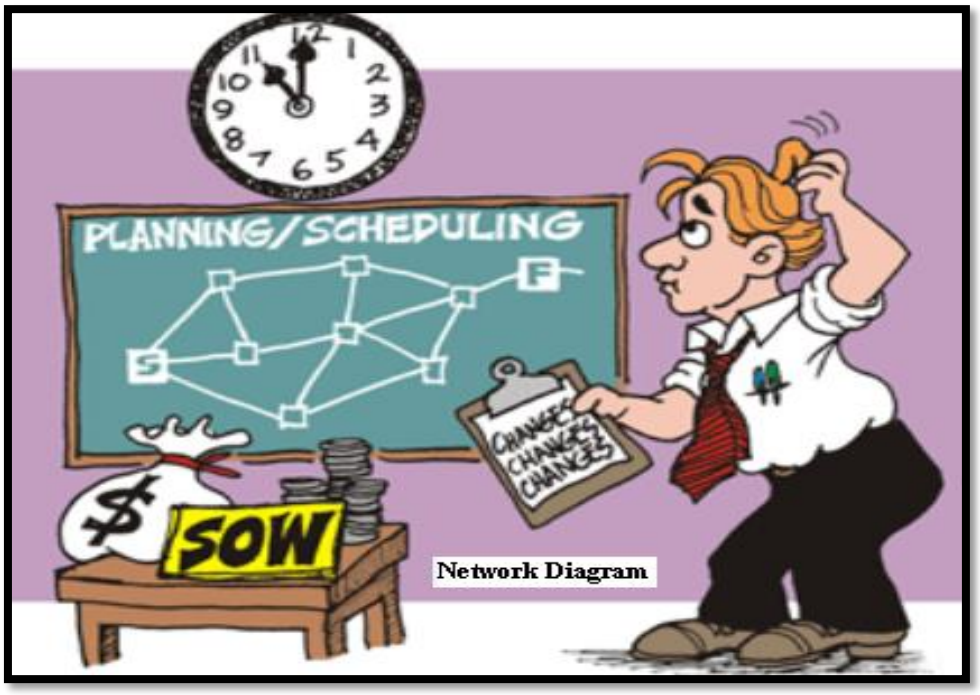
Electrical and Electronic Technical College
Computer Engineering Department

Mode Unit in study

Project Planning Techniques
Network analysis
Critical Path Method (CPM)

For

Students of Fourth Stage
Computer Department



By

Department Of computer Engineering

1. Overview

a. Target Population: For students of stage of second and fourth stage in technical Colleges and institutes in foundation of technical education.

b. Rationale: The most common and widely used project management technique that can be classified under the title of Network Analysis is Critical Path Method (CPM). It was developed in the 1950's to help managers schedule, monitor and control large and complex projects. CPM was first used in 1957 to assist in the development and building of chemical plants within the DuPont corporation.

a. Central Ideas: The basic purpose of a network analysis is to help managers schedule, monitor and control large and complex projects.

b. Objectives: The student will be able after finishing lecture on:

- Define Critical Path Method (CPM).
- Study steps of CPM.

2. Pre-Test:

1. Define the term 'CPM.
2. CPM predicts the time required to complete the project— State True or False
3. The time between its earliest and latest start time, or between its earliest and latest finish time of an activity is

   a) delay time b) slack time c) critical path d) start time
4. The path through the project network in which none of the activities have slack is called
   a) start time b) slack time c) critical path d) delay time
5. Activity is an ---------- needed for the completion of a project.

Note: Check your answers in "Answer Keys" in end of mode unit. If you obtain 75% of solution, you cannot need to this mode unit. If your answer is poor, you will transfer to next page.

3. Theory:

Introduction

Critical Path Method (CPM) or (Calculate Schedule) is a modeling process that defines all the project's critical activities which must be completed on time. CPM models the activities and events of a project as a network.

Steps in CPM Project Planning

1. Specify the individual activities.
2. Determine the sequence of those activities.
3. Draw a network diagram.
4. Estimate the completion time for each activity.
5. Identify the critical path (longest path through the network)
6. Update the CPM diagram as the project progresses.

CPM Benefits

- Provides a graphical view of the project.
- Predicts the time required to complete the project.
- Shows which activities are critical to maintaining the schedule and which are not.

Critical path is the longest-duration path through the network. The significance of the critical path is that the activities that lie on it cannot be delayed without delaying the project. Because of its impact on the entire project, critical path analysis is an important aspect of project planning. The critical path can be identified by determining the following four parameters for each activity:

1. Earliest Start time (ES): the earliest time at which the activity can start given that its precedent activities must be completed first.
2. Earliest Finish time (EF), equal to the earliest start time for the activity plus the time required completing the activity.
3. Latest Finish time (LF): the latest time at which the activity can be completed without delaying the project.
4. Latest Start time (LS), equal to the latest finish time minus the time required to complete the activity.

4. Self-Test:

1. Define the term “critical path”.
2. List the benefits of CPM.

The **slack time** or **Total float** for an activity is the time between its earliest and latest start time, or between its earliest and latest finish time. Slack is the amount of time that an activity can be delayed past its earliest start or earliest finish without delaying the project.

The critical path is the path through the project network in which none of the activities have slack, that is, the path for which ES=LS and EF=LF for all activities in the path. A delay in the critical path delays the project. Similarly, to accelerate the project it is necessary to reduce the total time required for the activities in the critical path.

