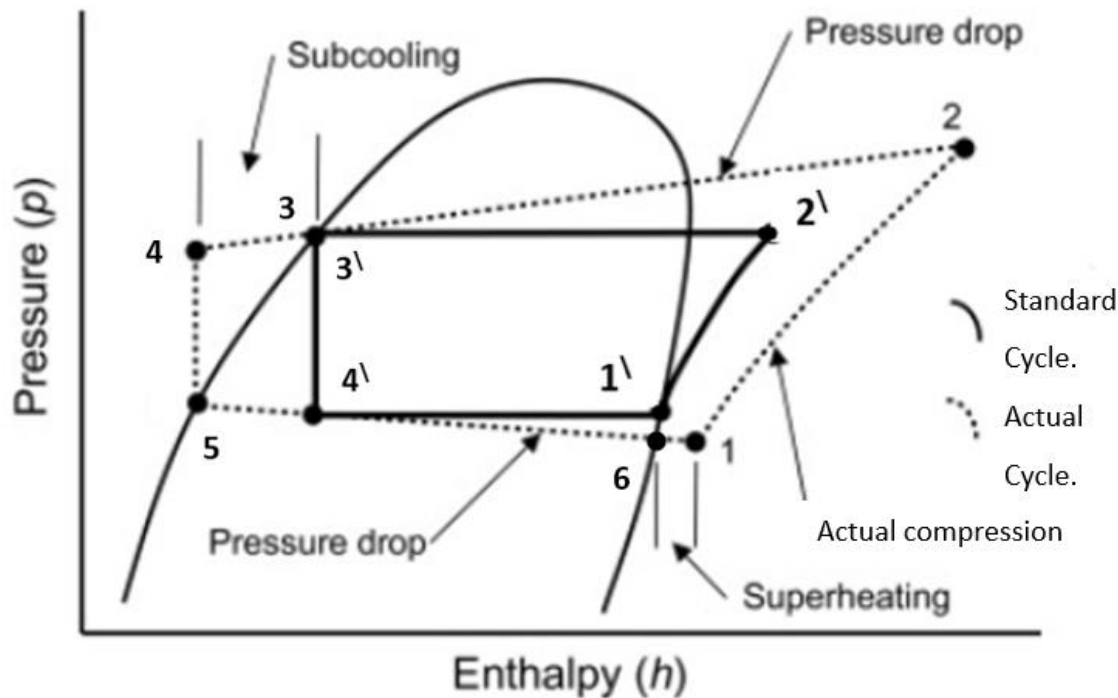




7- Actual vapour compression cycle:

— Standard cycle.

..... Actual cycle.



Main difference from standard cycle.

	Standard.	Actual.
1. Condensation.	Constant pressure.	Press. Drop due to friction in pipes.
2. Evaporation.	Constant pressure.	Press. Drop due to friction in pipes
3. Compression.	Isentropic.	Usually polytropic. (irreversible).
4. Expansion.	Isentropic.	Isentropic.



Also, liquid subcooling and vapour superheating is always insured in the actual cycle.

Points (1 & 2) determined by pressure and temperature.

Point (4) determined by temperature and subcooled liquid.

Compressor work from ideal gas relations:

The refrigerant vapour being compressed may be considered an ideal gas, where:

$$P.V^n = \text{constant.}$$

Recall the work done in a steady flow process.

$$W = \int_{P_1}^{P_2} V \cdot dP$$

$$V = c^n P^{-\frac{1}{n}} \rightarrow W = \int_{P_e}^{P_d} c^n P^{-\frac{1}{n}} \cdot dP = c^n \int_{P_e}^{P_d} P^{-\frac{1}{n}} \cdot dP$$

$$= c^n \left[\frac{P^{-\frac{1}{n}+1}}{-\frac{1}{n}+1} \right]_{P_e}^{P_d} = c^n \frac{n}{n-1} \left[P^{\frac{n-1}{n}} \right]_{P_e}^{P_d}$$

Where:

P_e =evaporator pressure.

P_d = condenser pressure.

