Exact First order differential equation:

الصيغة العامة :

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

ان هذه المعادلة تكون (Exact) أذا كان

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

وخلاف ذلك فأن المعادلة تكون غير (Exact)

بعد أختبار المعادلة وكانت (Exact) فيكون الحل كالاتي :

$$\int_a^x M(x,y)\,dx + \int_b^y N(x,y)\,dy = \int 0$$

قبل أجراء التكامل اعلاه يجب ان نعوض في الحد الثاني ($\int N(x,y)\,dy$)
بدل كل (x) بـ (a) ثم نجري التكامل.

Ex(1): show that the equation is exact and solve:

$$(2x+3y-2)\,dx + (3x-4y+1)\,dy = 0$$

$$M(x,y) = 2x+3y-2 \Rightarrow \frac{\partial M}{\partial y} = 3.$$

$$N(x,y) = 3x-4y+1 \Rightarrow \frac{\partial M}{\partial x} = 3.$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3 \quad \text{Exact.}$$

$$\int_a^x M\,dx + \int_b^y N\,dy = c$$

$$\int_a^x (2x+3y-2)\,dx + \int_b^y (3a-4y+1)\,dy = 0$$

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$$\Big[x^2+3yx-2x\Big]_a^x+\Big[3ay-2y^2+y\Big]_b^y=0$$

$$\Big[(x^2+3yx-2x)-(a^2+3ya-2a)\Big]+\Big[(3ay-2y^2+y)-(3ab-2b^2+b)\Big]=0$$

$$x^2+3xy-2x-a^2-\cancel{3ay}+2a+\cancel{3ay}-2y^2+y-3ab+2b^2-b=0$$

$$x^2+3xy-2x-2y^2+y=a^2-2a+3ab-2b^2-b$$

$$\therefore \boxed{x^2+3xy-2x-2y^2+y=k}$$

where

$$k=a^2-2a+3ab-2b^2-b$$

Ex(2):- Show that the equation is exact and solve:-

$$\left(\frac{1}{x^2}+\frac{2y}{x^3}\right)dx-\frac{1}{x^2}\,dy=0$$

$$M=\frac{1}{x^2}+\frac{2y}{x^3}\Rightarrow\frac{\partial M}{\partial y}=\frac{2}{x^3}$$

$$N=-\frac{1}{x^2}\Rightarrow\frac{\partial N}{\partial x}=+\frac{2}{x^3}$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=\frac{2}{x^3}\Rightarrow \text{Exact.}$$

$$\int_a^x\left(\frac{1}{x^2}+\frac{2y}{x^3}\right)dx+\int_b^y-\frac{1}{x^2}\,dy=0$$

$$\left[\left(-\frac{1}{x}-\frac{y}{x^2}\right)-\left(-\frac{1}{a}-\frac{y}{a^2}\right)\right]-\left[\left(\frac{y}{a^2}\right)-\left(\frac{b}{a^2}\right)\right]=0$$

$$-\frac{1}{x}-\frac{y}{x^2}+\frac{1}{a}+\cancel{\frac{y}{a^2}}-\cancel{\frac{y}{a^2}}+\frac{b}{a^2}=0$$

$$\frac{1}{x}+\frac{y}{x^2}=\frac{1}{a}+\frac{b}{a^2} \Rightarrow \frac{1}{x}+\frac{y}{x^2}=k$$

$$\boxed{k=\frac{1}{a}+\frac{b}{a^2}}.$$

Ex(3): show that the equation is exact and solve:

$$(2xy^4+\sin y)\,dx+(4x^2y^3+x\cos y)\,dy=0$$

$$M=2xy^4+\sin y \Rightarrow \frac{\partial M}{\partial y}=8xy^3+\cos y.$$

$$N=4x^2y^3+x\cos y \Rightarrow \frac{\partial N}{\partial x}=8xy^3+\cos y.$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}=8xy^3+\cos y \Rightarrow \text{Exact.}$$

$$\int_a^x (2xy^4+\sin y)\,dx+\int_b^y (4x^2y^3+x\cos y)\,dy=0$$

$$\left[x^2y^4+x\sin y\right]_a^x+\left[a^2y^4+a\sin y\right]_b^y=0$$

$$\left[(x^2y^4+x\sin y)-(a^2y^4+a\sin y)\right]+\left[(a^2y^4+a\sin y)-(a^2b^4+a\sin b)\right]=0$$

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$$x^2y^4 + x\sin y - a^2y^4 - a\cancel{\sin y} + a^2y^4 + a\cancel{\sin y} - a^2b^2 - a\sin b = 0$$

$$x^2y^4 + x\sin y = a^2b^4 + a\sin b$$

$$x^2y^4 + x\sin y = k \qquad \boxed{k = a^2b^4 + a\sin b}.$$

H.W: show that the following equations are exact and solve each one:

1- $(y^2-1)\,dx + (2xy - \sin y)\,dy = 0.$

Ans.
$y^2x - x + \cos y = k.$

2- $(2xe^y + e^x)\,dx + (x^2+1)\,e^y\,dy = 0.$

Ans.
$x^2e^y + e^x + e^y = k.$

3- $$\frac{y\ln x^2\,dy + \frac{y^2}{x}\,dx}{x\ln y^2\,dx + \frac{x^2}{y}\,dy} = 1.$$

Ans. $y^2\ln x - \frac{x^2}{2}\ln y^2 = k.$

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