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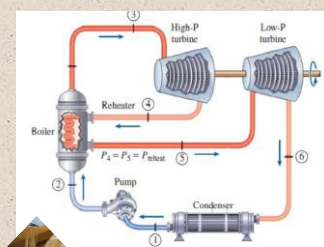
Thermodynamic II

LECTURE 6
High speed flow



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High speed flow

- A compressible flow is that flow in which the density of the fluid changes during flow.
- All real fluids are compressible to some extent and therefore their density will change with change in pressure or temperature.
- If the relative change in density $\Delta\rho/\rho$ is small, the fluid can be treated as incompressible.
- A compressible fluid, such as air, can be considered as incompressible with constant ρ if changes in elevation are small, acceleration is small, and/or temperature changes are negligible.
- In other words, if Mach's number U/C, where C is the sonic velocity, is small, compressible fluid can be treated as incompressible.

- The gases are treated as compressible fluids and study of this type of flow is often referred to as 'Gas dynamics'.

- Some important problems where compressibility effect has to be considered are:
  - (i) Flow of gases through nozzles, orifices;
  - (ii) Compressors;
  - (iii) Flight of aero planes and projectiles moving at higher altitudes;
  - (iv) Water hammer and acoustics.

- Compressibility' affects the drag coefficients of bodies by formation of shock waves, discharge coefficients of measuring devices such as orifice meters, venturi meters and pitot tubes, stagnation pressure and flows in converging-diverging sections.

The Mach number

- The theory of high-speed flow is concerned with flows of fluid at speeds high enough that account must be taken of the fluid's compressibility. The theory finds application in many branches of science and technology, from which we may single out, as being of unrivalled importance in the modern world, the applications to high-speed flight.
- The dimensionless parameter that measures the importance of a fluid's compressibility in high-speed flow is the *Mach number*. Suppose that, at a given point in space and time, the speed of the fluid is u and the speed of sound is c. The Mach number M is defined as the ratio u/c. Thus

$$M = \frac{u}{c}$$

The speed of sound in a fluid is a function of the thermodynamic properties of that fluid. When the fluid is an ideal gas ($P = \rho RT$), the speed of sound can easily be calculate from the following equation:

$$c = \sqrt{\gamma RT}$$

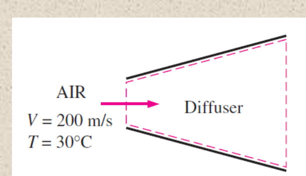
Where: γ = specific heat ratio
R = gas constant
T = Fluid temperature in K

Noting that the gas constant R has a fixed value for a specified ideal gas and the specific heat ratio γ of an ideal gas is, at most, a function of temperature, we see that the speed of sound in a specified ideal gas is a function of temperature alone

In general, the value of the Mach number varies with position and time. However, in many problems, we may choose a representative flow speed, say ***U***, and a representative sound speed, say ***C***. Then the quantity ***U/c*** is a single number measuring the importance of compressibility in the flow, and we may say that the flow is taking place at a Mach number ***M = U/c***.

Example 1

Air enters a diffuser shown in Fig. with a velocity of 200 m/s. The gas constant of air is R = 0.287 kJ/kg · K, and its specific heat ratio at 30°C is 1.4. Determine (*a*) the speed of sound and (*b*) the Mach number at the diffuser inlet when the air temperature is 30°C.



Solution

(*a*) The speed of sound in air at 30°C is determined from:

T = 30 + 273 = 303 K

$$c = \sqrt{\gamma RT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(303\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = \mathbf{349\ m/s}$$

(*b*) Then the Mach number becomes

$$\text{Ma} = \frac{V}{c} = \frac{200\ \text{m/s}}{349\ \text{m/s}} = \mathbf{0.573}$$

Discussion The flow at the diffuser inlet is subsonic since Ma < 1.

Flow Regimes

Flows corresponding to different ranges of the Mach number ***M*** have very different properties. We shall distinguish five regimes, namely

a) Incompressible flow
b) Subsonic flow
c) Transonic flow
d) Supersonic flow
e) Hypersonic flow.

