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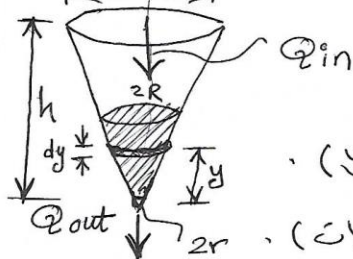
Application of First order Differential equations

أولاً :- حساب التسرب من الخزانات باستخدام معادلات تورشلي :-

إذا كان الخزان مملوء بالماء وهذا الخزان يحتوي على ثقب
فإن الماء يبدأ بالتسرب خلال الزمن وبمرور الزمن يبدأ عمق الماء
داخل الخزان بالنقصان لذلك فإن تورشلي وضع معادلة يتم من
حدها حساب مقدار ارتفاع الماء داخل الخزان المكتوب مع الزمن .

$$A(y) \frac{dy}{dt} = \sum Q_{in} - \sum Q_{out}$$

معادلة تورشلي .



حيث أن :-

$A(y)$:- مساحة الشريحة عند العمق (y) .

Q_{in} :- التصريف الداخل إلى الخزان (التورشلي) .

Q_{out} :- التصريف الخارج من الخزان ويحسب من خلال المعادلة
الآتية :-

$$Q_{out} = \pi r^2 \sqrt{2gy}$$

حيث أن :-

r :- نصف قطر الفتحة التي يتسرب منها الماء .

g :- التجهيل الأرضي ويساوي :-

١- إذا كانت الانحدار الكعك بالتر :- $g = 9.8 \text{ m/sec}^2$

٢- إذا كانت الانحدار الكعك بالقدم :-

$$g = 32.2 \text{ m/sec}^2$$



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Ex(1) :- A cylindrical tank with radius of (2m) and height of 4m has initially filled with water. In the bottom of the tank there is a hole with diameter of (2cm) In which the water drain under the influence of gravity. Find :-

- (1) The depth of water at any time.
- (2) The time of reach the height of water one meter.
- (3) The time to empty the tank.

Solution :-

$$A(y) = \pi r^2$$

$$= 4\pi \text{ (مساحة الشريحة)}$$

$$A(y) \frac{dy}{dt} = Q_{in} - Q_{out}$$

$$4\pi \frac{dy}{dt} = 0 - \pi (0.01)^2 \sqrt{2 * 9.81 * y}$$

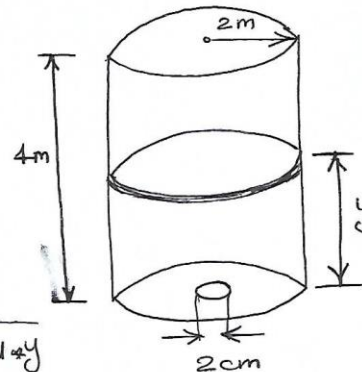
$$4 \frac{dy}{dt} = - (0.01)^2 \sqrt{19.6} \sqrt{y}$$

$$\therefore 4 \frac{dy}{dt} = - 4.427 * 10^{-4} \sqrt{y}$$

$$\frac{4}{4.427 * 10^{-4} \sqrt{y}} dy = - dt \quad (\text{by separable}).$$

$$\int \frac{4}{4.427 * 10^{-4} \sqrt{y}} dy = - \int dt$$

$$\frac{4}{4.427 * 10^{-4}} \int (y)^{-1/2} dy = - \int dt$$





$$18070.9 \sqrt{y} = -t + C$$

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To find c use (Boundary Condition)

$$\text{at } t=0 \quad y=H=4\text{m}$$

$$18070.9 \sqrt{4} = -(0) + C$$

$$\Rightarrow C = 36141.8$$

$$18070.9 \sqrt{y} = 36141.8 - t \quad (1)$$

$$\Rightarrow t = 18070.9 \text{ sec when } y = 1 \quad (2)$$

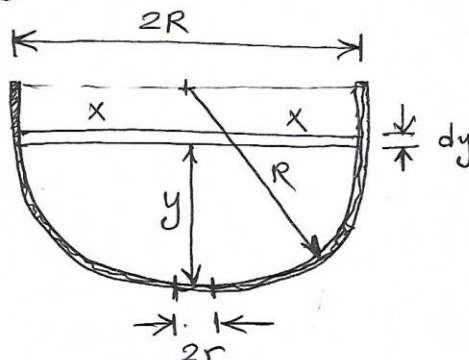
$$18070.9 \sqrt{0} = 36141.8 - t \Rightarrow t = 36141.8 \text{ sec when } y = 0$$

Ex (2) :- A hemispherical tank of radius ($R=3\text{m}$) in initially filled with water. At the bottom of the tank, there is a hole of radius ($r=1\text{cm}$) through which the water drains under the influence of gravity. Find the depth of water in the tank at any time t , and determine how long it will take the tank to drain completely.

Solution :-

$$A(y) \frac{dy}{dt} = Q_{in} - Q_{out}$$

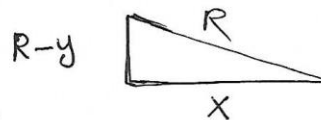
$$A(y) = \pi x^2 \quad (1)$$





$$X^2 + (R-y)^2 = R^2 \quad (33)$$

$$X^2 = R^2 - (R^2 - 2Ry + y^2)$$



$$X^2 = 2Ry - y^2 \Rightarrow X^2 = 6y - y^2$$

sub.in eq. (1)

$$\therefore A(y) = \pi (6y - y^2),$$

$$Q_{in} = 0$$

$$Q_{out} = \pi r^2 \sqrt{2gy} = \pi (0.01)^2 \sqrt{2 \times 9.81 \times y}$$

$$= 4.429 \times 10^{-4} y^{1/2}.$$

$$\therefore \cancel{\pi} (6y - y^2) \cdot \frac{dy}{dt} = 0 - 4.429 \times 10^{-4} \cancel{\pi} y^{1/2}$$

$$(6y - y^2) \frac{dy}{dt} = -4.429 \times 10^{-4} y^{1/2}$$

$$\frac{(6y - y^2)}{y^{1/2}} dy = -4.429 \times 10^{-4} dt$$

$$y^{-1/2} (6y - y^2) dy = -4.429 \times 10^{-4} dt$$

$$\int (6y^{1/2} - y^{3/2}) dy = \int -4.429 \times 10^{-4} dt$$

$$6 \frac{y^{3/2}}{3/2} - \frac{y^{5/2}}{5/2} = -4.429 \times 10^{-4} t + C$$

$$4 y^{3/2} - \frac{2}{5} y^{5/2} = -4.429 \times 10^{-4} t + C$$

Apply boundary condition at $t=0 \Rightarrow y=h=R=3m$



$$4 (3)^{3/2} - \frac{2}{5} (3)^{5/2} = -4.429 \cdot 10^{-4} \cdot 0 + C \Rightarrow (34)$$

$$C = 14.549 .$$

$$\therefore 4 y^{3/2} - \frac{2}{5} y^{5/2} = -4.429 \cdot 10^{-4} t + 14.549 .$$

at any time.

The time to empty the tank at $y=0$:-

$$\therefore 4 (0)^{3/2} - \frac{2}{5} (0)^{5/2} = -4.429 \cdot 10^{-4} t + 14.549$$

$$t = 32849.9 \text{ sec.}$$