Since $P_e V_e^n = P_d V_d^n = \text{constant.}$

$$\therefore W = \frac{n}{n-1} P_e V_e \left[\left(\frac{P_d}{P_e} \right)^{\frac{n-1}{n}} - 1 \right] \dots\dots\dots(15).$$

For isentropic compression $n = \gamma = \frac{C_p}{C_v}$

$$\text{Then } W = \frac{\gamma}{\gamma-1} P_e V_e \left[\left(\frac{P_d}{P_e} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \dots\dots\dots(15a).$$

For non-adiabatic compression (i.e. $Q \neq 0$), that is some is lost from the vapour to the compressor jacket.

$$W_C = \dot{m}[(h_2-h_1)+h_j].$$



Where h_j is the heat lost to the compressor jacket in kJ/kg or kJ/kg of refrigerant.

$$h_j = \left[\frac{n}{n-1} - \frac{\gamma}{\gamma-1} \right] P_e V_e \left[\left(\frac{P_d}{P_e} \right)^{\frac{n-1}{n}} - 1 \right] \dots\dots\dots(16).$$

Total work for (m) kg of refrigerant.

$$\dot{m} = \frac{X}{h_1 - h_4} = \frac{X}{Q_{ref}}$$

$$W = \left(\frac{X}{h_1 - h_4} \right) \left(\frac{n}{n-1} \right) P_e V_e \left[\left(\frac{P_d}{P_e} \right)^{\frac{n-1}{n}} - 1 \right] \dots\dots\dots(17).$$

Also $(P_e V_e = RT_e)$ can be used in equation (17). The temperature of discharged gas after compression is obtained from:

$$\frac{T_d}{T_e} = \left(\frac{P_d}{P_e} \right)^{\frac{n-1}{n}} \quad \text{if adiabatic } (n=\gamma).$$

.....(18).

e.g.: A refrigerant behaves as an ideal gas has a molecular mass of 64.06 per mole and specific heat ratio ($\gamma=1.26$). If the compression ratio is (3.119) and the refrigerating effect is 322kJ/kg with evaporation at 4.5°C. find:

- a) the W.D. per kw of refrigeration. (i.e. $X=1kw$).
- b) the enthalpy gains during compression.
- c) the temperature (T_d) [for the two cases of 1) isentropic compression & 2) $n=1.22$ (reversible non-adiabatic compression)].

Sol:

i) Isentropic compression.

a) $R_g = \frac{R_o}{M} = \frac{8314.66}{64.06} \text{ J/kg}^\circ\text{K} \quad (R_o \text{ in J/kg.mole.}^\circ\text{K})$

$$W_{Comp.} = \left(\frac{X}{h_1 - h_4} \right) \left(\frac{\gamma}{\gamma-1} \right) P_e V_e \left[\left(\frac{P_d}{P_e} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad W = \left(\frac{X}{h_1 - h_4} \right) \left(\frac{n}{n-1} \right) RT_e \left[\left(\frac{P_d}{P_e} \right)^{\frac{n-1}{n}} - 1 \right]$$

.....(17).



$$= \left(\frac{1}{322}\right) \left(\frac{1.26}{1.26-1}\right) \left(\frac{8314.66}{64.06}\right) \left(\frac{277.5}{1000}\right) [(3.119)^{0.26} - 1] = 0.1438 \text{kw/kw of refrigeration.}$$

b) Enthalpy gain = $h_2 - h_1$

$$W_C = \dot{m}(h_2 - h_1)$$

$$(h_2 - h_1) = \frac{W_C}{\dot{m}} = \frac{0.143}{\frac{X}{Q_{ref.}}} = \frac{0.143 \text{kw/kw ref.}}{\frac{1 \text{kw}}{322 \text{kJ/kg}}} = \frac{0.143 \text{kw/kw ref.}}{\frac{1 \text{kJ/sec}}{322 \text{kJ/kg}}} = 46.3 \text{kJ/kg.}$$

c) $T_d = T_e \left(\frac{P_d}{P_e}\right)^{\frac{\gamma-1}{\gamma}} = 277.5 * (3.119)^{(0.26/1.26)} = 351.5^\circ\text{K} = 78.5^\circ\text{C.}$