(a) Incompressible Flow

A flow is said to be incompressible if the *density* of a fluid element does not change during its motion. It is a property of the flow and not of the fluid. The rate of change of density of a material fluid element is given by the material derivative

$$\frac{\partial \rho}{\partial t} = 0$$

Hence, the flow is incompressible if the divergence of the velocity field is identically zero. Note that the density field need not be uniform in an incompressible flow. All that is required is that the density of a fluid element should not change in time as it moves through space. For example, flow in the ocean can be considered to be incompressible even though the density of water is not uniform due to stratification.

Compressible flow can with good accuracy be approximated as incompressible for steady flow if the *Mach number* is below 0.3

(b) Subsonic Flow

This regime is defined by $\mathbf{0} < \boldsymbol{M} < \mathbf{1}$, subject to the restriction that ***M*** is not too close to 0 or 1.

- The flow speed is high enough for the fluid's compressibility to be important, but low enough for the speed to be comfortably clear of the speed of sound.
- Since most aircraft fly well below the speed of sound, the regimes of incompressible flow and subsonic flow include most of standard aeronautics. The subsonic regime includes much of acoustics.

(c) Transonic Flow

This regime is defined by ***M*** being close to **1** (**Ma ≅ 1**) in some important part of the flow. The regime raises difficult and interesting mathematical questions, because the governing partial differential equations are then of mixed type. That is, in some regions, the equations are elliptic, in other regions, they are hyperbolic, and on the separating lines or surfaces, they are parabolic. The transonic regime is of vital importance to an aircraft or a land vehicle that "breaks the sound barrier."

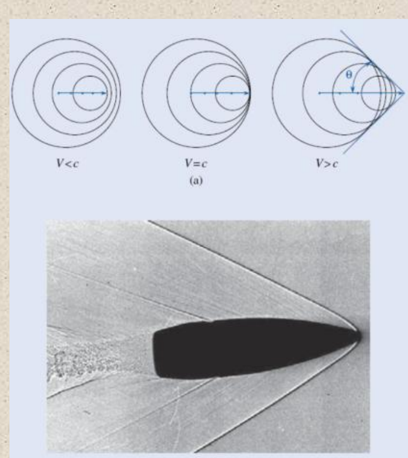
(d) Supersonic Flow

This regime is defined by $\boldsymbol{M > 1}$, subject to the restriction that M is not too close to 1 nor too large. Parts of the mathematical theory of supersonic flow can be obtained from that for subsonic flow by replacing $(1 \gg M^{2})^{1/2}$ and an elliptic equation by $(M^{2}c_{1})^{1/2}$ and a hyperbolic equation. The theory of the supersonic regime was of importance to the design of the civil supersonic airplane *Concorde*.

(e)Hypersonic Flow

This regime is defined by $M \gg 1$ the flow speed is so high that compressibility is all important, particularly in producing very high temperatures and ionization. The hypersonic regime is of importance for rockets and for the civil hypersonic aircraft “Orient Express.”

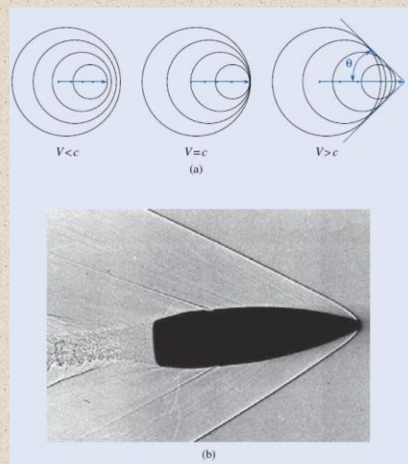
Mach began a study of the flight of artillery shells using the new technology of photography. In this research, he discovered that the angle θ of the shock cone radiating from the leading edge of a supersonic object was related to the speed of sound c and the velocity of the object V by $\sin \theta = c/V$ (see Figure), and θ was later called the Mach angle.



The ratio of the local fluid velocity **V** to the speed of sound in the fluid ***c*** came to be of fundamental value in the study of high-speed aerodynamics, and after1930, it was called the Mach number (***M = V/c***).

(a) The representation of a sound waves produced when an object moves from subsonic ($V < c$) to sonic ($V = c$) to supersonic ($V > c$) velocity. Generally, a conical shock wave sweeps back from the leading edge of the object with the cone angle proportional to the ratio of the sonic velocity of the fluid to the supersonic velocity of the object, c/V.

(b) The shadowgraph of a supersonic bullet showing shock waves.