Activity is an individual task needed for the completion of a project.
Duration is the length of time (hours, days, weeks, months) needed to complete an activity.

Float is the amount of time that an activity can slip past its duration without delaying the rest of the project.

Free float is the excess time available before the start of the following activity.

3. The time between its earliest and latest start time, or between its earliest and latest finish time of an activity is
   a) delay time b) slack time c) critical path d) start time
4. ---------- is the longest-duration path through the network.

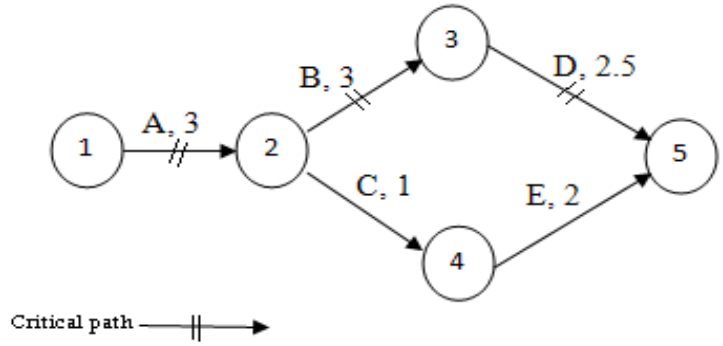
Activity on arrow (A.O.A)

Examples

Ex1: Determine the critical path by using CPM of the following Table (project),

Activities	Path	Duration (day)	Description
A	1 ─ 2	3	وصف مختصر لكل فعالية
B	2 ─ 3	3	
C	2 ─ 4	1	
D	3 ─ 5	2.5	
E	4 ─ 5	2	

Ans:

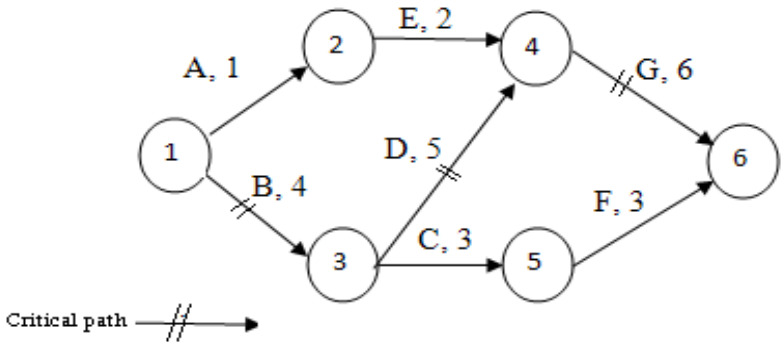


Critical path =3+ 3+ 2.5= 8.5 days

Ex2: Determine the critical path by using CPM of the following Table (project),

Activities	Path	Duration (week)	Description
A	1 — 2	1	وصف مختصر لكل فعالية
B	1 — 3	4	
C	3 — 5	3	
D	3 — 4	5	
E	2 — 4	2	
F	5 — 6	3	
G	4 — 6	6	

Ans:



Critical path= 4+ 5+ 6= 15 weeks

Dummy Activity: An imaginary activity that requires no time and is used to correctly maintain the appropriate precedence relationships.

5. Determine the critical path by using CPM of the following Table (project),

Activities	Path	Duration (day)	Description
A	1 – 2	2	تحضير الموقع
B	2 – 3	3	تخطيط الموقع
C	3 – 4	8	انشاء الاعمدة
D	3 – 5	5	بناء الجدران الخارجية
E	5 – 6	4	بناء السقف

5. Post- Test

1. What is slack time in critical path in CPM?
2. Which of the following statements about critical path analysis (CPA) is true?
   a) The critical path is the longest path through the network
   b) The critical path is the shortest path through the network
   c) Tasks with float will never become critical
   d) The network should remain constant throughout the project
3. In Critical Path of CPM used in project planning techniques indicates----------.
   a) time require for the completion of the project
   b) delays in the project
   c) early start and late end of the project
   d) none of the above
4. Dummy Activity is an imaginary activity that requires no time and is used to correctly maintain the appropriate precedence relationships. State True or False
5. Determine the critical path by using CPM of the following Table (project),

Activities	Path	Duration (day)	Description
A	1 – 2	3	وصف مختصر لكل فعالية
B	2 – 3	3	
C	2 – 4	1	
D	3 – 5	2.5	
E	4 – 5	2	

Note: Check your answers in "Answer Keys" in end of mode unit.

6. References

1. Principles of Project Management, NPC publication
2. S. Choudhury "Project Management", Tata McGraw Hill – 2003
3. W. Durfee and T. Chase, "Project Management - Gantt Chart Tutorial" University of Minnesota, 2003
4. http://www.projectmanagement.com/main.htm

Answer Keys

Pre- Test

1. **Critical Path Method (CPM)** or (Calculate Schedule) is a modeling process that defines all the project's critical activities which must be completed on time. CPM models the activities and events of a project as a network.
2. True.
3. b) slack time
4. c) critical path
5. individual task

Self-Test

1. Critical path is the longest-duration path through the network. The main objective of a Gantt chart is to assess how long a project should take and to establish the order in which tasks need to be carried out by the ending of the project.
2.

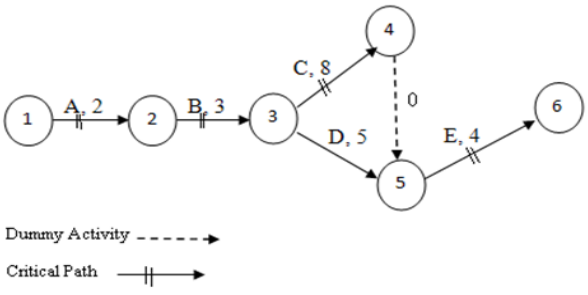
• Provides a graphical view of the project.