ii) Polytropic compression (n=1.22)

a)

$$W_C = \left(\frac{1}{322}\right) \left(\frac{1.22}{1.22-1}\right) \left(\frac{8314.66}{64.06}\right) \left(\frac{277.5}{1000}\right) [(3.119)^{(0.22/1.22)} - 1] = 0.1417 \text{kw/kw of refrigeration.}$$

$$W_C = \dot{m}[(h_2-h_1) + h_j].$$

$$h_j = \left[\frac{n}{n-1} - \frac{\gamma}{\gamma-1}\right] P_e V_e \left[\left(\frac{P_d}{P_e}\right)^{\frac{n-1}{n}} - 1\right] = h_j = \left[\frac{n}{n-1} - \frac{\gamma}{\gamma-1}\right] R T_e \left[\left(\frac{P_d}{P_e}\right)^{\frac{n-1}{n}} - 1\right]$$

$$h_j = \left[\frac{1.22}{0.22} - \frac{1.26}{0.26}\right] \left(\frac{8314.66}{64.06}\right) \left(\frac{277.5}{1000}\right) [(3.119)^{(0.22/1.22)} - 1] = 5.825 \text{kJ/kg.}$$

$$(h_2 - h_1) = \frac{W_C}{\dot{m}} - h_j = \frac{0.1417}{\frac{X}{Q_{ref.}}} - h_j = \frac{0.1417 \text{kw/kw ref.}}{\frac{1 \text{kw}}{322 \text{kJ/kg}}} - h_j = \frac{0.1417 \text{kw/kw ref.}}{\frac{1 \text{kJ/sec}}{322 \text{kJ/kg}}} - 5.825 = 39.78 \text{kJ/kg.}$$

$$T_d = T_e \left(\frac{P_d}{P_e}\right)^{\frac{n-1}{n}} = 277.5 * (3.119)^{(0.22/1.22)} = 341.5^\circ\text{K} = 68.5^\circ\text{C.}$$



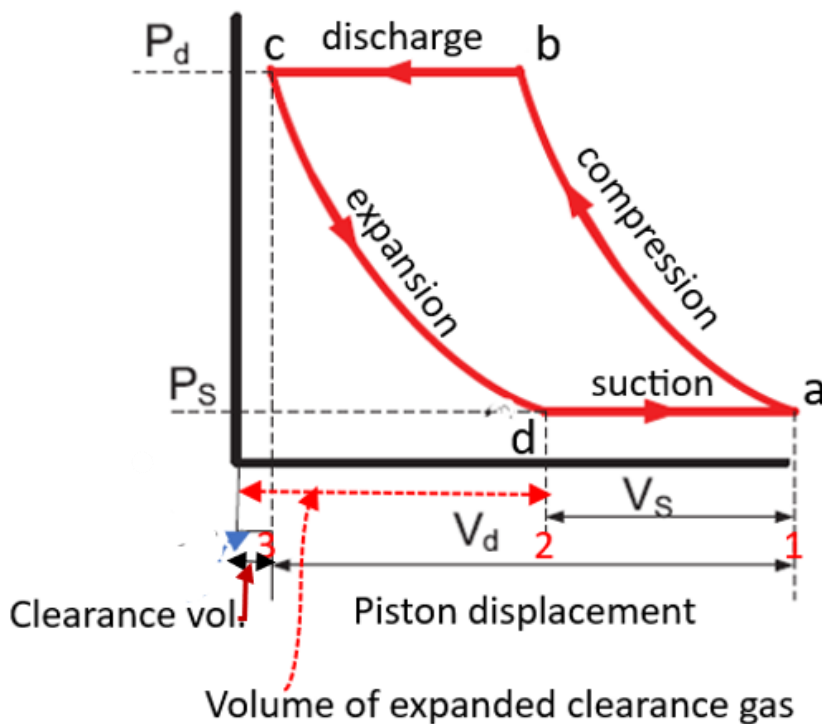
8) Volumetric efficiency:

Due to losses in the compression process, a decrease in the capacity of the ideal compressor occurs, and an increase in power is required as a result of a decrease in the amount of gas pumped.

