Lecture 4
Moments, skewness and kurtosis

Prof . Dr.Saad abed madhi

1. Mean (Average)

Example dataset (lesion sizes in mm):

$$X = [10, 12, 15, 18, 20]$$

Formula:

$$\bar{X} = \frac{1}{N} \sum X_i$$

Calculation:

$$\bar{X} = \frac{10 + 12 + 15 + 18 + 20}{5} = \frac{75}{5} = 15$$

Interpretation: The average lesion size is **15 mm**.

2. Mean Deviation (MD)

Dataset (CT attenuation in HU):

$$X = [40, 42, 45, 50, 55]$$

Formula:

$$MD = \frac{1}{N} \sum |X_i - \bar{X}|$$

Step 1: Find mean:

$$\bar{X} = \frac{40 + 42 + 45 + 50 + 55}{5} = 46.4$$

Step 2: Absolute deviations:

X_i	$X_i - \bar{X}$	$\lvert X_i - \bar{X} \rvert$
40	-6.4	6.4
42	-4.4	4.4
45	-1.4	1.4
50	3.6	3.6
55	8.6	8.6

$$MD = \frac{6.4 + 4.4 + 1.4 + 3.6 + 8.6}{5} = \frac{24.4}{5} = 4.88$$

Interpretation: Average deviation from mean is **4.88 HU**.

3. Variance and Standard Deviation

Dataset (MRI signal intensity):

$$X = [100, 110, 120, 130, 150]$$

Step 1: Mean:

$$\bar{X} = \frac{100 + 110 + 120 + 130 + 150}{5} = 122$$

Step 2: Deviations squared:

$X_i - \bar{X}$	$(X_i - \bar{X})^2$
-22	484
-12	144
-2	4
8	64
28	784

$$\sigma^2 = \frac{484 + 144 + 4 + 64 + 784}{5} = \frac{1480}{5} = 296$$

$$\sigma = \sqrt{296} \approx 17.2$$

Interpretation: MRI intensity varies with SD ≈ **17.2** units.

4. Skewness

Dataset (lesion volumes in mL):

$$X = [2, 3, 3, 4, 10]$$

Step 1: Mean:

$$\bar{X} = \frac{2 + 3 + 3 + 4 + 10}{5} = 4.4$$

Step 2: Standard deviation:

$$\sigma^2 = \frac{(2 - 4.4)^2 + (3 - 4.4)^2 + (3 - 4.4)^2 + (4 - 4.4)^2 + (10 - 4.4)^2}{5} = \frac{5.76 + 1.96 + 1.96 + 0.16 +}{5}$$

$$\sigma = \sqrt{8.24} \approx 2.87$$

Step 3: Skewness formula:

$$\text{Skewness} = \frac{\frac{1}{N}\sum(X_i - \bar{X})^3}{\sigma^3}$$

$X_i - \bar{X}$	$(X_i - \bar{X})^3$
-2.4	-13.824
-1.4	-2.744
-1.4	-2.744
-0.4	-0.064
5.6	175.616

$$\text{Skewness} = \frac{(-13.824 - 2.744 - 2.744 - 0.064 + 175.616)/5}{2.87^3} = \frac{156.24/5}{23.6} \approx 1.32$$

Interpretation: Right-skewed distribution due to **one large lesion**.

5. Kurtosis

Dataset (lesion diameters in mm):

$$X = [10, 12, 12, 13, 20]$$

Step 1: Mean:

$$\bar{X} = \frac{10 + 12 + 12 + 13 + 20}{5} = 13.4$$

Step 2: Std deviation:

$$\sigma^2 = \frac{(10 - 13.4)^2 + (12 - 13.4)^2 + (12 - 13.4)^2 + (13 - 13.4)^2 + (20 - 13.4)^2}{5}$$

$$\sigma^2 = \frac{11.56 + 1.96 + 1.96 + 0.16 + 43.56}{5} = 11.84$$

$$\sigma = \sqrt{11.84} \approx 3.44$$

Step 2: Std deviation:

$$\sigma^2 = \frac{(10-13.4)^2 + (12-13.4)^2 + (12-13.4)^2 + (13-13.4)^2 + (20-13.4)^2}{5}$$

$$\sigma^2 = \frac{11.56 + 1.96 + 1.96 + 0.16 + 43.56}{5} = 11.84$$

$$\sigma = \sqrt{11.84} \approx 3.44$$

Step 3: Fourth moment:

$X_i - \bar{X}$	$(X_i - \bar{X})^4$
-3.4	133.63
-1.4	3.84
-1.4	3.84
-0.4	0.0256
6.6	1897.0

$$\text{Kurtosis} = \frac{(133.63 + 3.84 + 3.84 + 0.0256 + 1897)/5}{3.44^4} - 3 = \frac{407.87}{140.2} - 3 \approx 2.91 - 3 \approx -0.09$$

Interpretation: Slightly **platykurtic**, nearly normal.