• Predicts the time required to complete the project.

• Shows which activities are critical to maintaining the schedule and which are not.

3. b) slack time
4. Critical path
5.

Ans:

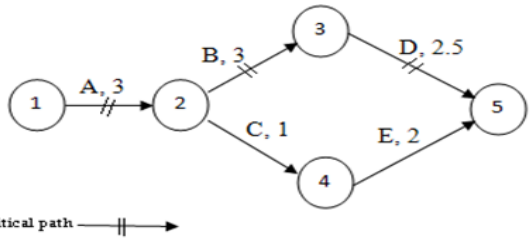


Critical path= 2+ 3+ 8+4= 17 days

Post- Test

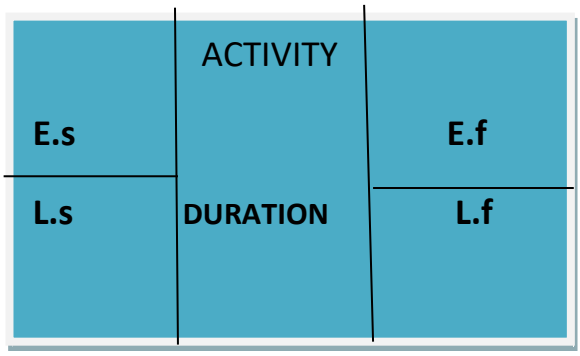
1. The slack time or Total float for an activity is the time between its earliest and latest start time, or between its earliest and latest finish time
2. a) The critical path is the longest path through the network
3. a) time require for the completion of the project
4. True
5.

Ans:



Critical path =3+ 3+ 2.5= 8.5 days

Activity on anode (A.O.N)

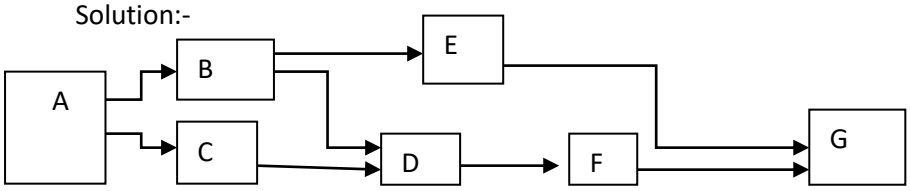


Example 3:- by using activity on Node(A.O.N)

Determine the critical path by using CPM of the following Table (project),

Activities	Preceded by	Duration (week)	Description
A	None	1	DESCIPTION OF ACTIVITY
B	A	4	
C	A	3	
D	B, C	5	
E	B	2	
F	D	3	
G	E,F	6	

Solution:-



PATH1:-A,B,E,G=1,4,2,6=13 WEEKS

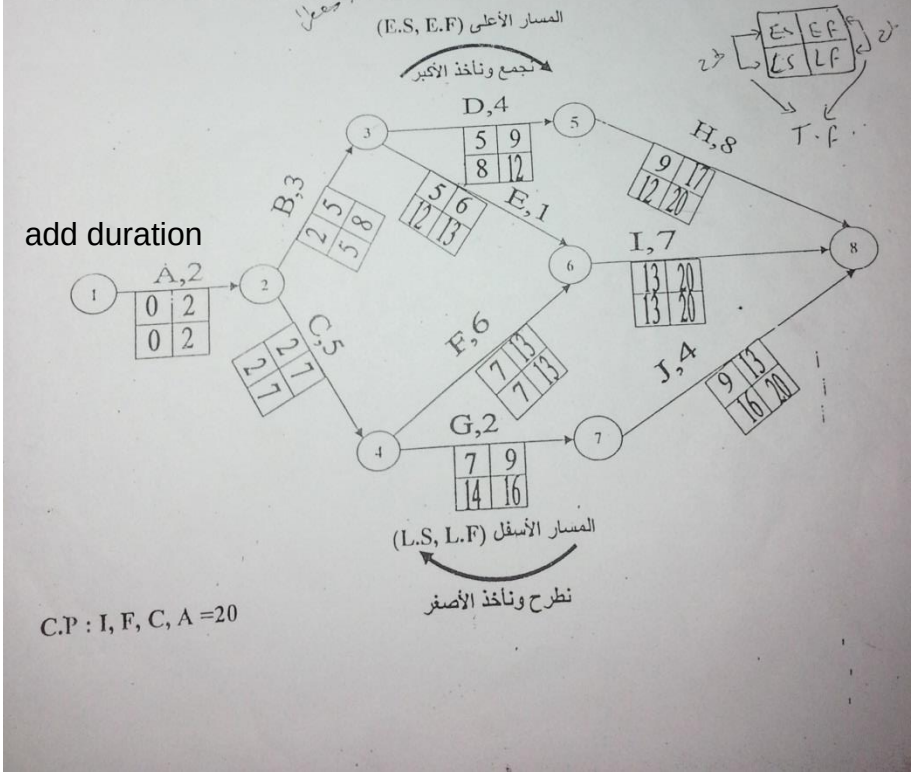
PATH 2:-A,B,D,F,G=1,4,5,3,6=19 WEEKS

PATH3:- A,C,D,F,G=1,3,5,3,6=18 WEEKS CRTICAL PATH=19 WEEKS

Example

Data for a small project are given, draw the net work and determine the E.S, E.F, L.S, L.F, T.F, and C.P?