Losses are mainly due to:

- i. Clearance volume (V_c).
- ii. Pressure losses through suction & discharge valves.
- iii. Super heating of suction gases upon contact with the cylinder walls.
- iv. Leakage past the piston.

$$\text{Clearance volumetric efficiency} = \frac{\text{the theoretical volume of fresh gas.}(\dot{V})}{\text{piston displacement (swept volume)}}$$



Clearance volumetric efficiency is obtained when the effect of clearance only is considered. This applied to the ideal compressor and gives good approximation.



c-d expansion of clearance gases.

d-a suction of fresh gases.

a-b compression.

b-c discharge [exhaust valve open to discharge compressed gas].

Clearance (C): is the ratio in percentage of the clearance volume to the piston displacement.

$$C = \frac{V_3}{V_1 - V_3} = \frac{V_c}{V_s} \dots\dots\dots(19).$$

Clearance volumetric efficiency (η_{vc}): is the volume of resh gases drawn by compressor to the piston displacement (swept volume).

$$\eta_{vc} = \frac{V_1 - V_2}{V_1 - V_3} = \frac{V_1 + V_3 - V_3 - V_2}{V_1 - V_3} = \frac{(V_1 - V_3) - (V_2 - V_3)}{V_1 - V_3} = 1 + \frac{(V_3 - V_2)}{V_1 - V_3} = 1 + \frac{V_3}{V_1 - V_3} \left(1 - \frac{V_2}{V_3}\right)$$

$$\eta_{vc} = 1 + C \left(1 - \frac{V_2}{V_3}\right)$$

$V_2 = V_s$ specific volume at suction pressure.

$V_3 = V_d$ specific volume at discharge pressure.

$$\eta_{vc} = 1 + C - C \left(\frac{V_s}{V_d}\right) \dots\dots\dots(20).$$

Since expansion of clearance gases is polytropic, then

$$P_s \cdot V_s^n = P_d \cdot V_d^n$$

$$\frac{V_s}{V_d} = \left(\frac{P_d}{P_s}\right)^{\frac{1}{n}}$$

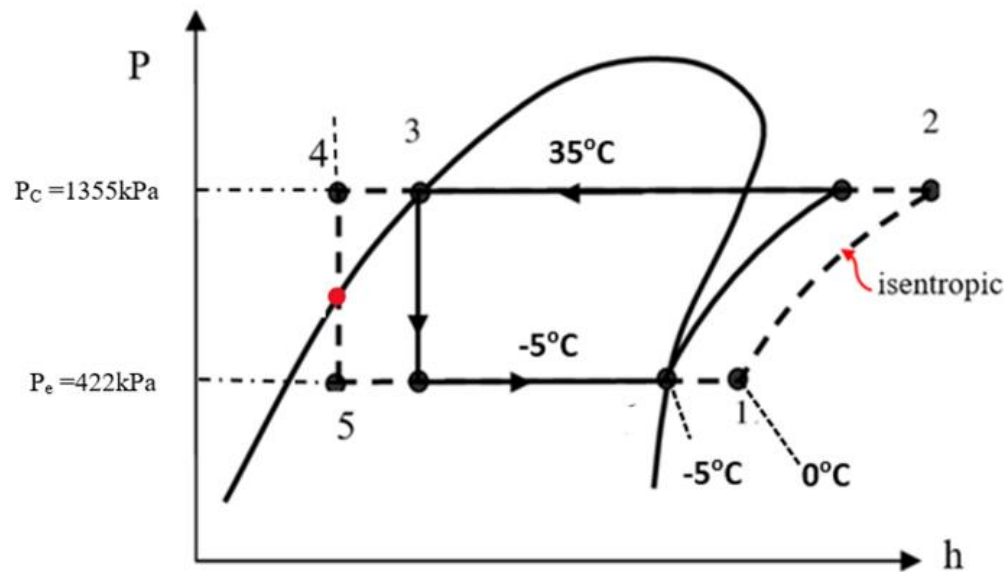
$$\eta_{vc} = 1 + C - C \left(\frac{P_d}{P_s}\right)^{\frac{1}{n}} \dots\dots\dots(20a)$$



