436. Heat and vapour transfer in a wetted-wall column.
39. CHAMBERS, F.S. and SHERWOOD, T.K.: *Ind. Eng. Chem.* **29** (1937) 579. *Trans. Am. Inst. Chem. Eng.* **33** (1937) 579. Absorption of nitrogen dioxide by aqueous solutions.
40. HINCHLEY, J.W. and HIMUS, G.W.: *Trans. Inst. Chem. Eng.* **2** (1924) 57. Evaporation in currents of air.
41. POWELL, R.W. and GRIFFITHS, E.: *Trans. Inst. Chem. Eng.* **13** (1935) 175. The evaporation of water from plane and cylindrical surfaces.
42. WADE, S.H.: *Trans. Inst. Chem. Eng.* **20** (1942) 1. Evaporation of liquids in currents of air.
43. PASQUILL, F.: *Proc. Roy. Soc. A* **182** (1943) 75. Evaporation from a plane free liquid surface into a turbulent air stream.
44. POWELL, R.W.: *Trans. Inst. Chem. Eng.* **18** (1940) 36. Further experiments on the evaporation of water from saturated surfaces.
45. LURIE, M. and MICHAILOFF, M.: *Ind. Eng. Chem.* **28** (1936) 345. Evaporation from free water surfaces.
46. MAISEL, D.S. and SHERWOOD, T.K.: *Chem. Eng. Prog.* **46** (1950) 131. Evaporation of liquids into turbulent gas streams.
47. SHERWOOD, T.K.: *Trans. Am. Inst. Chem. Eng.* **36** (1940) 817. Mass transfer and friction in turbulent flow.
48. SHERWOOD, T.K.: *Ind. Eng. Chem.* **42** (1950) 2077. Heat transfer, mass transfer, and fluid friction.
49. GAMSON, B.W., THODOS, G. and HOUGEN, O.A.: *Trans. Am. Inst. Chem. Eng.* **39** (1943) 1. Heat mass and momentum transfer in the flow of gases through granular solids.
50. PRATT, H.R.C.: *Trans. Inst. Chem. Eng.* **28** (1950) 77. The application of turbulent flow theory to transfer processes in tubes containing turbulence promoters and packings.
51. FRÖSSLING, N.: *Gerlands Beitr. Geophys.* **52** (1938) 170. Über die Verdunstung fallender Tropfen.
52. GARNER, F.H. and KEYE, R.B.: *Chem. Eng. Sci.* **9** (1958) 119. Mass transfer from single solid spheres—I. Transfer at low Reynolds numbers.
53. GARNER, F.H. and KEYE, R.B.: *Chem. Eng. Sci.* **9** (1959) 218. Mass transfer from single solid spheres—II. Transfer in free convection.
54. RANZ, W.E. and MARSHALL, W. R.: *Chem. Eng. Prog.* **48** (1952) 173. Evaporation from drops Part II.
55. ROWE, P.N., CLAXTON, K.T., and LEWIS, J.B.: *Trans. Inst. Chem. Eng.* **41** (1965) T14. Heat and mass transfer from a single sphere in an extensive flowing fluid.
56. BRIAN, P.L.T. and HAYES, H.B.: *A.I.Ch.E.Jl.* **15** (1969) 419. Effects of transpiration and changing diameter on heat and mass transfer.
57. GUPTA, A.S. and THODOS, G.: *A.I.Ch.E.Jl.* **9** (1963) 751. Direct analogy between heat and mass transfer in beds of spheres.
58. KRAMERS, H.: *Physica* **12** (1946) 61. Heat transfer from spheres to flowing media.

10.11. NOMENCLATURE

		Units in SI system	Dimensions in M, N, L, T, θ
A_p	External surface area of particle	m^2	L^2
a	Interfacial area per unit volume	m^2/m^3	L^{-1}
B_1, B_2	Integration constants	$kmol\ s/m^3$	$NL^{-3}T$
B'_1, B'_2	Integration constants	$kmol/m^3$	NL^{-3}
C	Molar concentration	$kmol/m^3$	NL^{-3}
C_A, C_B	Molar concentration of A, B	$kmol/m^3$	NL^{-3}
C_p	Specific heat at constant pressure	$J/kg\ K$	$L^2T^{-2}\theta^{-1}$
C_T	Total molar concentration	$kmol/m^3$	NL^{-3}
C_{Bm}	Logarithmic mean value of C_B	$kmol/m^3$	NL^{-3}
C'	$C_A - C_{A0}$	$kmol/m^3$	NL^{-3}
C''_A, C''_B	Molar concentration of A, B in film	$kmol/m^3$	NL^{-3}
\bar{C}'	Laplace transform of C'	$kmol\ s/m^3$	$NL^{-3}T$
c	Mass concentration	kg/m^3	ML^{-3}
D	Diffusivity	m^2/s	L^2T^{-1}
D_e	Effective diffusivity within catalyst particle	m^2/s	L^2T^{-1}
D_f	Diffusivity of fluid in film	m^2/s	L^2T^{-1}
DK_n	Knudsen diffusivity	—	L^2T^{-1}