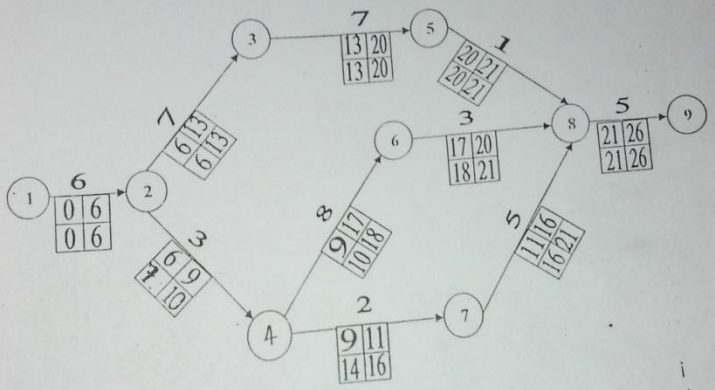
Act.	Initial & finish	D	E.S	E.F	L.S	L.F	T.F
A	1-2	2	0	2	0	2	0
B	2-3	3	2	5	5	8	3
C	2-4	5	2	7	2	7	0
D	3-5	4	5	9	8	12	3
E	3-6	1	5	6	12	13	7
F	4-6	6	7	13	9	13	0
G	4-7	2	7	9	14	16	7
H	5-8	8	9	17	12	20	3
I	6-8	7	13	20	13	20	0
J	7-8	4	9	13	16	20	7



C.P : I, F, C, A =20

Activity	Duration	E.S	E.F	L.S	L.F	Total Float
1-2	6	0	6	0	6	0
2-3	7	6	13	6	13	0
2-4	3	6	9	7	10	1
3-5	7	13	20	13	20	0
4-6	8	9	17	10	18	1
4-7	2	9	11	14	16	5
5-8	1	20	21	20	21	0
6-8	3	17	20	18	21	1
7-8	5	11	16	16	21	5
8-9	5	21	26	21	26	0