e.g: A system using R-22 produces 15 kw of refrigeration at an evaporator temperature of -5°C and condenser temperature of 35°C . The liquid refrigerant is subcooled 5°C and the vapour is superheated 5°C before leaving the evaporator and entering the compressor. Compression is reversible adiabatic, valve throttling and clearance are to be disregarded. A two-cylinder vertical single acting compressor with stroke equal to 1.5 times the bore operating at 900 rpm is used. Take ($\gamma = 1.18$ for R22). Determine:

- i) a) theoretical power required. b) theoretical bore and stroke of compressor.
- ii) if the compressor has 2% clearance, determine:
 - a) η_{vc} . b) corrected piston displacement. c) bore and stroke of compressor.

Sol:



From tables for R-22.

$P_e = 422 \text{ kPa}$. (Saturated vapour at -5°C).

$h_4 = h_5 = 236.61 \text{ kJ/kg}$. (Saturated liquid at 35°C).

$P_c = 1355 \text{ kPa}$. (Saturated liquid at 35°C). = P_d .

From chart:

$h_1 = 407 \text{ kJ/kg}$ (at p_e & 0°C).

$h_2 = 439 \text{ kJ/kg}$, from chart at point 2, (from $1 \rightarrow 2$ constant entropy line).



$v_1 = 0.057 \text{ m}^3/\text{kg}$ from chart at point 1.

$\gamma = 1.18$ for R22.

$X = 15 \text{ kW}$.

a)

$$W_{\text{Comp.}} = \left(\frac{X}{h_1 - h_5} \right) \left(\frac{\gamma}{\gamma - 1} \right) P_e V_e \left[\left(\frac{P_d}{P_e} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]$$
$$= \left(\frac{15}{407 - 236.6} \right) * \left(\frac{1.18}{1.18 - 1} \right) * 422 * 0.057 * \left[\left(\frac{1355}{422} \right)^{\frac{1.18 - 1}{1.18}} - 1 \right]$$
$$= 2.7035 \text{ kW.}$$

$$\text{Or } W = \dot{m} * (h_2 - h_1) = \frac{X}{(h_1 - h_5)} (h_2 - h_1)$$
$$= \frac{15}{407 - 236.6} (439 - 407) = 2.8169 \text{ kW.}$$

$$a) \dot{V} = \dot{m} \cdot v_1, \quad \dot{m} = \frac{X}{Q_{\text{ref}}}$$
$$= \frac{15}{h_1 - h_5} v_1 = \frac{15}{(407 - 236.6)} * 0.057 = 0.0050176 \text{ m}^3 / \text{sec.}$$

$$\text{Displacement per piston} = \frac{0.0050176}{2} = 0.0025088 \text{ m}^3 / \text{sec.}$$

$$\dot{V} = A * 1.5d * \frac{900}{60}$$

$$0.0025088 = \frac{\pi}{4} * d^2 * (1.5d) * \frac{900}{60}$$

$$d = 0.05216 \text{ m} = 52.16 \text{ mm.}$$

$$\text{stroke} = 1.5 * 52.16 = 78.24 \text{ mm.}$$

$$ii) \quad a) \eta_{vc} = 1 + C - C \left(\frac{P_d}{P_s} \right)^{\frac{1}{\gamma}} = 1 + 0.02 - 0.02 * \left(\frac{1355}{422} \right)^{\frac{1}{1.18}} = 0.96625$$

$$b) \text{Actual piston displacement} = \frac{\dot{V}}{\eta_{vc}} = \frac{0.0050176}{0.96625} = 0.0051928 \text{ m}^3 / \text{sec.}$$



c)

$$\frac{0.0051928}{2} = \frac{\pi}{4} * d^2 (1.5d) * \frac{900}{60}$$

$$d = 0.05276 \text{ m} = 52.76 \text{ mm.}$$

$$\text{stroke} = 1.5 d = 1.5 * 52.76 = 79.14 \text{ mm.}$$