Lecture 2
Central Tendency Measures

- I’ll use the same dataset for all examples: 2, 4, 4, 6, 8.

1. Arithmetic mean (average)

- Definition: Sum of the values divided by how many there are.
- Formula: mean = (x1 + x2 + … + xn) / n
- Example: (2 + 4 + 4 + 6 + 8) / 5 = 24 / 5 = 4.8

2. Median (middle value)

- Definition: The middle value when the data are sorted. If there are an even number of values, average the two middle ones.
- Example: Sorted: 2, 4, 4, 6, 8 → middle (3rd) value is 4 → median = 4
- Even-count note: For 2, 4, 6, 8 → median = (4 + 6) / 2 = 5

3. Mode (most frequent value)

- Definition: The value that appears most often. There can be more than one mode (bimodal, multimodal), or none if all counts are equal.
- Example: 4 appears twice; others once → mode = 4

4. Geometric mean (multiplicative average)

- Definition: The nth root of the product of n positive values. Use for growth rates and ratios.
- Formula: GM = (x1 × x2 × … × xn)^(1/n) [requires all values > 0]
- Example: Product = 2 × 4 × 4 × 6 × 8 = 1536 → GM = 1536^(1/5) ≈ 4.34

5. Harmonic mean (reciprocal average)

- Definition: The number of values divided by the sum of reciprocals. Great for averaging rates (like speed) over equal distances. Values must be nonzero.
- Formula: $HM = n / (\Sigma\ 1/xi)$
- Example: $HM = 5 / (1/2 + 1/4 + 1/4 + 1/6 + 1/8)$

$= 5 / (0.5 + 0.25 + 0.25 + 0.1667 + 0.125)$

$= 5 / 1.2916667 = 120/31 \approx 3.871$

6. Root mean square (RMS, quadratic mean)

- Definition: Square the values, average them, then take the square root. Weights larger values more.
- Formula: RMS = sqrt((Σ xi^2) / n)
- Example: Squares: 4, 16, 16, 36, 64 → sum = 136
- Mean of squares = 136/5 = 27.2 → RMS = sqrt(27.2) ≈ 5.215
- Quick comparison on this dataset (all positive): HM < GM < mean < RMS, and median/mode sit at 4.
- So: 3.871 < 4.34 < 4.8 < 5.215, with median = mode = 4.

Examples

1. Arithmetic mean (average)

- Example1: CT liver ROI Hounsfield Units (HU)
- Data (HU): 52, 55, 58, 53, 54, 57
- Sum = 329; n = 6
- Mean = 329 / 6 ≈ 54.83 HU

- Example2: Average DLP over four chest CTs
- DLP (mGy·cm): 280, 300, 320, 260
- Sum = 1160; n = 4
- Mean = 1160 / 4 = 290 mGy·cm

2. Median (middle value)

- Example1: CT ROI HU with a calcification outlier
- Data (HU): 52, 53, 54, 55, 56, 200
- Sorted already; middle two are 54 and 55
- Median = (54 + 55) / 2 = 54.5 HU
- Note how the 200 HU outlier barely affects the median.

3. Mode (most frequent value)

- Example: Dominant intensity in a small CT ROI
- Data (HU): 50, 50, 52, 52, 52, 54
- 52 appears 3 times (others fewer)
- Mode = 52 HU

4. Geometric mean (multiplicative average)

- Example 1: Nuclear medicine conjugate views (anterior/posterior counts)
- Left kidney counts: Anterior = 4000, Posterior = 2500
- Geometric mean count = sqrt(4000 × 2500) = sqrt(10,000,000) ≈ 3162 counts

- Example 2: Average multiplicative change in lesion SUV across intervals
- - Interval factors: +20%, −10%, +5% → 1.20, 0.90, 1.05
- Product = 1.20 × 0.90 × 1.05 = 1.134
- GM = 1.134^(1/3) ≈ 1.043 → average change ≈ +4.3% per interval
- Use GM for growth/decay rates (all factors must be > 0).