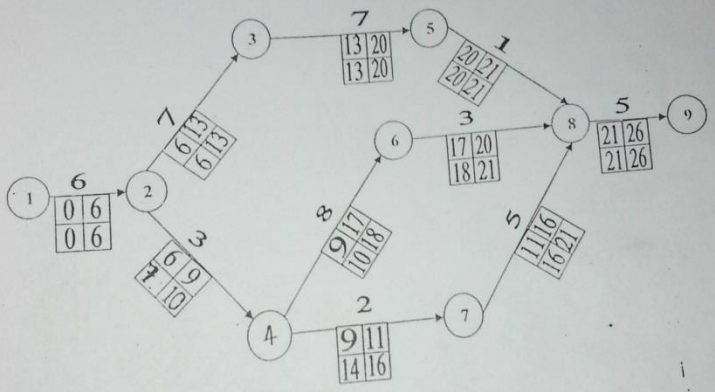
C.P. :6+7+7+1+5=26

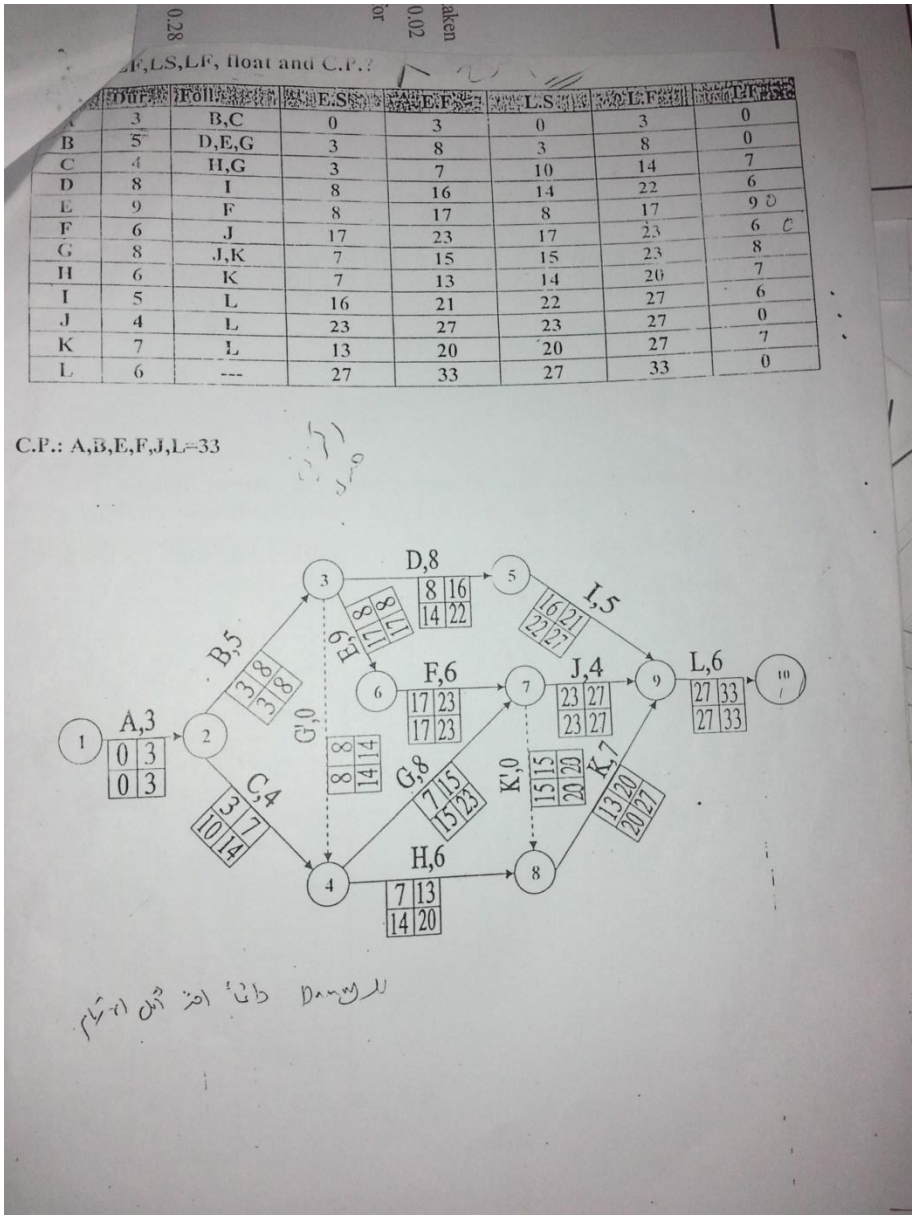


of n = 12 observations have been taken
ss. The average sample range was 0.02
ne upper and lower control limits for

Activity	Dur.	E.S	E.F	L.S	L.F	Total Float
1-2	6	0	6	0	6	0
2-3	7	6	13	6	13	0
2-4	3	6	9	7	10	1
3-5	7	13	20	13	20	0
4-6	8	9	17	10	18	1
4-7	2	9	11	14	16	5
5-8	1	20	21	20	21	0
6-8	3	17	20	18	21	1
7-8	5	11	16	16	21	5
8-9	5	21	26	21	26	0

C.P. :6+7+7+1+5=26





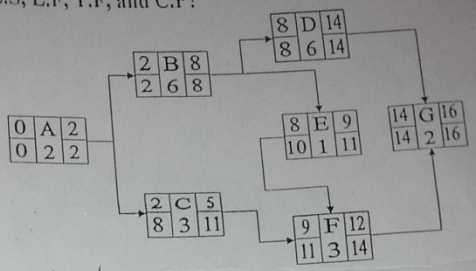
F,LS,LF, float and C.P.?

	Dur.	Foll.	E.S.	E.F.	L.S.	L.F.	T.F.
A	3	B,C	0	3	0	3	0
B	5	D,E,G	3	8	3	8	0
C	4	H,G	3	7	10	14	7
D	8	I	8	16	14	22	6
E	9	F	8	17	8	17	0
F	6	J	17	23	17	23	0
G	8	J,K	7	15	15	23	8
H	6	K	7	13	14	20	7
I	5	L	16	21	22	27	6
J	4	L	23	27	23	27	0
K	7	L	13	20	20	27	7
L	6	---	27	33	27	33	0

C.P.: A,B,E,F,J,L=33

a small project are given, draw the net work by using (A.O.N.) and ...ine the E.S, E.F, L.S, L.F, T.F, and C.P?

...ct.	D.	Pre.	T.F.
A	2	None	0
B	6	A	0
C	3	A	6
D	6	B	0
E	1	B	2
F	3	C,E	2
G	2	D,F	0



C.P. : G,D,B,A=16

H.W

Example

Data for a small project are given, draw the net work by using (A.O.N.) and A.O.A), determine the E.S, E.F, L.S, L.F, T.F, and C.P?

Activity On Node (A.O.N.)
Activity On Arrow (A.O.A)

...ct.	D.	Pre.	Foll.	T.F.
A	3	None	B,C,D	0
B	5	A	E	7
C	4	A	F,G	4
D	6	A	G,H	0
E	4	B	I	7
F	5	C	J	4
G	3	C,D	K	7
H	6	D	L	0
I	5	E	N	7
J	7	F	O	4
K	4	G	P	7
L	5	H	M,Q	0
M	3	L	P	0
N	4	I	S	7
O	5	J	S,T	4
P	6	K,M	T	0
Q	4	L	R	1
R	4	Q	T	1
S	5	N,O	U	7
T	4	O,P,R	U	0
U	3	T,S	---	0

Project Crashing and Time-Cost Trade-Off
General Relationship of Time and Cost (2 of 2)

