



(Mathematic 1)

Basic Fundamentals in Mathematics

The Rate of Change of a Function

Cartesian Coordinate إحداثيات ديكارتية has two number lines:

Horizontal line called x-axis has real numbers $(\dots, -3, -2, -1, 0, 1, 2, 3, \dots)$ + rational number اعداد نسبية

Vertical line called y-axis , has real numbers numbers $(\dots, -3, -2, -1, 0, 1, 2, 3, \dots)$ + rational number اعداد نسبية

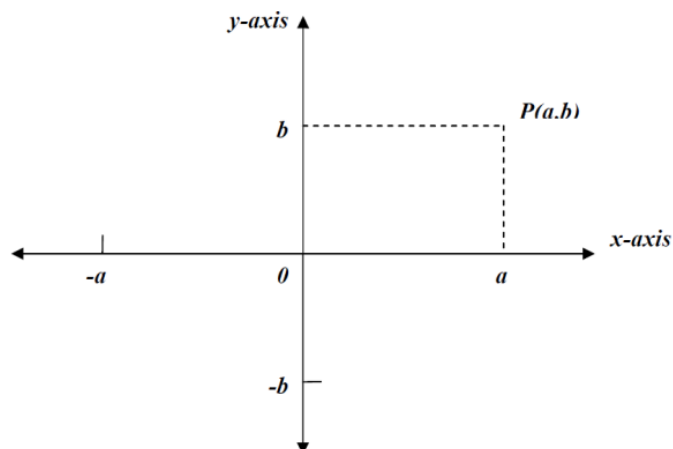
Real numbers included : الطبيعية Natural Numbers(N): $\{1, 2, 3, \dots\}$

الكاملة Whole Numbers (W): $\{0, 1, 2, 3, \dots\}$

الصحيحة Integers (Z): $\{\dots, -2, -1, 0, 1, 2, \dots\}$

النسبية Rational Numbers (Q): $(1/2, -0.75, 0.333\dots)$.

Intersection of two lines called origin point (0,0).



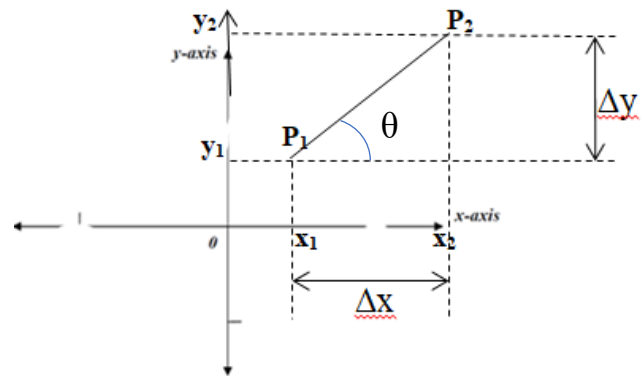


The Slope **الخط ميل of a line**

Increments **مقدار الزيادة** – When a particle moves from one position(x_1 , y_1) in the plane to another point (x_2 , y_2) , the net changes **التغير** in the particle's coordinates **احداثيات** are calculated by:

$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$



Slope of the line :

Let L the line in the plane has $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, then the slope (m) is:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } \Delta x \neq 0$$

❖ For the two perpendicular **متعامدان** lines L_1 and L_2 : $m_1 \cdot m_2 = -1$.

❖ For the parallel two lines: $m_1 = m_2$.

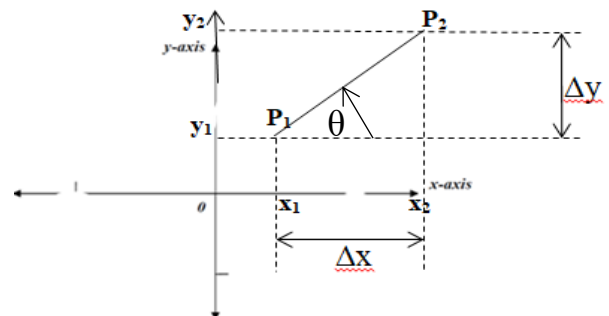
Angles of Inclination: **زوايا الميل**

The angle of inclination of a line measure **عكس عقرب تقاس** counter clockwise **عكس عقرب** from the x -axis around the point of intersection.

θ = inclination angle

$m = \tan \theta$ prove that?

Ans. : $m = \frac{\Delta y}{\Delta x} = \frac{\text{مقابل}}{\text{مجاور}} = \tan \theta$





Example: 1

Find the slope of the line determined by two points A(2,1) and B(-1,3) and find the slope of the line perpendicular to AB.

Solution:

Slope of AB is:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{-1 - 2} = -\frac{2}{3}$$

Slope of line perpendicular to AB is:

$$-\frac{1}{m_{AB}} = \frac{3}{2} \quad (m_1 \cdot m_2 = -1) \rightarrow m_{AB} \cdot m_{AB\perp} = -1$$

Equations for lines معادلات الخطوط

An equation for a line is an equation that is satisfied by the coordinates of the points that lies on the line and is not satisfied by the coordinates of the points that lie elsewhere.