(f) Stagnation properties

The stagnation state of a moving fluid is the state it would achieve if it underwent an adiabatic, aergonic deceleration to zero velocity. The energy rate balance (ERB) for an adiabatic, aergonic, steady state, steady flow, single inlet, single-outlet open system with negligible change in flow stream potential energy reduces to

$$h_{in} + V_{in}^2/(2g_c) = h_{out} + V_{out}^2/(2g_c)$$

$$V^2/(2g_c) = h_o - h = c_p(T_o - T)$$

If we let the subscript o refer to the stagnation (or zero velocity) state, then $V_o = 0$, and the preceding equation can be used to define the stagnation specific enthalpy ho as

$$h_o = h + V^2/(2g_c)$$

For an ideal gas or a low-pressure vapor with constant specific heats, this equation can be written as

$$\frac{T_o}{T} = 1 + \frac{V^2}{2g_c c_p T}$$

Where T_o is the stagnation temperature (the temperature at zero velocity).

$\boldsymbol{g_c}$ is a unit conversion factor used to convert mass to force or vice versa. It is defined as

$$g_c = \frac{ma}{F}$$

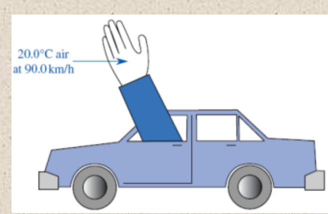
International System	English System 1	English System 2
g_c = 1 (kg·m)/(N·s^2)	g_c = 32.174 (lb·ft)/(lbf·s^2)	g_c = 1 (slug·ft)/(lbf·s^2)

Example 2

While driving in your new sports car at 90.0 km/h in still air at 20.0°C, you put your hand out the window with your palm toward the front of the car. What is the air temperature on the center of your palm?

Solution

When your hand is placed perpendicular to the air flow, you should feel the stagnation pressure and temperature of the air flow.



The stagnation temperature is given by Eq.

$$T_0 = T\left(1 + \frac{V^2}{2g_c C_p T}\right)$$

$$T_0 = (20 + 273)\left(1 + \frac{\left[\left(90\frac{km}{hr}\right)\left(\frac{1000m}{km}\right)\left(\frac{1h}{3600s}\right)\right]^2\left(\frac{\frac{1kJ}{kg}}{1000m^2/s^2}\right)}{2(1)(1.004\frac{kJ}{kg}.K)(20+273\ K)}\right)$$

T_0 = 294 K = 20.3 °C

Isentropic flow

- Constant entropy flow is called *isentropic* flow.
- During fluid flow through many devices such as nozzles, diffusers, and turbine blade passages, flow quantities vary primarily in the flow direction only, and the flow can be approximated as one-dimensional isentropic flow with good accuracy.
- From a consideration of the second law of thermodynamics, a reversible flow maintains a constant value of entropy.

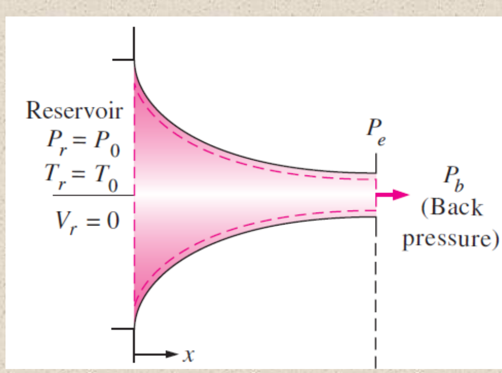
In the following all thermodynamic properties are related to their **properties *at rest*** (u = 0). From the energy equation for a frictionless, adiabatic flow you will get that the enthalpy at rest $\mathbf{h_t}$ is always the same regardless of an isentropic or non-isentropic state change.

$$\frac{u^2}{2} + h = h_t$$

In fact ***h = Cp T*** the temperature is it as well. In contrary, the pressure depends on how the gas is brought to rest. The pressure at rest is only obtained if the state change is isentropic. If the entropy changes the pressure at rest changes as well, e.g. when passing a shock.

