Boolean Algebra and Logic Gates

المحاضرة الثالثة

م.م. رؤى ستار جبار

Boolean Algebra

Developed by George Boole in 1854

Formal way to describe logic statements and determine truth of statements.

Only has two-values domain (0 and 1)

- Algebra with Binary (Boolean) Variable and Logic Operations.

George Boole
1815-1864

- Boolean Algebra is useful in Analysis and Synthesis of Digital Logic Circuits.

  Input and Output signals can be represented by Boolean Variables, and Function of Digital Logic Circuits can be represented by Logic Operations, i.e., Boolean Functions.

- From a Boolean function, a logic diagram can be constructed using AND, OR, and I .

Binary Logic and Gates

- **Binary variables take on one of two values.**
- **Logical operators operate on binary values and binary variables.**
- **Basic logical operators are the logic functions AND, OR and NOT.**
- **Logic gates implement logic functions.**
- **Boolean Algebra: a useful mathematical system for specifying and transforming logic functions.**
- **We study Boolean algebra as a foundation for designing and analyzing digital systems!**

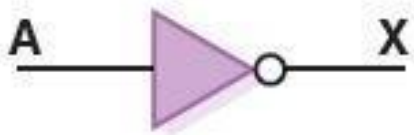
Basic Logic Gates

Six types of gates

–NOT –AND –OR –XOR –NAND –NOR

NOT Gate

A NOT gate accepts one input signal (0 or 1) and returns the complementary (opposite) signal as output

Boolean Expression	Logic Diagram Symbol	Truth Table
$X = A'$		

A	X
0	1
1	0

Truth Table:

The most elementary specification of the function of a Digital Logic Circuit is the Truth Table.

Table that describes the Output Values for all the combinations of the Input Values, called *MINTERMS*

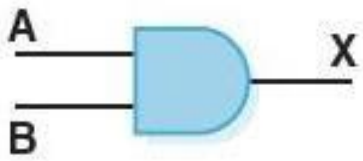
AND Gate

An AND gate accepts two input signals If both are 1, the output is 1; otherwise, the output is 0

Boolean Expression

$X = A \cdot B$

Logic Diagram Symbol



Truth Table

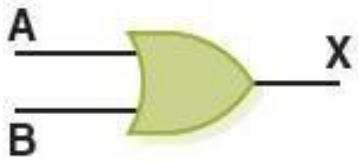
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate

An OR gate accepts two input signals ,If both are 0, the output is 0; otherwise,the output is 1

Boolean Expression	Logic Diagram Symbol	Truth Table

$X = A + B$



A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

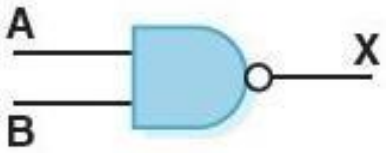
NAND Gate

The NAND (“NOT of AND”) gate accepts two input signals If both are 1, the output is 0; otherwise,the output is 1

Boolean Expression

$X = (A \cdot B)'$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate

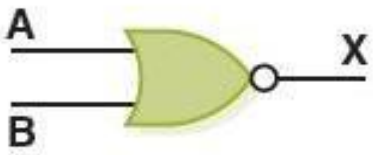
The NOR (“NOT of OR”) gate accepts two inputs

If both are 0, the output is 1; otherwise,the output is 0

Boolean Expression

$X = (A + B)'$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

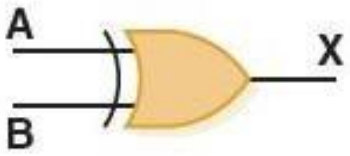
XOR Gate

An XOR gate accepts two input signals,If both are the same, the output is 0; otherwise,the output is 1

Boolean Expression

$X = A \oplus B$

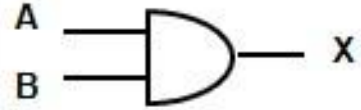
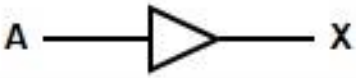
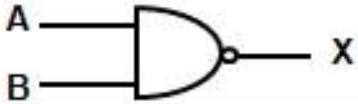
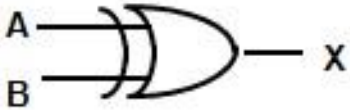
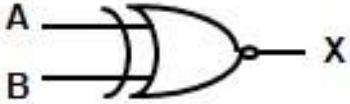
Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Combinational Logic Gates

Name	Symbol	Function	Truth Table
AND		$X = A \cdot B$ or $X = AB$	A B | X 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$X = A + B$	A B | X 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
I		$X = \overline{A}$	A | X 0 | 1 1 | 0
Buffer		$X = A$	A | X 0 | 0 1 | 1
NAND		$X = (\overline{AB})$	A B | X 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$X = (\overline{A + B})$	A B | X 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
XOR Exclusive OR		$X = A \oplus B$ or $X = \overline{A}B + A\overline{B}$	A B | X 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0
XNOR Exclusive NOR		$X = (\overline{A \oplus B})$ or $X = \overline{A}\overline{B} + AB$	A B | X 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1