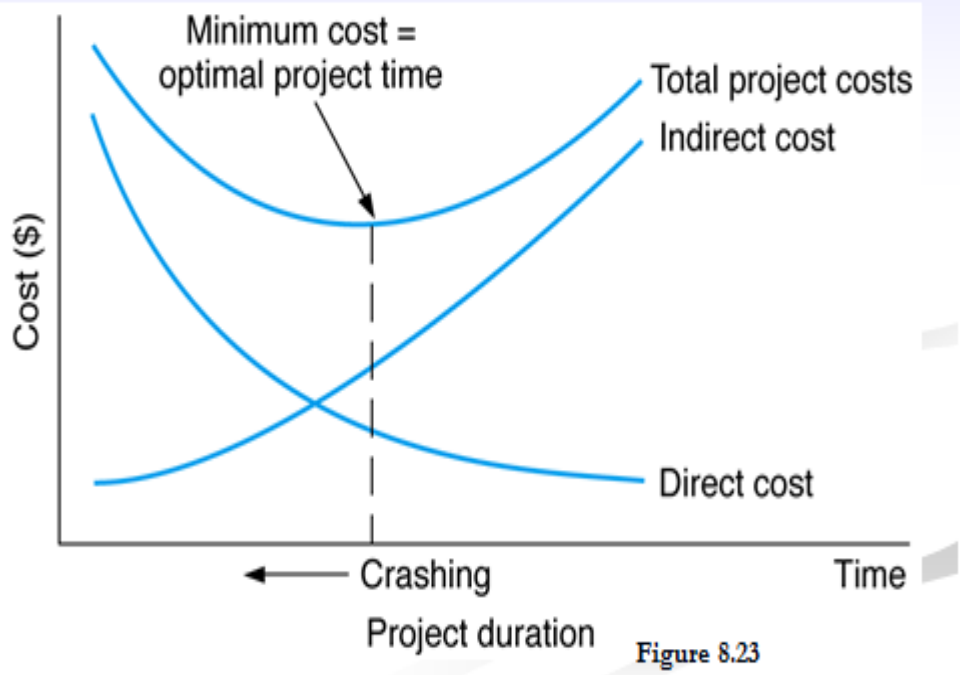


Figure 8.23
The Time-Cost Trade-Off

Project Crashing
Solution!

Yes, the project duration can be reduced **by assigning more resources** to project activities. But, doing this would somehow increase our project cost!

How do we strike a balance?

- ***Project crashing*** is a method for shortening project duration by reducing one or more ***critical activities*** to a time less than normal activity time.

3

8-3

Trade-Offs And Project Crashing

It is not uncommon to face the following situations:

- ☑ *The project is behind schedule*
- ☑ *The completion time has been moved forward*

Shortening the duration of the project is called project crashing

Factors to Consider When Crashing A Project

- ☑ *The amount by which an activity is crashed is, in fact, permissible*
- ☑ *Taken together, the shortened activity durations will enable us to finish the project by the due date*
- ☑ *The total cost of crashing is as small as possible*

Steps in Project Crashing

1. Compute the crash cost per time period. If crash costs are linear over time:

$$\text{Crash cost per period} = \frac{(\text{Crash cost} - \text{Normal cost})}{(\text{Normal time} - \text{Crash time})}$$

2. Using current activity times, find the critical path and identify the critical activities

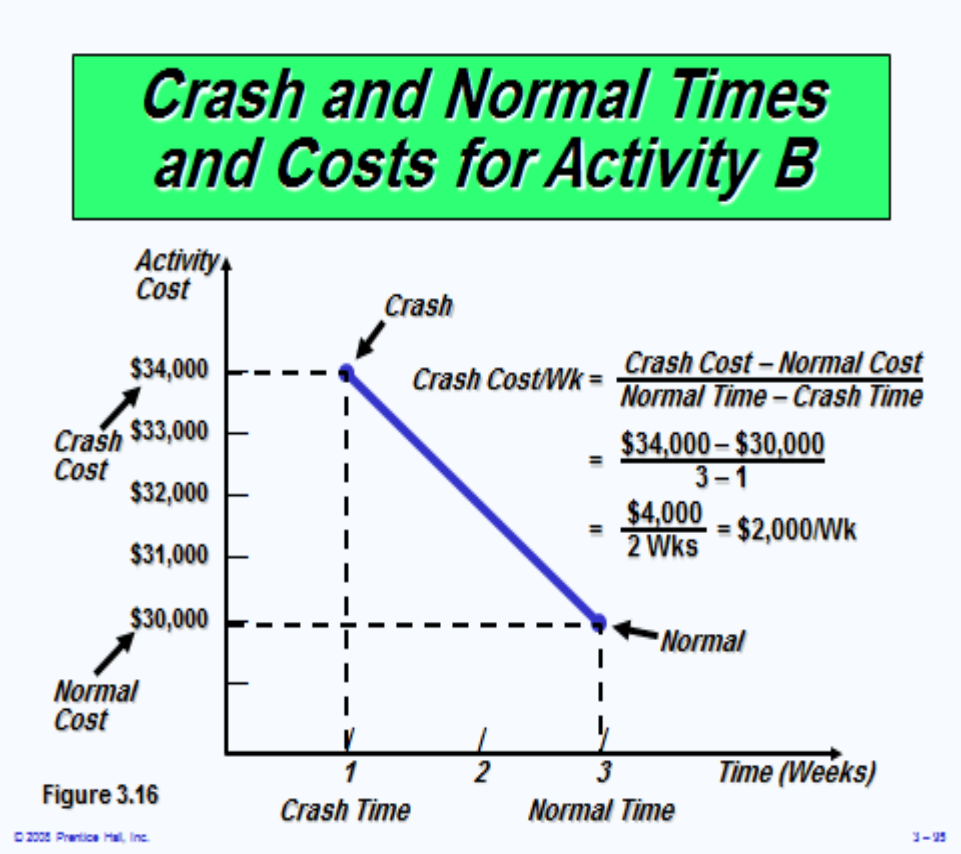
Steps in Project Crashing

3. If there is only one critical path, then select the activity on this critical path that (a) can still be crashed, and (b) has the smallest crash cost per period. If there is more than one critical path, then select one activity from each critical path such that (a) each selected activity can still be crashed, and (b) the total crash cost of all selected activities is the smallest. Note that the same activity may be common to more than one critical path.