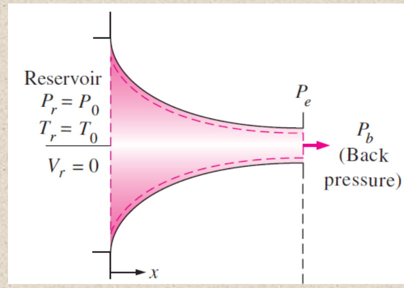
Isentropic flow through nozzles

- Converging or converging–diverging nozzles are found in many engineering applications including steam and gas turbines, aircraft and spacecraft propulsion systems, and even industrial blasting nozzles and torch nozzles.
- In this section we consider the effects of **back pressure** (i.e., the pressure applied at the nozzle discharge region) on the exit velocity, the mass flow rate, and the pressure distribution along the nozzle.

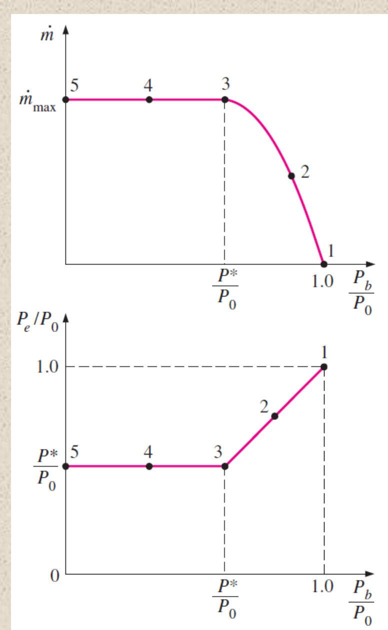


Converging Nozzles

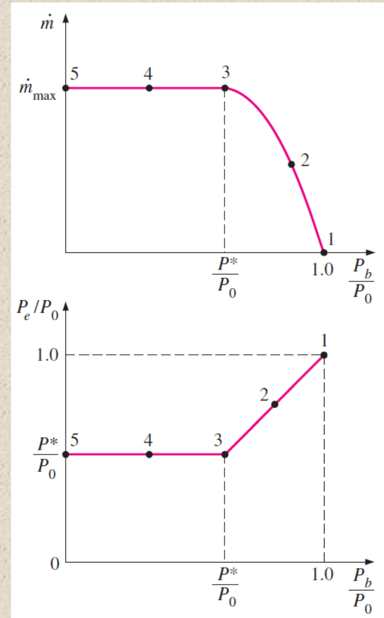
- Consider the subsonic flow through a converging nozzle as shown in Fig.
- The nozzle inlet is attached to a reservoir at pressure P_r and temperature Tr.
- The reservoir is sufficiently large so that the nozzle inlet velocity is negligible.
- Since the fluid velocity in the reservoir is zero and the flow through the nozzle is approximated as isentropic, the stagnation pressure and stagnation temperature of the fluid at any cross section through the nozzle are equal to the reservoir pressure and temperature, respectively.



- Now we begin to reduce the back pressure and observe the resulting effects on the pressure distribution along the length of the nozzle, as shown in Fig.
- If the back pressure P_b is equal to P_1, which is equal to Pr, there is no flow and the pressure distribution is uniform along the nozzle.
- When the back pressure is reduced to P_2, the exit plane pressure Pe also drops to P_2. This causes the pressure along the nozzle to decrease in the flow direction.



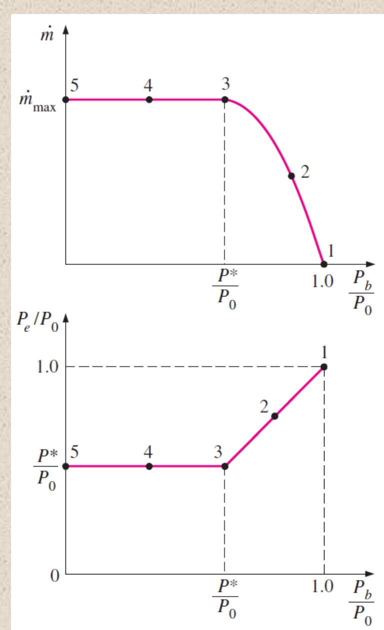
- When the back pressure is reduced to P_3 (=P^*, which is the pressure required to increase the fluid velocity to the speed of sound at the exit plane or throat), the mass flow reaches a maximum value and the flow is said to be **choked.**
- Further reduction of the back pressure to level P_4 or below does not result in additional changes in the pressure distribution, or anything else along the nozzle length.
- A plot of $\dot{m}$ versus P_b/P_0 for a converging nozzle is shown in Fig.



