Al-Mustaqbal University / College of Engineering & Technology
Department Computer of engineering techniques

Class (1)
Assist. Lect. Saja Mohsen Abood

AC circuit Theory

Lecture (2)

Alternating Current Circuit with Resistor and impedance

Al-Mustaqbal University / College of Engineering & Technology
Department Computer of engineering techniques

Class (1)
Assist. Lect. Saja Mohsen Abood

AC circuit Theory

1. Introduction

Impedance is the equivalent of resistance, but depends on frequency. Admittance is the equivalent of conductance and also depends on frequency. Impedance and admittance are defined for the resistor, inductor, and capacitor, and are used to analyze ac circuits. For a given frequency, the impedances for inductors and capacitors are complex numbers. If a circuit is driven by a sinusoidal signal, the steady-state response of the circuit can be found by transforming the circuit to the phasor domain first, and then applying the circuit laws and theorems. The circuit laws and theorems for resistive circuits apply to the phasor-transformed circuits also. The differences are that impedances are used instead of resistances, and the values are complex instead of real numbers

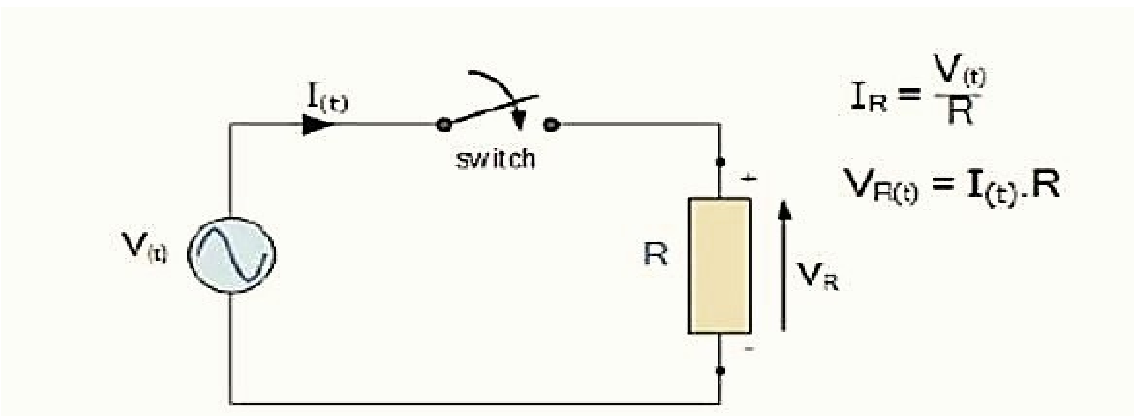


Fig.1

When the switch is closed, an AC voltage, V will be applied to resistor, R. This voltage will cause a current to flow which in turn will rise and fall as the applied voltage rises and falls sinusoidal. As the load is a resistance, the current and voltage will both reach their maximum or peak values and fall through zero at exactly the same time, i.e. they rise and fall simultaneously and are therefore said to be "in-phase".

Then the electrical current that flows through an AC resistance varies sinusoidal with time and is represented by the expression, **$I(t) = I_m \sin(\omega t + \theta)$**, where I_m is the maximum amplitude of the current and θ is its phase angle. In addition we can also say that for any given current, **I** flowing through the resistor the maximum or peak voltage across the terminals of **R** will be given by Ohm's Law as:

$$V_{(t)} = R \cdot I_{(t)} = R \cdot I_m \sin(\omega t + \theta)$$

Where:

Resistance (in Ohms, Ω) :**R**

The maximum (peak) current: I_m

Angular frequency (equal to 2πf) :**ω**

Time :**t**

The phase angle : θ

Instantaneous Current

$$i_{R(t)} = I_{R(\max)} \sin(\omega t)$$

So for a purely resistive circuit the alternating current flowing through the resistor varies in proportion to the applied voltage across it following the same sinusoidal pattern. As the supply frequency is common to both the voltage and current, their phasors will also be common resulting in the current being "in-phase" with the voltage, **(θ = 0).**
In other words, there is no phase difference between the current and the voltage when using an AC resistance as the current will achieve its maximum, minimum and zero

values whenever the voltage reaches its maximum, minimum and zero values as shown below