Steps in Project Crashing

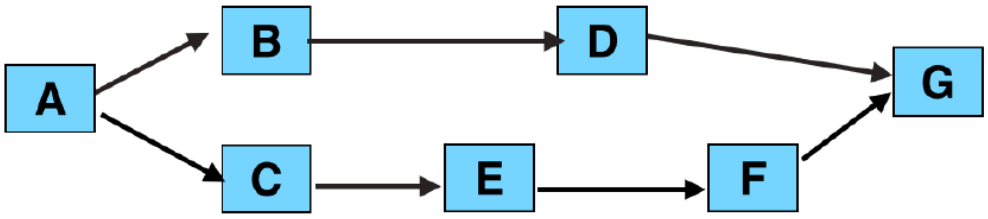
4. Update all activity times. If the desired due date has been reached, stop. If not, return to Step 2.

For example


Crash and Normal Times and Costs for Activity B
Activity Cost
Crash
$34,000
$33,000
$32,000
$31,000
$30,000
Crash Cost
Normal Cost
Crash Cost/Wk = Crash Cost – Normal Cost / Normal Time – Crash Time
= $34,000 – $30,000 / 3 – 1
= $4,000 / 2 Wks = $2,000/Wk
Normal
1
2
3
Time (Weeks)
Crash Time
Normal Time
Figure 3.16

Exercise 1, Part A.
You are given the following data about the project tasks, network, and crash times/costs. Calculate the cost of the project at all time durations until you can no longer crash the project any further.

ID	Direct costs				Slope	Maximum Crash Time
	Normal		Crash			
	Time	Cost	Time	Cost		
A	5	$500	4	$600	**$100**	**1**
B	10	$1200	6	$2000	**$200**	**4**
C	13	$3600	11	$4800	**$600**	**2**
D	13	$300	11	$600	**$150**	**2**
E	5	$1000	4	$1400	**$400**	**1**
F	10	$2400	8	$5400	**$1500**	**2**
G	5	$700	5	$700	**$0**	**0**
		$9700				

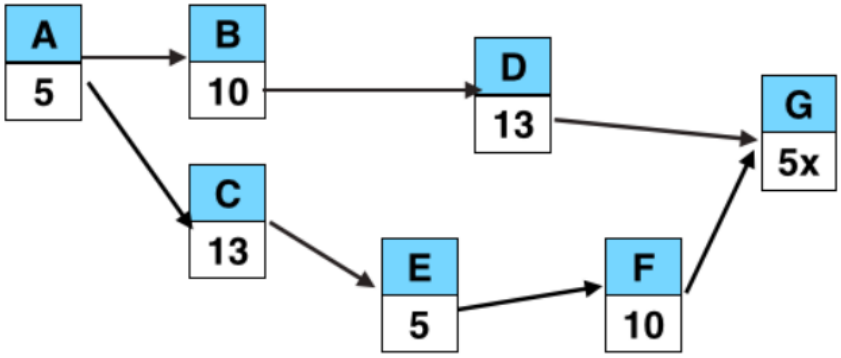


Exercise 1, Part B: Initial Network
Calculate the critical path(s) and the direct costs of this project:

Path 1: A->B->D->G Duration: **33** Project Dur: **38**

Path 2: A->C->E->F->G Duration: **38**

Total Direct Costs: $9700



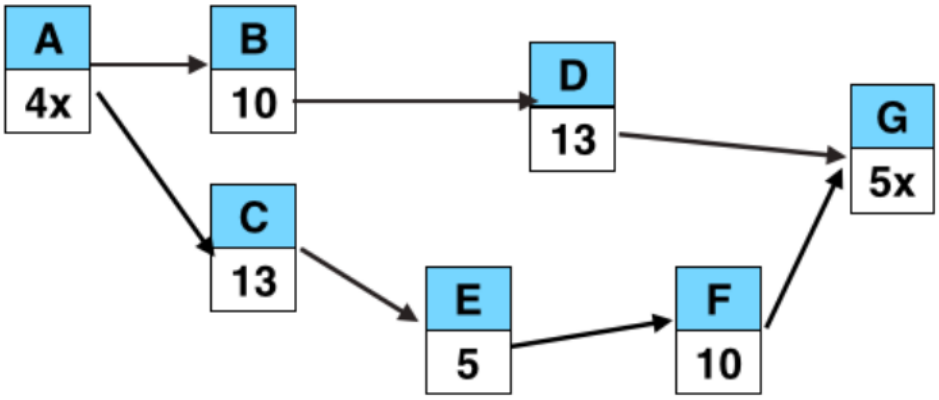
Exercise 1, Part C.
If possible, calculate the critical path(s), and the additional costs of crashing this project by another 1 day:

Path 1: A->B->D->G Duration: **32** Project Dur: **37**

Path 2: A->C->E->F->G Duration: **37**

Normal Duration Direct Costs: $9700 Additional Costs of Crashing: **$100 (A)**

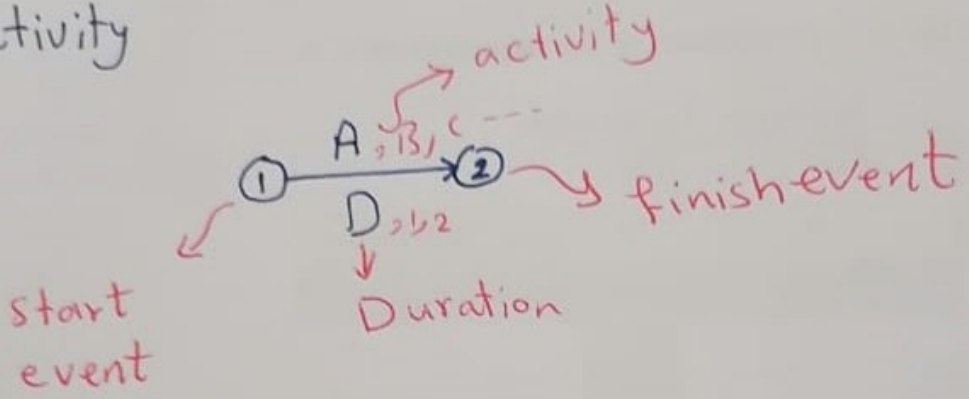
Total Cost at this time period: **$9800**