Notice that the mass flow rate increases with decreasing P_b/P_0, reaches a maximum at $P_b = P^*$, and remains constant for P_b/P_0 values less than this critical ratio. Also illustrated on this figure is the effect of back pressure on the nozzle exit pressure *Pe*. We observe that

$$P_e = \begin{cases} P_b & \text{for } P_b \geq P^* \\ P^* & \text{for } P_b < P^* \end{cases}$$

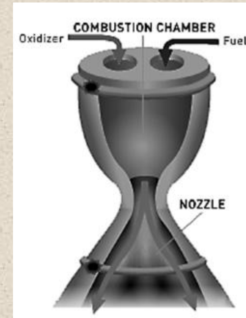
To summarize, for all back pressures lower than the critical pressure P^*, the pressure at the exit plane of the converging nozzle *Pe* is equal to P^*, the Mach number at the exit plane is unity, and the mass flow rate is the maximum (or choked) flow rate. Because the velocity of the flow is sonic at the throat for the maximum flow rate, a back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and does not affect the flow rate.



Converging–Diverging Nozzles

- When we think of nozzles, we ordinarily think of flow passages whose cross-sectional area decreases in the flow direction. However, the highest velocity to which a fluid can be accelerated in a converging nozzle is limited to the sonic velocity (Ma = 1), which occurs at the exit plane (throat) of the nozzle.

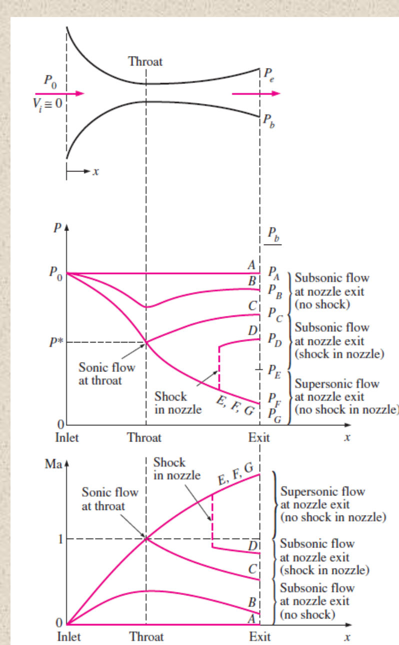
- Accelerating a fluid to supersonic velocities (Ma > 1) can be accomplished only by attaching a diverging flow section to the subsonic nozzle at the throat. The resulting combined flow section is a converging–diverging nozzle, which is standard equipment in supersonic aircraft and rocket propulsion.



Consider the converging–diverging nozzle shown in Fig.

- A fluid enters the nozzle with a low velocity at stagnation pressure P_0. When $P_b = P_0$ (case A), there will be no flow through the nozzle.

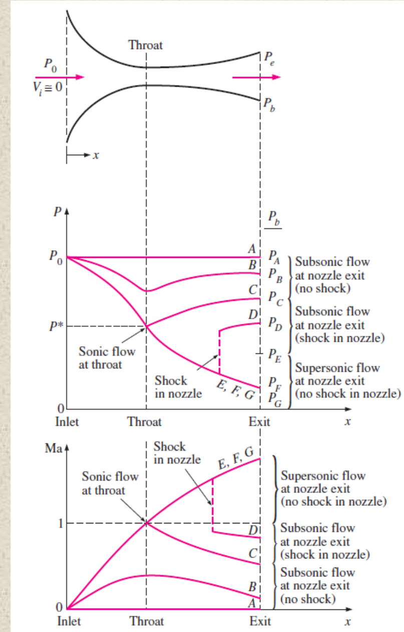
- This is expected since the flow in a nozzle is driven by the pressure difference between the nozzle inlet and the exit.



Now let us examine what happens as the back pressure is lowered.

1. When $P_0 > P_b > P_C$, the flow remains subsonic throughout the nozzle, and the mass flow is less than that for choked flow.

- The fluid velocity increases in the first (converging) section and reaches a maximum at the throat (but Ma < 1). However, most of the gain in velocity is lost in the second (diverging) section of the nozzle, which acts as a diffuser.
- The pressure decreases in the converging section, reaches a minimum at the throat, and increases at the expense of velocity in the diverging section.