		Units in SI system	Dimensions in M, N, L, T, θ
D_L	Liquid phase diffusivity	m^2/s	L^2T^{-1}
D_{AB}	Diffusivity of A in B	m^2/s	L^2T^{-1}
D_{Th}	Coefficient of thermal diffusion	$kmol/ms$	$NL^{-1}T^{-1}$
D_{BA}	Diffusivity of B in A	m^2/s	L^2T^{-1}
D'	Effective diffusivity in multicomponent system	m^2/s	L^2T^{-1}
d	Pipe diameter	m	L
d'	Diameter of sphere	m	L
e	Bed voidage	—	—
E_D	Eddy diffusivity	m^2/s	L^2T^{-1}
F	Coefficient in Maxwell's law of diffusion	$m^3/kmols$	$N^{-1}L^3T^{-1}$
f	Ratio $-N'_B/N'_A$	—	—
G	Molar flow of stream	$kmol/s$	NT^{-1}
G'	Molar flow per unit area	$kmol/m^2s$	$NL^{-2}T^{-1}$
	Ratio of equilibrium values of concentrations in two phases	—	—
H	Height of transfer unit	m	L
h	Heat transfer coefficient	$W/m^2 K$	$MT^{-3}\theta^{-1}$
h_D	Mass transfer coefficient	m/s	LT^{-1}
h'_D	Mass transfer coefficient enhanced by chemical reaction	m/s	LT^{-1}
\mathcal{H}	Henry's law constant, C''_{Ai}/C_{Ai} (equation 10.125)	—	—
j_d	j -factor for mass transfer	—	—
j_h	j -factor for heat transfer	—	—
K	Overall mass transfer coefficient	m/s	LT^{-1}
J	Flux as mass per unit area and unit time	kg/m^2s	$ML^{-2}T^{-1}$
K_{ABT}	Ratio of transport rate by thermal diffusion to that by Fick's law	—	—
k_G	Mass transfer coefficient for transfer through stationary fluid	$kmol s/kg m$	$NM^{-1}L^{-1}T$
k'_G	Mass transfer coefficient for equimolecular counterdiffusion	$kmol s/kg m$	$NM^{-1}L^{-1}T$
k_x	Mass transfer coefficient (mole fraction driving force)	$kmol/m^2s$	$NL^{-2}T^{-1}$
k	Reaction rate constant	first-order reaction s^{-1} n th -order reaction $kmol^{1-n} m^{n-3} s^{-1}$	T^{-1} $N^{1-n} L^{3n-3} T^{-1}$
L	Length of surface, or film thickness, or half-thickness of platelet or V_p/A_p	m	L
L	Thickness of film	m	L
L_f	Thickness of film	m	L
L_0	Length of surface, unheated	m	L
M	Molecular weight (Relative molecular mass)	$kg/kmol$	MN^{-1}
N	Molar rate of diffusion per unit area (average value)	$kmol/m^2s$	$NL^{-2}T^{-1}$
$(N)_t$	Molar rate of diffusion per unit area at time t	$kmol/m^2s$	$NL^{-2}T^{-1}$
N'	Total molar rate of transfer per unit area	$kmol/m^2s$	$NL^{-2}T^{-1}$
$(N)_{Th}$	Molar flux due to thermal diffusion	$kmol/m^2s$	$NL^{-2}T^{-1}$
N	Number of transfer units	—	—
n	Number of moles of gas	$kmol$	N
n	Order of reaction, or number of term in series	—	—
P	Total pressure	N/m^2	$ML^{-1}T^{-2}$
P_A, P_B	Partial pressure of A, B	N/m^2	$ML^{-1}T^{-2}$
P_s	Vapour pressure of water	N/m^2	$ML^{-1}T^{-2}$
P_w	Partial pressure of water in gas stream	N/m^2	$ML^{-1}T^{-2}$
P_{Bm}	Logarithmic mean value of P_B	N/m^2	$ML^{-1}T^{-2}$
p	Parameter in Laplace Transform	s^{-1}	T^{-1}
q	Concentration gradient dC_A/dy	$kmol/m^4$	NL^{-4}
R	Shear stress acting on surface	N/m^2	$ML^{-1}T^{-2}$

