



المحاضرة : Non- Exact First Order Differential Equation

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Non- Exact Differential Equation

المعادلة التفاضلية غير تامة

The equation which is not exact can be made exact by multiplying it by the integrating factor (I.F) :-

$$M(x,y) dx + N(x,y) dy = 0$$

يُضرب إذا كان $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ ، فإنها غير تامة

إذا كانت المعادلة التفاضلية غير تامة (Non - exact) فإننا نستخدم جعلها تامة بواسطة الضرب بعامل التكامل (I.F) وهو عامل نضرب به المعادلة الغير تامة فيحولها الى معادلة تامة.

هنا يكون لدينا حالتين

$$(1) f(x) = \frac{1}{N} \left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right]$$

$$I.F = e^{\int f(x) dx}$$

$$(2) f(y) = \frac{1}{M} \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right]$$

$$I.F = e^{\int f(y) dy}$$



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Ex(1) :- Find a general solution for $y \cdot dx + (3+3x-y) dy = 0$?

$$\begin{aligned} M(x,y) &= y \Rightarrow \frac{\partial M}{\partial y} = 1 \\ N(x,y) &= (3+3x-y) \Rightarrow \frac{\partial N}{\partial x} = 3 \end{aligned} \quad \left. \vphantom{\begin{aligned} M(x,y) &= y \\ N(x,y) &= (3+3x-y) \end{aligned}} \right\} \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

∴ Non-Exact.

⇒ Find integrating factor ⇒

$$f(x) = \frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$= \frac{1}{3+3x-y} (1-3) = \frac{-2}{3+3x-y}$$

$$\begin{aligned} (I.F)_x &= e^{\int f(x) dx} = e^{\int \frac{-2}{3+3x-y} dx} \\ &= e^{-\frac{2}{3} \ln(3+3x-y)} = e^{\ln(3+3x-y)^{-2/3}} \\ &= (3+3x-y)^{-2/3} \end{aligned}$$

يسهل هذا العمل لأنه دالة في x و y



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$$f(y) = \frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$$
$$= \frac{1}{y} (3 - 1) = \frac{2}{y}$$

$$\therefore (I.F)_y = e^{\int f(y) dy} = e^{\int \frac{2}{y} dy}$$
$$= e^{2 \ln y} = e^{\ln y^2} = y^2.$$

$\therefore y^2$ is integrating factor.

$$\Rightarrow y \cdot dx + (3 + 3x - y) \cdot dy = 0 \quad] * y^2$$

$$y^3 \cdot dx + (3y^2 + 3xy^2 - y^3) \cdot dy = 0$$

$$M(x, y) = y^3 \Rightarrow \frac{\partial M}{\partial y} = 3y^2.$$

$$N(x, y) = 3y^2 + 3xy^2 - y^3 \Rightarrow \frac{\partial N}{\partial x} = 3y^2.$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3y^2 \Rightarrow \therefore \text{Exact.}$$

To solve the equation

$$\int_a^x y dx + \int_b^y (3 + 3x - y) dy = 0$$



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$$\int_a^x y^3 dx + \int_b^y (3y^2 + 3xy^2 - y^3) dy = 0$$

$$\left[y^3 x \right]_a^x + \left[\cancel{\frac{3y^3}{3}} + \cancel{\frac{3ay^3}{3}} - \frac{y^4}{4} \right]_b^y = C$$

$$\left[y^3 x - y^3 a \right] + \left[y^3 + ay^3 - \frac{y^4}{4} - b^3 - ab^3 + \frac{b^4}{4} \right] = C$$

$$\cancel{y^3 x} - \cancel{y^3 a} + y^3 + \cancel{ay^3} - \frac{y^4}{4} - b^3 - ab^3 + \frac{b^4}{4} = C$$

$$-\frac{y^4}{4} + y^3 x + y^3 + \left(\frac{b^4}{4} - b^3 - ab^3 \right) = C$$

$$-\frac{y^4}{4} + y^3 x + y^3 + K = 0$$

$$K = \frac{b^4}{4} - b^3 - ab^3$$

Ex(2) : Find a general solution for

$$(x^2 + y^2 + x) dx + xy dy = 0$$

$$M(x, y) = x^2 + y^2 + x \Rightarrow \frac{\partial M}{\partial y} = 2y$$

$$N(x, y) = xy \Rightarrow \frac{\partial N}{\partial x} = y$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow \infty \text{ Non-exact.}$$



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We must find I.F

$$f(x) = \frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) = \frac{1}{xy} (2y - y) = \frac{y}{xy} = \frac{1}{x}$$

$$(I.F)_x = e^{\int f(x) dx} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$\circ (x^2 + y^2 + x) dx + xy dy = 0 \quad] * x$$

$$(x^3 + xy^2 + x^2) dx + x^2 y dy = 0$$

$$\left. \begin{array}{l} \frac{\partial M}{\partial y} = 2xy \\ \frac{\partial N}{\partial x} = 2xy \end{array} \right\} \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \circ \text{ exact.}$$

$$\int_a^x (x^3 + xy^2 + x^2) dx + \int_b^y x^2 y dy = 0$$

$$\left[\frac{x^4}{4} + y^2 \frac{x^2}{2} + \frac{x^3}{3} \right]_a^x + \left[\frac{a^2 y^2}{2} \right]_b^y = 0$$

$$\frac{x^4}{4} + y^2 \frac{x^2}{2} + \frac{x^3}{3} - \left(\frac{a^4}{4} + \cancel{y^2 \frac{a^2}{2}} + \frac{a^3}{3} \right)$$

$$+ \cancel{a^2 \frac{y^2}{2}} - a^2 \frac{b^2}{2} = 0$$



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$$\frac{x^4}{4} + y^2 \frac{x^2}{2} + \frac{x^3}{3} - \frac{a^4}{4} - \frac{a^3}{3} - a^2 \frac{b^3}{2} = 0 \quad (25)$$

$$\left(\frac{x^4}{4} + y^2 \frac{x^2}{2} + \frac{x^3}{3} \right) - k = 0$$

$$k = \frac{a^4}{4} + \frac{a^3}{3} + a^2 \frac{b^3}{2}$$

H.W Find a general solution for

$$(2xy^4 e^y + 2xy^3 + y) dx + (x^2 y^4 e^y - x^2 y^2 - 3x) dy$$