5. Harmonic mean (rate average over equal distances/work)

- Example: Doppler ultrasound velocities over equal-length vessel segments
- Velocities (cm/s) over three equal 1-cm segments: 20, 30, 60
- HM = 3 / (1/20 + 1/30 + 1/60) = 3 / (0.05 + 0.0333 + 0.0167) = 3 / 0.10 = 30 cm/s
- Harmonic mean is the correct “overall speed” when distances are equal.

6. Root mean square (RMS, quadratic mean)

- Example 1: RMS of MRI signal within a small ROI (intensity “energy”)
- Intensities: 120, 122, 121, 124, 118
- Squares: 14,400; 14,884; 14,641; 15,376; 13,924 → sum = 73,225
- Mean of squares = 73,225 / 5 = 14,645
- RMS = sqrt(14,645) ≈ 121.0

- Example 2: RMSE of lesion diameter measurements vs. reference (same RMS idea on errors)
- Reference (mm): 20, 25, 30, 35
- Measured (mm): 22, 24, 29, 40
- Errors: +2, −1, −1, +5 → squared: 4, 1, 1, 25 → sum = 31
- RMSE = sqrt(31 / 4) = sqrt(7.75) ≈ 2.78 mm

additional detailed worked examples

- 1. Arithmetic Mean
- Example: A student scored 85, 90, 78, 92, and 88 on five exams. What is their average score? Solution:
- Sum = 85 + 90 + 78 + 92 + 88 = 433
- Number of exams (n) = 5
- Mean = 433 ÷ 5 = 86.6

- 2. Median
- Example: Find the median of: 12, 15, 9, 20, 18, 10, 14.
  Solution:
- Step 1: Sort the data → 9, 10, 12, 14, 15, 18, 20
- Step 2: n = 7 (odd) → Median = 4th term = 14
- Even-numbered case: Data = 5, 11, 3, 8
- Sorted: 3, 5, 8, 11
- Median = (5 + 8) ÷ 2 = 6.5

- 3. Mode
- Example: Ages of participants in a workshop: 22, 25, 22, 30, 25, 22, 28, 25, 25
  Solution:
- Frequency count:
  - 22 → 3 times
  - 25 → 4 times
  - 28 → 1 time
  - 30 → 1 time
- Mode = 25 (highest frequency)
- *Note*: This data is unimodal (one mode).
- Bimodal example: 4, 6, 6, 7, 8, 8, 9
- 6 and 8 each appear twice → Modes = 6 and 8

- 4. Geometric Mean
- Example: A company's revenue grew by factors of 1.2, 1.5, and 2.0 over three years. What is the average growth factor?
  Solution:
- Use geometric mean for multiplicative growth.
- GM = $\sqrt[3]{(1.2 \times 1.5 \times 2.0)} = \sqrt[3]{(3.6)} \approx 1.53$
- Interpretation: Average annual growth factor ≈ 1.53 (or 53% per year)
- Another example: Find GM of 1, 4, 16
- Product = $1 \times 4 \times 16 = 64$
- GM = $\sqrt[3]{64} = 4$

- 5. Harmonic Mean
- Example: A car travels 100 km at 50 km/h and another 100 km at 25 km/h. What is the average speed for the entire 200 km trip?
  Solution:
- Since distances are equal, use harmonic mean of speeds.
- HM = 501+2512=501+5022=5032=3100≈**33.33 km/h**
- *Note*: Arithmetic mean (37.5 km/h) would be incorrect here.
- Simple HM: Find HM of 2, 4, 8
- Reciprocals: 1/2, 1/4, 1/8 → Sum = 0.5 + 0.25 + 0.125 = 0.875
- HM = 3 / 0.875 = 3.43 (approx)

