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### Exact First order differential equation :-

الصيغة العامة :-

$$M(x,y) dx + N(x,y) dy = 0$$

ان هذه المعادلة تكون (Exact) اذا كان

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

وخلاف ذلك فان المعادلة تكون غير (Exact)  
بعد اختبار المعادلة كانت (Exact) فيكون الحل كالتالي :-

$$\int_a^x M(x,y) dx + \int_b^y N(x,y) dy = \int_0$$

قبل اجراء التكامل اعلاه يجب ان نعوض في احد المتغيرين (  $\int N(x,y) dy$  )  
بدل كل (x) بـ (a) ثم يجري التكامل .

Ex(1) :- show that the equation is exact and solve :-

$$(2x+3y-2) dx + (3x-4y+1) dy = 0$$

$$M(x,y) = 2x+3y-2 \Rightarrow \frac{\partial M}{\partial y} = 3.$$

$$N(x,y) = 3x-4y+1 \Rightarrow \frac{\partial N}{\partial x} = 3.$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3 \quad \text{Exact.}$$

$$\int_a^x M dx + \int_b^y N dy = c$$

$$\int_a^x (2x+3y-2) dx + \int_b^y (3a-4y+1) dy = 0$$



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$$\left[ x^2 + 3yx - 2x \right]_a^x + \left[ 3ay - 2y^2 + y \right]_b^y = 0$$
$$\left[ (x^2 + 3yx - 2x) - (a^2 + 3ya - 2a) \right] + \left[ (3ay - 2y^2 + y) - (3ab - 2b^2 + b) \right] = 0$$

$$x^2 + 3xy - 2x - a^2 - 3ay + 2a + 3ay - 2y^2 + y - 3ab + 2b^2 - b = 0$$

$$x^2 + 3xy - 2x - 2y^2 + y = a^2 - 2a + 3ab - 2b^2 - b$$
$$\therefore \boxed{x^2 + 3xy - 2x - 2y^2 + y = k} \quad \text{where} \quad k = a^2 - 2a + 3ab - 2b^2 - b$$

Ex(2) :- show that the equation is exact and solve :-

$$\left( \frac{1}{x^2} + \frac{2y}{x^3} \right) dx - \frac{1}{x^2} dy = 0$$

$$M = \frac{1}{x^2} + \frac{2y}{x^3} \Rightarrow \frac{\partial M}{\partial y} = \frac{2}{x^3}$$

$$N = -\frac{1}{x^2} \Rightarrow \frac{\partial N}{\partial x} = +\frac{2}{x^3}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{2}{x^3} \Rightarrow \text{Exact.}$$

$$\int_a^x \left( \frac{1}{x^2} + \frac{2y}{x^3} \right) dx + \int_b^y -\frac{1}{x^2} dy = 0$$

$$\left[ \left( -\frac{1}{x} - \frac{y}{x^2} \right) - \left( -\frac{1}{a} - \frac{y}{a^2} \right) \right] - \left[ \left( \frac{y}{a^2} \right) - \left( \frac{b}{a^2} \right) \right] = 0$$



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$$-\frac{1}{x} - \frac{y}{x^2} + \frac{1}{a} + \frac{y}{a^2} - \frac{y}{a^2} + \frac{b}{a^2} = 0$$

$$\frac{1}{x} + \frac{y}{x^2} = \frac{1}{a} + \frac{b}{a^2} \Rightarrow \frac{1}{x} + \frac{y}{x^2} = k$$

$$k = \frac{1}{a} + \frac{b}{a^2}.$$

Ex(3): show that the equation is exact and solve it

$$(2xy^4 + \sin y) dx + (4x^2y^3 + x \cos y) dy = 0$$

$$M = 2xy^4 + \sin y \Rightarrow \frac{\partial M}{\partial y} = 8xy^3 + \cos y.$$

$$N = 4x^2y^3 + x \cos y \Rightarrow \frac{\partial N}{\partial x} = 8xy^3 + \cos y.$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 8xy^3 + \cos y \Rightarrow \text{Exact.}$$

$$\int_a^x (2xy^4 + \sin y) dx + \int_b^y (4x^2y^3 + x \cos y) dy = 0$$

$$\left[ x^2 y^4 + x \sin y \right]_a^x + \left[ a^2 y^4 + a \sin y \right]_b^y = 0$$

$$\left[ (x^2 y^4 + x \sin y) - (a^2 y^4 + a \sin y) \right] + \left[ (a^2 y^4 + a \sin y) - (a^2 b^4 + a \sin b) \right] = 0$$



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$$x^2 y^4 + x \sin y - a^2 y^4 - a \sin y + a^2 y^4 + a \sin y - a^2 b^2 - a \sin b = 0$$

$$x^2 y^4 + x \sin y = a^2 b^4 + a \sin b$$

$$x^2 y^4 + x \sin y = k \quad \boxed{k = a^2 b^4 + a \sin b}$$

How & show that the following equations are exact and solve each one &

1-  $(y^2 - 1) dx + (2xy - \sin y) dy = 0$ .

Ans.

$$y^2 x - x + \cos y = k.$$

2-  $(2x e^y + e^x) dx + (x^2 + 1) e^y dy = 0$ .

Ans.

$$x^2 e^y + e^x + e^y = k.$$

3-  $\frac{y \ln x^2 dy + \frac{y^2}{x} dx}{x \ln y^2 dx + \frac{x^2}{y} dy} = 1$ .

Ans.  $y^2 \ln x - \frac{x^2}{2} \ln y^2 = k.$