		Units in SI system	Dimensions in in M, N, L, T, θ
R	Universal gas constant	8314 J/kmol K	$MN^{-1}L^2T^{-2}\theta^{-1}$
r	Radius within sphere or cylinder	m	L
r_c	External radius of cylinder	m	L
r_0	External radius of sphere	m	L
\mathfrak{R}_p	Reaction rate per particle for first-order reaction	kmol/s	$N T^{-1}$
\mathfrak{R}_v	Reaction rate per unit volume of particle for first-order reaction	kmol/m ³ s	$N L^{-3}T^{-1}$
\mathfrak{R}'_{vn}	Reaction rate per unit volume of particle for n th-order reaction (no mass transfer resistance)	kmol/m ³ s	$N L^{-3}T^{-1}$
S	Cross-sectional area of flow	m ²	L ²
s	Rate of production of fresh surface per unit area	s ⁻¹	T ⁻¹
T	Absolute temperature	K	θ
t	Time	s	T
t_e	Time of exposure of surface element	s	T
u	Mean velocity	m/s	LT ⁻¹
u_c	Superficial velocity (volumetric flowrate/ total area)	m/s	LT ⁻¹
u_A, u_B	Mean molecular velocity in direction of transfer	m/s	LT ⁻¹
u_{DA}, u_{DB}	Diffusional velocity of transfer	m/s	LT ⁻¹
u_F	Velocity due to bulk flow	m/s	LT ⁻¹
u_m	Molecular velocity	m/s	LT ⁻¹
u_s	Stream velocity	m/s	LT ⁻¹
V	Volume	m ³	L ³
V_p	Volume of catalyst particle	m ³	L ³
V	Molecular volume	m ³ /kmol	$N^{-1}L^3$
V_o	Correction term in equation 10.96	m ³ /kmol	$N^{-1}L^3$
W	Mass rate of evaporation	kg/s	MT ⁻¹
x	Distance from leading edge of surface or in X-direction	m	L
	or mole fraction	—	—
y	Distance from surface or in direction of diffusion	m	L
y'	Mol fraction in stationary gas	—	—
Z	Height of column	m	L
z	Distance in Z-direction	m	L
α	Thermal diffusion factor	—	—
α'	Term in equation 10.234	—	—
β', β''	coefficient in equations 10.234 and 10.233	—	—
η	Effectiveness factor	—	—
λ	$\sqrt{(k/D_e)}$	s ⁻¹	L ⁻¹
λ_m	Mean free path of molecules	m	L
μ	Viscosity of fluid	N s/m ²	ML ⁻¹ T ⁻¹
ρ	Density of fluid	kg/m ³	ML ⁻³
ϕ	Friction factor ($R/\rho u^2$)	—	—
Φ	Thiele modulus based on L, r_c or r_0	—	—
Φ_L	Thiele modulus based on length term $L = V_p/A_p$	—	—
ψ	rC_A	kmol/m ²	NL ⁻²
ω	Mass fraction	—	—
Nu	Nusselt number hd/k	—	—
Nu'	Nusselt number for sphere hd'/k	—	—
Pr	Prandtl number $C_p\mu/k$	—	—
Re	Reynolds number $ud\rho/\mu$	—	—
Re'	Reynolds number for sphere $ud'\rho/\mu$	—	—
Re'_c	Reynolds number for particle in packed bed $u_c d'\rho/\mu$	—	—
Re_L	Reynolds number for flat plate $u_s L\rho/\mu$	—	—
Sc	Schmidt number $\mu/\rho D$	—	—
Sh'	Sherwood number for sphere $h_D d'/D$	—	—

Units in SI
system

Dimensions in
in **M, N, L, T, θ**

St Stanton number $h/C_p \rho u$

* Dimensions depend on order of reaction.

Suffixes

- 0 Value in bulk of phase
- 1 Phase 1
- 2 Phase 2
- A Component A
- B Component B
- AB Of A in B
- b Bottom of column
- e Value in equilibrium with bulk of other phase
- G Gas phase
- i Interface value
- L Liquid phase
- o Overall value (for height and number of transfer units)
or value in bulk of phase
- t Top of column