6. Summary Table

Measure	Dataset Example	Value	Interpretation
Mean	Lesion sizes [10,12,15,18,20]	15 mm	Average size
Mean Deviation	CT HU [40,42,45,50,55]	4.88 HU	Average deviation
Variance/SD	MRI [100,110,120,130,150]	296 / 17.2	Spread of intensities
Skewness	Lesion volume [2,3,3,4,10]	1.32	Right-skewed
Kurtosis	Diameters [10,12,12,13,20]	-0.09	Nearly normal, slightly flat

Exercises and Solutions: Radiology Data Analysis

Exercise on Moments, skewness and kurtosis

1. Data (mm): 8, 12, 15, 20, 25 .Calculate the mean lesion size.
2. Data (CT HU): 40, 45, 50, 55, 60 .Calculate mean deviation from the mean.
3. Data (MRI intensity): 100, 110, 120, 130, 140 .Calculate variance and standard deviation.
4. Data (lesion volume in mL): 1, 2, 2, 3, 10 .Calculate skewness.
5. Data (lesion diameters in mm): 5, 6, 6, 7, 15 .Calculate excess kurtosis.
6. Data (CT HU of liver): 45, 50, 55, 60, 65. Calculate mean, mean deviation, variance, SD.
7. Data (lesion size in mm): 2, 3, 3, 4, 12 .Determine skewness.
8. Data (lesion diameter in mm): 5, 6, 6, 6, 7. Calculate kurtosis.
9. Data (mL): 0.5, 0.8, 1, 1.2, 1.5 .Compute variance and SD.
10. Data (lesion volumes in mL): 1, 1, 2, 2, 8. Calculate skewness and kurtosis.

MCQ on Moments, skewness and kurtosis

- **1. The mean of lesion sizes [10, 12, 15, 20, 25] is:**
- A) 16 mm
  B) 17 mm
  C) 18 mm
  D) 19 mm
  E) 15 mm
- **2. Which measure is least affected by extreme values?**
- A) Mean
  B) Median
  C) Variance
  D) Standard deviation
  E) Skewness
- **3. The variance of data [5, 7, 9, 11, 13] is:**
- A) 8
  B) 10
  C) 6
  D) 4
  E) 12
- **4. Which measure describes asymmetry of a distribution?**
- A) Mean
  B) Kurtosis
  C) Skewness
  D) Variance
  E) Mean deviation

- **5. Kurtosis measures:**
- A) Center of distribution
  B) Spread of data
  C) Peakedness or tail weight
  D) Median
  E) Mode
- **6. Mean deviation is calculated using:**
- A) Squared deviations from mean
  B) Absolute deviations from mean
  C) Cubic deviations
  D) Fourth power deviations
  E) Ratio of max/min
- **7. A right-skewed distribution has:**
- A) Skewness = 0
  B) Skewness < 0
  C) Skewness > 0
  D) Kurtosis > 3
  E) Kurtosis < 0
- **8. A platykurtic distribution is:**
- A) Sharply peaked
  B) Flat-topped
  C) Symmetric
  D) Right-skewed
  E) Left-skewed

- **9. The standard deviation is:**
- A) Square of variance
  B) Square root of variance
  C) Mean deviation
  D) Difference between max and min
  E) Sum of deviations
- **10. In radiology, histograms are used to:**
- A) Calculate variance
  B) Show frequency distribution of measurements
  C) Compute skewness formula
  D) Find mean deviation
  E) Measure kurtosis
- **11. If a dataset has one extremely large value, the mean will:**
- A) Decrease
  B) Remain same
  C) Increase
  D) Become median
  E) Become mode

- **12. For the dataset [2, 4, 6, 8, 10], the mean deviation is:**
- A) 2
  B) 2.4
  C) 2.5
  D) 3
  E) 1.6
- **13. Which measure is useful for detecting outliers?**
- A) Mean
  B) Variance
  C) Skewness
  D) Mode
  E) Median
- **14. The sum of deviations from the mean is always:**
- A) Positive
  B) Negative
  C) Zero
  D) Equal to variance
  E) Equal to SD
- **15. The mean of [10, 15, 20, 25, 30] is:**
- A) 18
  B) 20
  C) 22
  D) 25
  E) 15
- **16. Which measure can be negative?**
- A) Mean
  B) Variance
  C) Skewness
  D) Standard deviation
  E) Mean deviation

- **17. Excess kurtosis of a normal distribution is:**
- A) 1
  B) 0
  C) -1
  D) 3
  E) -3
- **18. A dataset with a very sharp peak and heavy tails is:**
- A) Platykurtic
  B) Leptokurtic
  C) Symmetric
  D) Left-skewed
  E) Right-skewed
- **19. In radiology, which measure summarizes average deviation of pixel intensity from mean?**
- A) Variance
  B) Mean deviation
  C) Skewness
  D) Kurtosis
  E) Mode
- **20. Which of the following is NOT a measure of central tendency or dispersion?**
- A) Mean
  B) Median
  C) Mode
  D) Skewness
  E) Variance