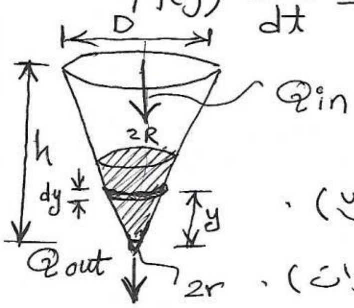
Application of first order Differential equations:

أولاً :- حساب التسرب من الخزانات بأستخدام معادلة تورشلي :-

أذا كان الخزان مملوء بالماء وهذا الخزان يحتوي على ثقب فأن الماء يبدأ بالتسرب خلال الزمن وبمرور الزمن يبدأ عمق الماء داخل الخزان بالنقصان لذلك فأن تورشلي وضع معادلة يتم من خلالها حساب مقدار أرتفاع الماء داخل الخزان المثقوب مع الزمن .

$$A(y)\,\frac{dy}{dt} = \sum Q_{in} - \sum Q_{out}$$

معادله تورشلي .



حيث ان :-

$A(y)$:- مساحه الشريحة عند العمق (y) .

Q_{in} :- التصريف الداخل الى الحوض (الخزان) .

Q_{out} :- التصريف الخارج من الخزان ويحسب من خلال المعادله الأتيه :-

$$Q_{out} = \pi r^2 \sqrt{2gy}$$

حيث ان :-

r :- نصف قطر الفتحه التي يتسرب منها الماء .

g :- التعجيل الأرضي ويساوي :-

١- اذا كانت الأبعاد المعطاه بالمتر :- $g = 9.8 \ m/sec^2$

٢- اذا كانت الأبعاد المعطاه بالقدم :-

$g = 32.2 \ m/sec^2$

E-mail : marwan.abbas.madhloom@uomus.edu.iq

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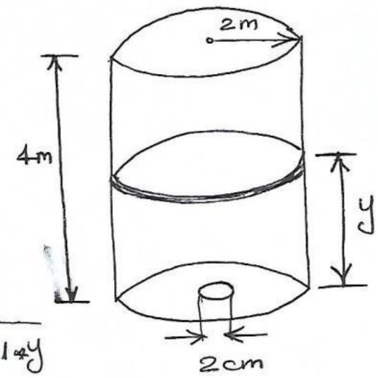
Ex(1) :- A cylindrecial tank with raduis of (2m) and height of 4m has initially filled with water. In the bottom of the tank there is a hole with diameter of (2cm) In which the water drain under the influee of gravity.

Find :-

(1) The depth of water at any time.

(2) The time of reach the hight of water one meter.

(3) The time to empty the tank.



Solution :-

$$A(y) = \pi r^2$$
$$= 4\pi \quad (\text{مساحه الشريحه})$$

$$A(y)\,\frac{dy}{dt} = Q_{in} - Q_{out}$$

$$4\cancel{\pi}\,\frac{dy}{dt} = 0 - \cancel{\pi}(0.01)^2\sqrt{2 * 9.81 * y}$$

$$4\,\frac{dy}{dt} = -(0.01)^2\sqrt{19.6}\,\sqrt{y}$$

$$\therefore\ 4\,\frac{dy}{dt} = -4.427 * 10^{-4}\sqrt{y}$$

$$\frac{4}{4.427 * 10^{-4}\sqrt{y}}\,dy = -\,dt \quad (\text{by seperable}).$$

$$\int \frac{4}{4.427 * 10^{-4}\sqrt{y}}\,dy = -\int dt$$

$$\frac{4}{4.427 * 10^{-4}}\int (y)^{-1/2}\,dy = -\int dt$$

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$$\boxed{18070.9\sqrt{y} = -t + C}$$