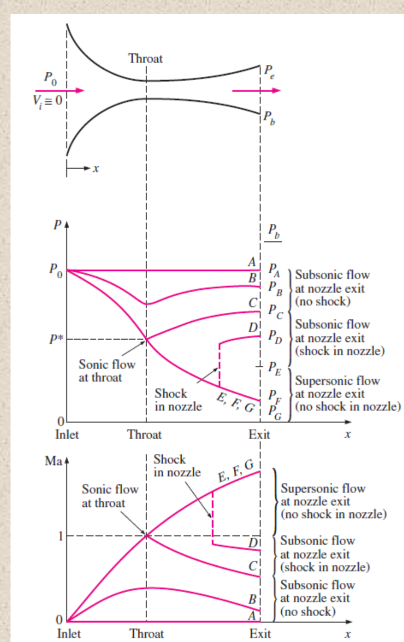


23

2. When $P_b = P_C$, the throat pressure becomes P^* and the fluid achieves sonic velocity at the throat. But the diverging section of the nozzle still acts as a diffuser, slowing the fluid to subsonic velocities.

- The mass flow rate that was increasing with decreasing P_b also reaches its maximum value.

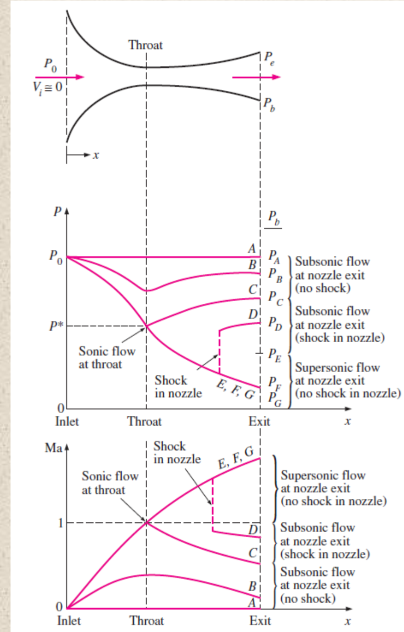
Recall that P^* is the lowest pressure that can be obtained at the throat, and the sonic velocity is the highest velocity that can be achieved with a converging nozzle. Thus, lowering P_b further has no influence on the fluid flow in the converging part of the nozzle or the mass flow rate through the nozzle. However, it does influence the character of the flow in the diverging section.



24

3. When $P_C > P_b > P_E$, the fluid that achieved a sonic velocity at the throat continues accelerating to supersonic velocities in the diverging section as the pressure decreases. This acceleration comes to a sudden stop, however, as a **normal shock** develops at a section between the throat and the exit plane, which causes a sudden drop in velocity to subsonic levels and a sudden increase in pressure.

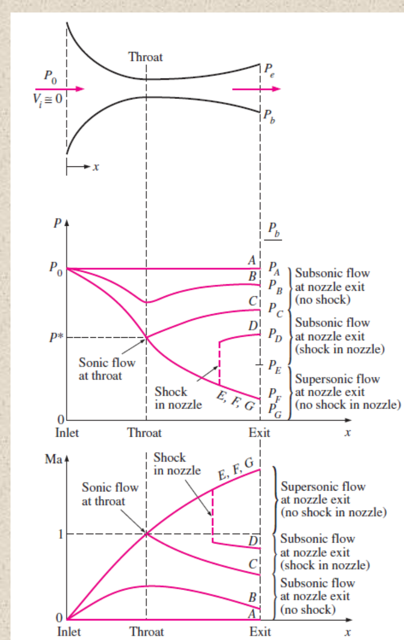
- The fluid then continues to decelerate further in the remaining part of the converging–diverging nozzle. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The normal shock moves downstream away from the throat as P_b is decreased, and it approaches the nozzle exit plane as P_b approaches P_E.



25

4. When $P_E > P_b > 0$, the flow in the diverging section is supersonic, and the fluid expands to P_F at the nozzle exit with no normal shock forming within the nozzle. Thus, the flow through the nozzle can be approximated as isentropic.

