Non-Exact Differential Equation

المعادلات التفاضلية الغير تامة :-

The equation which is not exact can be made exact by multiplying it by the integrating factor (I.F) :-

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

يعني اذا كان $\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$ فانها غير تام .

أذا كانت المعادلة التفاضلية غير تامة (Non-exact) فأننا نستطيع جعلها تامة بواسطة ضربها بعامل التكميل (I.F) وهو عامل نضرب به المعادلة الغير تامة فيحولها الى معادلة تامة.

هنا يكون لدينا حالتين

(1) $$f(x) = \frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right]$$

$$I.F = e^{\int f(x)\,dx}.$$

(2) $$f(y) = \frac{1}{M}\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right]$$

$$I.F = e^{\int f(y)\,dy}.$$

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Ex(1): Find a general solution for $y \cdot dx + (3+3x-y)\,dy = 0$?

$$M(x,y) = y \Rightarrow \frac{\partial M}{\partial y} = 1$$

$$N(x,y) = (3+3x-y) \Rightarrow \frac{\partial N}{\partial x} = 3$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

∴ Non-Exact.

$\Rightarrow$ Find integrating factor $\Rightarrow$

$$f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$$

$$= \frac{1}{3+3x-y}(1-3) = \frac{-2}{3+3x-y}$$

$$(I \cdot F)_x = e^{\int f(x)\,dx} = e^{\int \frac{-2}{3+3x-y}\,dx}$$

$$= e^{-\frac{2}{3}\ln(3+3x-y)} = e^{\ln(3+3x-y)^{-2/3}}$$

$$= (3+3x-y)^{-2/3}$$

يهمل هذا العامل لانه داله من x و y

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$$f(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$$

$$= \frac{1}{y}(3-1) = \frac{2}{y}$$

$$\therefore (I.F)_y = e^{\int f(y)\,dy} = e^{\int \frac{2}{y}\,dy}$$

$$= e^{2\ln y} = e^{\ln y^2} = y^2.$$

$\therefore$ y^2 is integrating factor.

$$\Rightarrow y\cdot dx + (3+3x-y)\cdot dy = 0 \;\Big]\, * y^2$$

$$y^3\cdot dx + (3y^2 + 3xy^2 - y^3)\cdot dy = 0$$

$$M(x,y) = y^3 \Rightarrow \frac{\partial M}{\partial y} = 3y^2.$$

$$N(x,y) = 3y^2 + 3xy^2 - y^3 \Rightarrow \frac{\partial N}{\partial x} = 3y^2.$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3y^2 \Rightarrow \therefore \text{Exact}.$$

To solve the equation

$$\int_a^x y\,dx + \int_b^y (3+3x-y)\,dy = 0$$

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$$\int_a^x y^3\,dx + \int_b^y (3y^2 + 3ay^2 - y^3)\,dy = 0$$

$$\left[y^3 x\right]_a^x + \left[\cancel{3}\frac{y^3}{\cancel{3}} + \cancel{3}a\frac{y^3}{\cancel{3}} - \frac{y^4}{4}\right]_b^y = c$$

$$\left[y^3x - y^3a\right] + \left[y^3 + ay^3 - \frac{y^4}{4} - b^3 - ab^3 + \frac{b^4}{4}\right] = c$$

$$y^3x - \cancel{y^3a} + y^3 + \cancel{ay^3} - \frac{y^4}{4} - b^3 - ab^3 + \frac{b^4}{4} = c$$

$$-\frac{y^4}{4} + y^3x + y^3 + \left(\frac{b^4}{4} - b^3 - ab^3\right) = c$$

$$-\frac{y^4}{4} + y^3x + y^3 + k = 0 \qquad \boxed{k = \frac{b^4}{4} - b + ab^3}$$

Ex(2): Find a general solution for

$$(x^2 + y^2 + x)\,dx + xy\,dy = 0$$

$$M(x,y) = x^2 + y^2 + x \Rightarrow \frac{\partial M}{\partial y} = 2y.$$

$$N(x,y) = xy \Rightarrow \frac{\partial N}{\partial x} = y.$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow \therefore \text{Non-exact.}$$

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We must find I.F

$$f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{xy}(2y - y) = \frac{y}{xy} = \frac{1}{x}$$

$$(I.F)_x = e^{\int f(x)\,dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

$$\therefore\ (x^2 + y^2 + x)\,dx + xy\,dy = 0 \ ] * x$$

$$(x^3 + xy^2 + x^2)\,dx + x^2 y\,dy = 0$$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 2xy \\ \frac{\partial N}{\partial x} &= 2xy \end{aligned}\right\} \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \therefore \text{ exact.}$$

$$\int_a^x (x^3 + xy^2 + x^2)\,dx + \int_b^y x^2 y\,dy = 0$$

$$\left[\frac{x^4}{4} + y^2\frac{x^2}{2} + \frac{x^3}{3}\right]_a^x + \left[a^2 y^2/2\right]_b^y = 0$$

$$\frac{x^4}{4} + y^2\frac{x^2}{2} + \frac{x^3}{3} - \left(\frac{a^4}{4} + \cancel{y^2\frac{a^2}{2}} + \frac{a^3}{3}\right)$$

$$+ \cancel{a^2\frac{y^2}{2}} - a^2\frac{b^2}{2} = 0$$

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$$\frac{x^4}{4} + y^2\frac{x^2}{2} + \frac{x^3}{3} - \frac{a^4}{4} - \frac{a^3}{3} - a^2\frac{b^2}{2} = 0$$

$$\left(\frac{x^4}{4} + y^2\frac{x^2}{2} + \frac{x^3}{3}\right) - k = 0$$

$$\boxed{k = \frac{a^4}{4} + \frac{a^3}{3} + a^2\frac{b^2}{2}}$$

H.W Find a general solution for

$$(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy$$

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