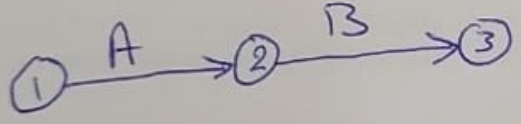
Network analysis

i) Activity on Arrow. (A.O.A)

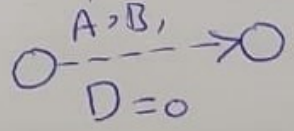
1- activity

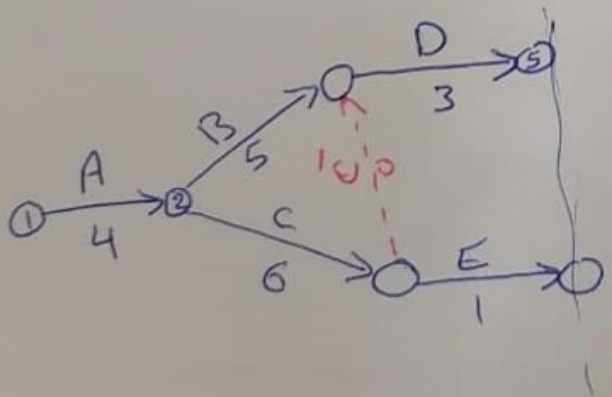


② Connection



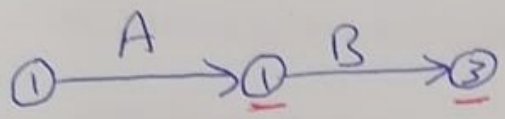
③ Dummy activity

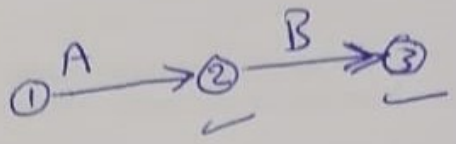


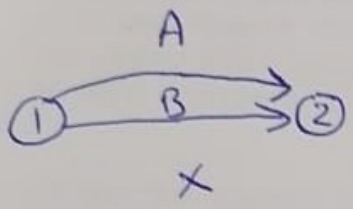


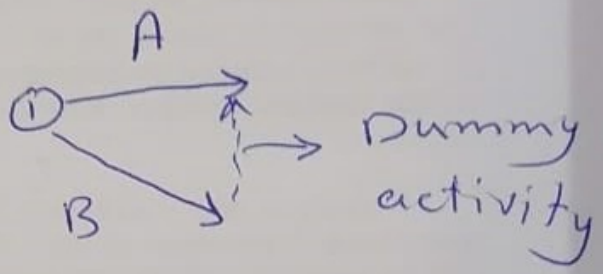
activity	D	Preceded by
A	4	None
B	5	A
C	6	A
D	3	B, C
E	1	C

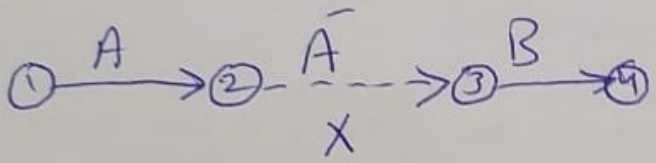
* errors for Drawing (A-o-A)

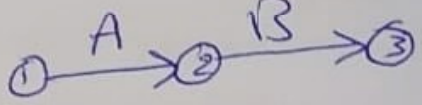
1- 

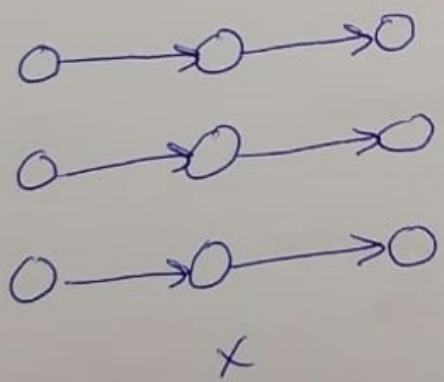


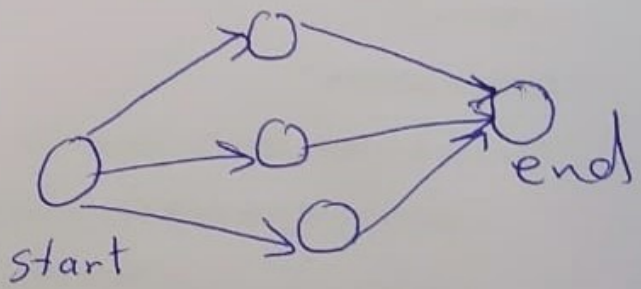
2- 



3- 



4- 



Ex:- Draw the Network by using (A-O-A)

activity	Duration	Preceded by
A	2	None
B	6	A
C	6	B
D	1	B
E	3	A
F	3	E, D
G	2	C, F

Sol.