To find C use (Boundary Condition)

at $t=0$ $y = H = 4\,m$

$$18070.9\sqrt{4} = -(0) + C$$

$$\Rightarrow C = 36141.8 .$$

$$18070.9\sqrt{y} = 36141.8 - t \quad (1).$$

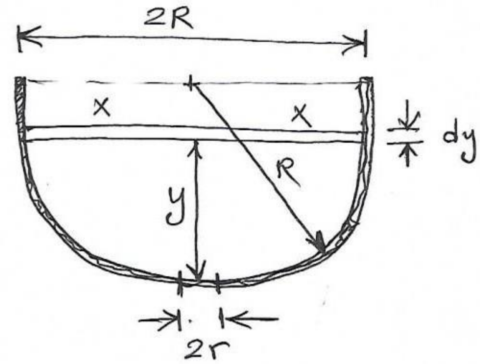
$$\Rightarrow t = 18070.9 \text{ sec} \quad \text{when } y = 1. \quad (2).$$

$$18070.9\sqrt{0} = 36141.8 - t \Rightarrow t = 36141.8 \text{ sec}$$

when $y = 0$.

Ex (2) :- A hemispherical tank of radius ($R=3m$) in initially filled with water. At the bottom of the tank, there is a hole of radius ($r = 1cm$) through which the water drains under the influence of gravity. Find the depth of water in the tank at any time t, and determine how long it will take the tank to drain completely.

Solution :



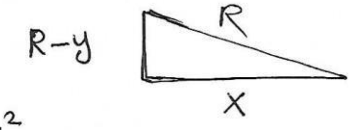
$$A(y)\frac{dy}{dt} = Q_{in} - Q_{out}$$

$$A(y) = \pi x^2 \quad (1)$$

E-mail : marwan.abbas.madhloom@uomus.edu.iq

$$X^2 + (R-y)^2 = R^2$$

$$X^2 = \cancel{R^2} - (\cancel{R^2} - 2Ry + y^2)$$



$$X^2 = 2Ry - y^2 \Rightarrow X^2 = 6y - y^2$$

sub. in eq. (1)

$$\therefore A(y) = \pi\,(6y - y^2).$$

$$Q_{in} = 0$$

$$Q_{out} = \pi r^2 \sqrt{2gy} = \pi\,(0.01)^2 \sqrt{2*9.81*y}$$

$$= 4.429 * 10^{-4} \pi y^{1/2}.$$

$$\therefore \cancel{\pi} \cdot (6y - y^2) \cdot \frac{dy}{dt} = 0 - 4.429 * 10^{-4} \cancel{\pi}\, y^{1/2}$$

$$(6y - y^2)\,\frac{dy}{dt} = -4.429\ 10^{-4}\ y^{1/2}$$

$$\frac{(6y - y^2)}{y^{1/2}}\,dy = -4.429\ 10^{-4}\ dt$$

$$y^{-1/2}(6y - y^2)\,dy = -4.429\ 10^{-4}\ dt$$

$$\int (6y^{1/2} - y^{3/2})\,dy = \int -4.429\ 10^{-4}\ dt$$

$$6\,\frac{y^{3/2}}{3/2} - \frac{y^{5/2}}{5/2} = -4.429\ 10^{-4}\ t + c$$

$$4y^{3/2} - \frac{2}{5}y^{5/2} = -4.429\ 10^{-4}\ t + c$$

Apply boundary condition at $t=0 \Rightarrow y = h = R = 3m$

E-mail : marwan.abbas.madhloom@uomus.edu.iq

$$4\,(3)^{3/2} - \frac{2}{5}(3)^{5/2} = -4.429\ 10^{-4} * 0 + C \Rightarrow \quad (34)$$

$$C = 14.549.$$

$$\therefore\ 4\ y^{3/2} - \frac{2}{5}y^{5/2} = -4.429\ 10^{-4}\,t + 14.549.$$

at any time.

The time to empty the tank at $y = 0$:-

$$\therefore\ 4\,(0)^{3/2} - \frac{2}{5}(0)^{5/2} = -4.429\ 10^{-4}\,t + 14.549$$

$$t = 32849.9\ \text{sec.}$$