- When $P_b = P_F$, no shocks occur within or outside the nozzle. When $Pb < PF$, irreversible mixing and expansion waves occur downstream of the exit plane of the nozzle.
- When $Pb > PF$, however, the pressure of the fluid increases from P_F to P_b irreversibly in the wake of the nozzle exit, creating what are called *oblique shocks.*



26

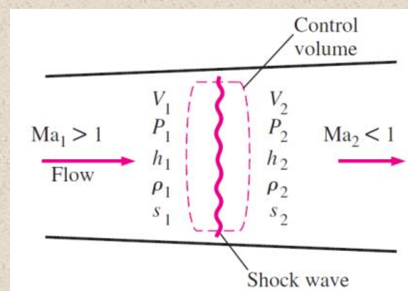
Shock wave

- In Physics, a shock wave is also known as shock waves. It is a strong pressure wave in an elastic medium such as air, water, or any solid material ejected from explosions or lightning, or other phenomena that create variations in pressure.
- It is a type of disturbance that propagates at a speed greater than the speed of sound in the medium.
- As a rule, like ordinary waves, shock waves carry energy and can propagate through a medium.
- Above all, we characterize a shock wave by a sudden, nearly discontinuous, change in pressure, temperature, and density of the medium.

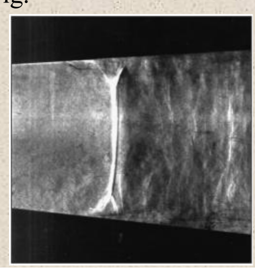
- Across a shock there is always an extremely rapid rise in pressure, temperature and density of the flow.
- In supersonic flows, expansion is achieved through an expansion fan.
- A shock wave travels through most media at a higher speed than an ordinary wave.

Normal Shocks

- In some range of back pressure, the fluid that achieved a sonic velocity at the throat of a converging-diverging nozzle and is accelerating to supersonic velocities in the diverging section experiences a ***normal shock***, which causes a sudden rise in pressure and temperature and a sudden drop in velocity to subsonic levels.
- Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic.
- We develop relationships for the flow properties before and after the shock, and we do this by applying the conservation of mass, momentum, and energy relations as well as some property relations to a stationary control volume that contains the shock, as shown in Fig.



The normal shock waves are extremely thin, so the entrance and exit flow areas for the control volume are approximately equal as shown in Fig.



Conservation of mass:

$$\rho_1 A V_1 = \rho_2 A V_2$$

or

$$\rho_1 V_1 = \rho_2 V_2$$

Conservation of energy:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

or

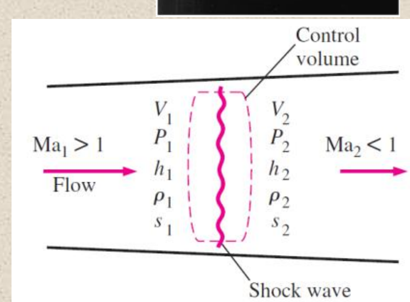
$$h_{01} = h_{02}$$

Conservation of momentum:

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1)$$

Increase of entropy:

$$s_2 - s_1 \geq 0$$



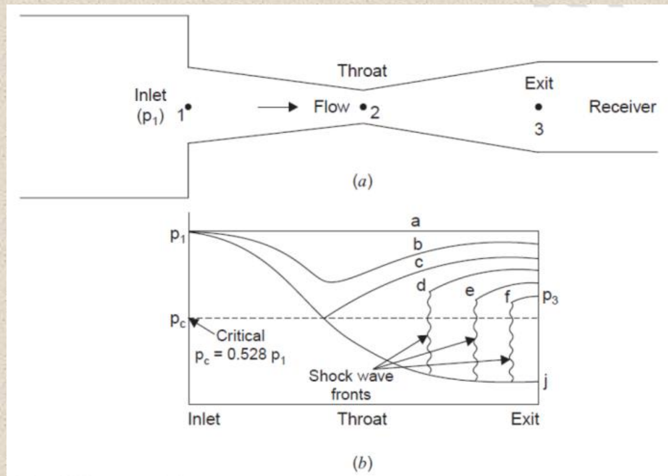
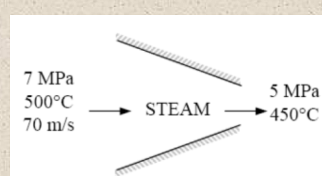


Fig.17 (a) Laval nozzle (convergent-divergent nozzle); (b) Pressure distribution through a convergent-divergent nozzle with flow of compressible fluid.