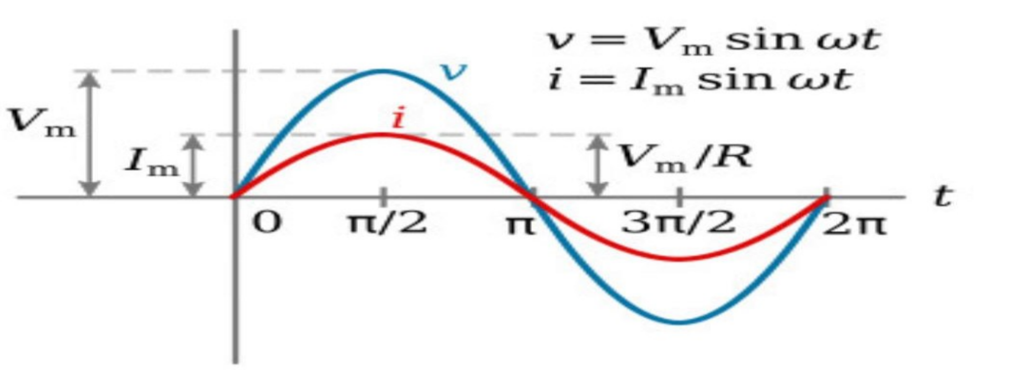


Fig.2

2. This “**in-phase**” effect can also be represented by a phasor diagram. In the complex domain, resistance is a real number only meaning that there is no “j” or imaginary component. Therefore, as the voltage and current are both in-phase with each other, there will be no phase difference (θ = 0) between them, so the vectors of each quantity are drawn super-imposed upon one another along the same reference axis. The transformation from the sinusoidal time-domain into the phasor-domain is given as.

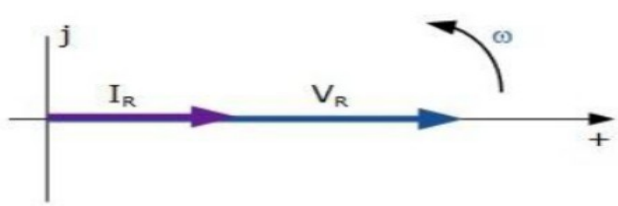


Fig.3

3. **Phase difference**: refers to the angular displacement between different waveforms of the same frequency. Consider Figure. If the angular

displacement is 0° as in (a), the waveforms are said to be in phase; otherwise, they are out of phase. When describing a phase difference, select one waveform as reference. Other waveforms then lead, lag, or are in phase with this reference. For example, in (b), for reasons to be discussed in the next paragraph, the current waveform is said to lead the voltage waveform, while in (c) the current waveform is said to lag

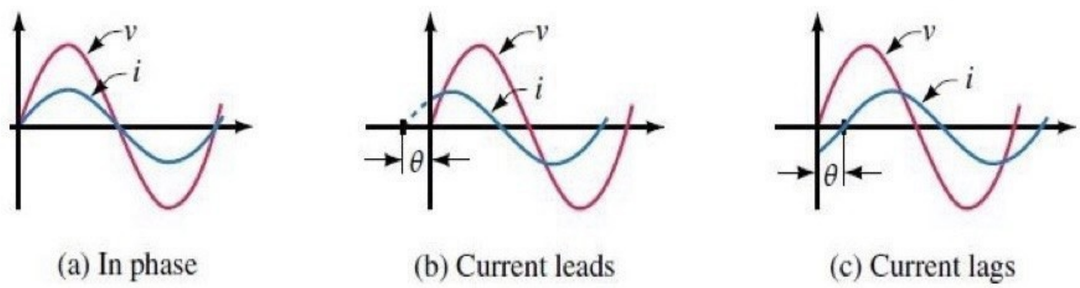


Fig.4

Examples: What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. v = 10 sin(ωt + 30°), i = 5 sin(ωt + 70°)

b. i = 15 sin(ωt + 60°), v = 10 sin(ωt - 20°)

c. i = 2 cos(ωt + 10°), v = 3 sin(ωt - 10°)

d. i = - sin(ωt + 30°), v = 2 sin(ωt + 10°) …. homework

e. i = - 2 cos(ωt - 60°), v = 3 sin(ωt - 150°) ….homework

Al-Mustaqbal University / College of Engineering & Technology
Department Computer of engineering techniques

Class (1)
Assist. Lect. Saja Mohsen Abood

AC circuit Theory

Solution:

a. See Fig. **5**

i leads v by 40°, or v lags i by 40°.

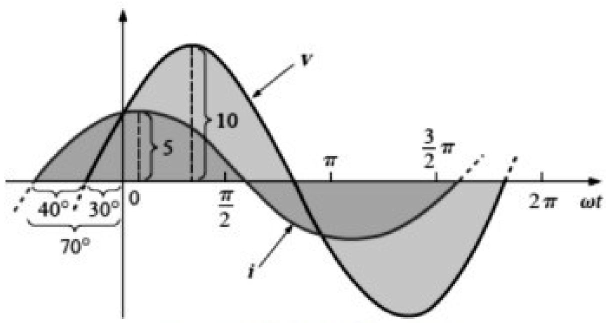


Fig. 5 i leads v by 40°.

b. See Fig. **6**

i leads v by 80°, or v lags i by 80°.