- 6. Root Mean Square (RMS)
- Example: Find the RMS of the values: –2, 0, 2, 4
  Solution:
- Step 1: Square each value → $(-2)^2 = 4$, $0^2 = 0$, $2^2 = 4$, $4^2 = 16$
- Step 2: Sum of squares = 4 + 0 + 4 + 16 = 24
- Step 3: Mean of squares = 24 ÷ 4 = 6
- Step 4: RMS = $\sqrt{6} \approx 2.45$
- *Note*: RMS handles negative values correctly because squaring removes the sign.
- Physics application: AC voltage values over time: 0, 5, 0, –5, 0
- Squares: 0, 25, 0, 25, 0 → Sum = 50
- Mean of squares = 50 / 5 = 10
- RMS = $\sqrt{10} \approx 3.16$ V (this is the effective voltage)

multiple-choice questions (MCQs) on measures of central tendency

- 1. Which measure of central tendency is most affected by extreme values (outliers)?
- A) Median
  B) Mode
  C) Geometric Mean
  D) Harmonic Mean
  E) Arithmetic Mean

- 2. The median is best described as a:
- A) Mathematical average
  B) Positional average
  C) Reciprocal average
  D) Multiplicative average
  E) Quadratic average
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  3. Which of the following cannot be calculated if any value in the data set is zero?
- A) Arithmetic Mean
  B) Median
  C) Mode
  D) Geometric Mean
  E) Both D and Harmonic Mean

- 4. For the data set: 3, 5, 7, 9, 11, what is the median?
- A) 5
  B) 6
  C) 7
  D) 8
  E) 9

- 5. What is the mode of: 2, 4, 4, 6, 8, 8, 8?
- A) 2
  B) 4
  C) 6
  D) 8
  E) No mode

- 6. The geometric mean of 2 and 8 is:
- A) 3
  B) 4
  C) 5
  D) 6
  E) 10

- 7. The harmonic mean of 4 and 4 is:
- A) 2
  B) 4
  C) 8
  D) 16
  E) Undefined

- 8. Which average is most appropriate for calculating average speed over equal distances?
- A) Arithmetic Mean
  B) Median
  C) Mode
  D) Geometric Mean
  E) Harmonic Mean

- 9. RMS of the values 0, 3, –3 is:
- A) 0
  B) 2
  C) √6
  D) 3
  E) 6

- 10. In a perfectly symmetrical distribution, which is true?
- A) Mean > Median > Mode
  B) Mean < Median < Mode
  C) Mean = Median = Mode
  D) Mode = 2 × Median – Mean
  E) None of the above

- 11. Which measure can be used for nominal data?
- A) Mean
  B) Median
  C) Mode
  D) Geometric Mean
  E) Harmonic Mean

- 12. The arithmetic mean of 10, 20, 30, 40 is:
- A) 20
  B) 25
  C) 30
  D) 35
  E) 100

- 13. If all values in a data set are the same (e.g., 5, 5, 5), then:
- A) Mean = Median = Mode
  B) GM < HM
  C) RMS = 0
  D) Median is undefined
  E) Mode does not exist

- 14. Which of the following is always less than or equal to the arithmetic mean for positive numbers?
- A) Median
  B) Mode
  C) Geometric Mean
  D) RMS
  E) None

- 15. The harmonic mean is most useful when dealing with:
- A) Heights of students
  B) Test scores
  C) Temperature readings
  D) Fuel efficiency (km/l)
  E) Shoe sizes

- 16. What is the geometric mean of 1, 2, 8?
- A) 2
  B) 2.5
  C) $\sqrt[3]{16} \approx 2.52$
  D) 4
  E) 11/3

- 17. Which measure is NOT a measure of central tendency?
- A) Mean
  B) Median
  C) Mode
  D) Standard Deviation
  E) Geometric Mean

- 18. For data: 1, 1, 2, 3, 5, 8, the median is:
- A) 2
  B) 2.5
  C) 3
  D) 3.5
  E) 5

- 19. RMS is always:
- A) Less than the mean
  B) Equal to the mean
  C) Greater than or equal to the arithmetic mean
  D) Negative
  E) Zero

- 20. Which pair of averages cannot be computed if one observation is zero?
- A) Mean and Median
  B) Median and Mode
  C) Geometric Mean and Harmonic Mean
  D) Arithmetic Mean and RMS
  E) Mode and RMS