Example 3

Steam is usually accelerated in the nozzle of a turbine before it strikes the turbine blades. Steam enters an adiabatic nozzle at 7 MPa and 500°C with a velocity of 70 m/s and exits at 5 MPa and 450°C. Assuming the surroundings to be at 25°C, determine *the* exit velocity of the steam

Solution



$$\left.\begin{array}{l} P_1 = 7\text{ MPa} \\ T_1 = 500°\text{C} \end{array}\right\} \begin{array}{l} h_1 = 3411.4\text{ kJ/kg} \\ s_1 = 6.8000\text{ kJ/kg}\cdot\text{K} \end{array}$$

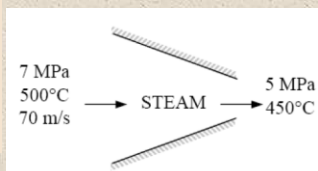
$$\left.\begin{array}{l} P_2 = 5\text{ MPa} \\ T_2 = 450°\text{C} \end{array}\right\} \begin{array}{l} h_2 = 3317.2\text{ kJ/kg} \\ s_2 = 6.8210\text{ kJ/kg}\cdot\text{K} \end{array}$$

$$\left.\begin{array}{l} P_{2s} = 5\text{ MPa} \\ s_{2s} = s_1 \end{array}\right\} h_{2s} = 3302.0\text{ kJ/kg}$$

(*a*) We take the nozzle to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed in in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$



$$\dot{m}(h_1 + V_1^2/2) = \dot{m}(h_2 + V_2^2/2) \quad (\text{since } \dot{W} = \dot{Q} \cong \Delta pe \cong 0)$$

$$0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

Then the exit velocity becomes

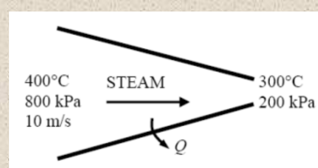
$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2} = \sqrt{2(3411.4 - 3317.2)\text{ kJ/kg}\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right) + (70\text{ m/s})^2} = \mathbf{439.6\ m/s}$$

33

Lec. Hassan Rijabo

Example 4

Steam enters a nozzle at 400°C and 800 kPa with a velocity of 10 m/s, and leaves at 300°C and 200 kPa while losing heat at a rate of 25 kW. For an inlet area of 800 cm², determine the velocity and the volume flow rate of the steam at the nozzle exit.



Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy change is negligible. **3** There are no work interactions.

Analysis We take the steam as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

Energy balance:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{Q}_{out} \quad \text{since } \dot{W} \cong \Delta pe \cong 0)$$

34

Lec. Hassan Rijabo

or $$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} + \frac{\dot{Q}_{out}}{\dot{m}}$$

The properties of steam at the inlet and exit are

$$\left.\begin{aligned} P_1 &= 800 \text{ kPa} \\ T_1 &= 400°\text{C} \end{aligned}\right\} \begin{aligned} v_1 &= 0.38429 \text{ m}^3/\text{kg} \\ h_1 &= 3267.7 \text{ kJ/kg} \end{aligned}$$

$$\left.\begin{aligned} P_2 &= 200 \text{ kPa} \\ T_1 &= 300°\text{C} \end{aligned}\right\} \begin{aligned} v_2 &= 1.31623 \text{ m}^3/\text{kg} \\ h_2 &= 3072.1 \text{ kJ/kg} \end{aligned}$$

The mass flow rate of the steam is

$$\dot{m} = \frac{1}{v_1} A_1 V_1 = \frac{1}{0.38429 \text{ m}^3/\text{s}} (0.08 \text{ m}^2)(10 \text{ m/s}) = 2.082 \text{ kg/s}$$

Substituting,

$$3267.7 \text{ kJ/kg} + \frac{(10 \text{ m/s})^2}{2}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 3072.1 \text{ kJ/kg} + \frac{V_2^2}{2}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) + \frac{25 \text{ kJ/s}}{2.082 \text{ kg/s}}$$

$$\longrightarrow V_2 = \mathbf{606 \text{ m/s}}$$

The volume flow rate at the exit of the nozzle is

$$\dot{V}_2 = \dot{m} v_2 = (2.082 \text{ kg/s})(1.31623 \text{ m}^3/\text{kg}) = \mathbf{2.74 \text{ m}^3/\text{s}}$$

Any Questions???