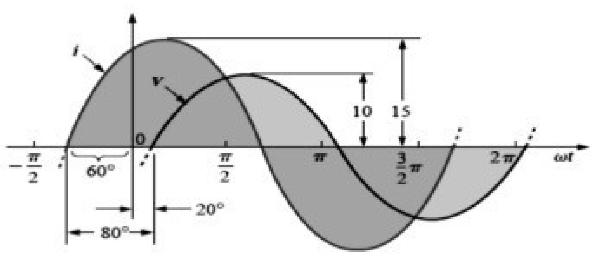


Fig. 6 i leads v by 80°.

c. See Fig. **7**

$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$

$= 2\sin(\omega t + 100°)$

Al-Mustaqbal University / College of Engineering & Technology
Department Computer of engineering techniques

Class (1)
Assist. Lect. Saja Mohsen Abood

AC circuit Theory

i leads v by 110°, or v lags i by 110°.

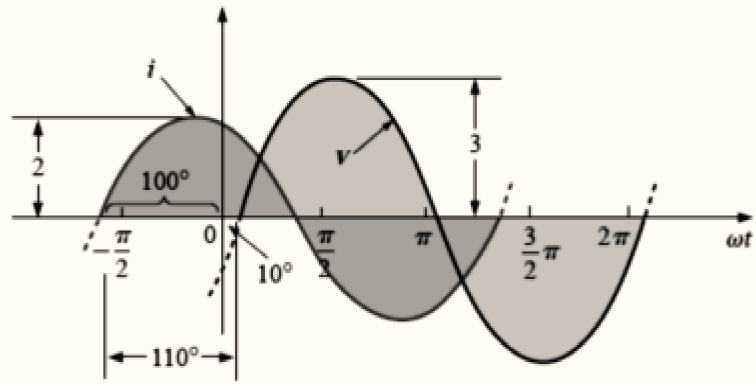


Fig. 7 i leads v by 110°.

4. Effective Root-Mean-Square (R.M.S.) Value

The R.M.S. value of an alternating current is given by that steady (D.C.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time. It is also known as the effective or virtual value of the alternating current, the former term being used more extensively.
For computing the R.M.S. value of symmetrical sinusoidal alternating currents, either mid-ordinate method or analytical method may be used, although for symmetrical but non-sinusoidal waves, the mid- ordinate method would be found more convenient

The standard form of a sinusoidal alternating current is $i = I_m \sin \omega t = I_m \sin \theta$. The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do.

$$I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

4.2 RMS Voltage

Al-Mustaqbal University / College of Engineering & Technology
Department Computer of engineering techniques

Class (1)
Assist. Lect. Saja Mohsen Abood
AC circuit Theory

Similarly, the effective voltage is the peak voltage (V_m) divided by $\sqrt{2}$. Since $V_m = R \cdot I_m$:

$$V = \frac{R \cdot I_m}{\sqrt{2}} \angle\theta \text{ V}$$

4.3 Ohm's Law for RMS Values

$$\therefore R = \frac{V}{I} = \frac{(R \cdot I_m/\sqrt{2})\angle\theta}{(I_m/\sqrt{2})\angle\theta}$$

EXAMPLE The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for v and i.

a. $v = 100 \sin 377t$
b. $v = 25 \sin(377t + 60°)$

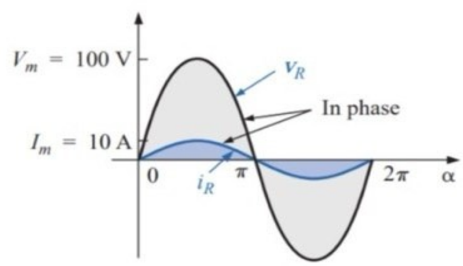
Solutions:

a. Eq. $$I_m = \frac{V_m}{R} = \frac{100\ \text{V}}{10\ \Omega} = 10\ \text{A}$$

(v and i are in phase), resulting in

$$i = \mathbf{10 \sin 377t}$$

The curves are sketched in Fig.



b. $$I_m = \frac{V_m}{R} = \frac{25\ \text{V}}{10\ \Omega} = 2.5\ \text{A}$$

(v and i are in phase), resulting in

$$i = \mathbf{2.5 \sin(377t + 60°)}$$

The curves are sketched in Fig.