Vertical lines الخطوط العمودية

Every vertical line L has to cross the x-axis at some point. $x = a, y = 0$.

Horizontal lines الخطوط الأفقية

The horizontal line cross the y-axis at point, $x = 0, y = b$.

Non vertical and non-horizontal lines

Point – slope equation of the line through the point (x_1, y_1) with slope m is :

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} \quad y - y_1 = m(x - x_1)$$

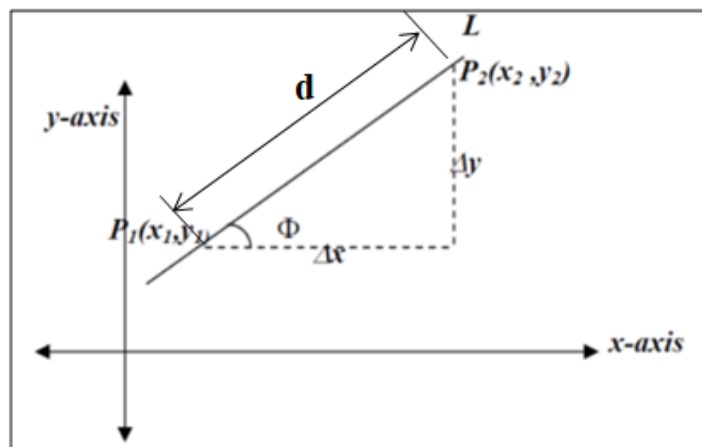
إذا كانت نقطة واحدة معروفة



The distance from a point to a line : المسافة بين نقطة وخط او بين نقطتين

To calculate the distance d between the point $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 2: Write an equation for the line that passes through point :

a) $P(-1, 3)$ with slope $m = -2$.

$$y - y_1 = m(x - x_1) \rightarrow y - 3 = -2(x - (-1)) \rightarrow y + 2x = 1 \rightarrow y = -2x + 1$$

Example 3: Write the distance between the points $P_1(-2, 0)$ and $P_2(2, -2)$, and the equation for the line that passes through the points :

Solution : = $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - (-2))^2 + (-2 - 0)^2} \approx 4.47$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{2 - (-2)} = -\frac{1}{2}$$

$$y - y_1 = m(x - x_1) \Rightarrow y - 0 = -\frac{1}{2}(x - (-2)) \Rightarrow 2y + x + 2 = 0$$



Example 4 : Find the slope of the line : $3x + 4y = 12$

Solution:

$$y = -\frac{3}{4}x + 3 \Rightarrow \text{the slope is } m = -\frac{3}{4}$$

H.W.

Find :

- a) an equation for the line through P(2 ,1) parallel to L: $y = x + 2$.
- b) an equation for the line through P(2 ,1) perpendicular to L: $y = x + 2$.

Functions

Function is any rule that assigns to dependent and independent variables:

الدالة هي أي قاعدة تُسند إلى المتغيرات المستقلة والغير مستقلة:

$$y = f(x)$$

Domain المجال and Range المدى

The symbol f represents the function, the letter x is the independent variable representing the input value to f , and y is the dependent variable or output value of f at x .

$$\begin{array}{ccc} y & = & f(x) \\ \downarrow & & \downarrow \\ \text{output} & & \text{input} \\ \downarrow & & \downarrow \\ \text{Range} & & \text{Domain} \end{array}$$



Function is given by a formula to calculate the output value from the input variable, for example :

$$y=x+2 \quad , \quad x=0 \rightarrow y=2 \quad , \quad x=1 \rightarrow y=3$$

Domain = largest set of real x-values gives real y-values.

If we want to restrict تحديد the domain:

Q / find domain and range for the equation $y = x^2$? If:

1. $x \geq 0$ 2. $x \geq 2$ 3. Domain not restricted

Solution:

The function $y = x^2$

1. when $x \geq 0$, i.e $x = 0, 1, 2, 3, \dots, \infty$
substitute التعويض in function $y = x^2$, then $y = 0, 1, 4, 9, \dots, \infty$
domain is $x \geq 0$ and range is $[0, \infty)$.
2. when $x \geq 2$, i.e $x = 2, 3, \dots, \infty$
substitute التعويض in function $y = x^2$, then $y = 4, 9, \dots, \infty$
domain is $x \geq 2$ and range is $[4, \infty)$ or $\{x^2: x \geq 2\}$ or $\{y: y \geq 4\}$.
3. Domain not restricted غير محددة قيمها , i.e domain of x is $(-\infty, \infty)$

x	$-\infty$		-2	-1	0	1	2		∞
y	∞		4	1	0	1	4		∞

Then , the range of (y) is $[0, \infty)$.



Example 5 : find domain and range for equation $y=\sqrt{x}$, if domain not restricted?

Solution : domain not restricted for x , but must be positive value inside root then:
domain is $x \geq 0$ i.e $[0, \infty)$ and $y \geq 0$ i.e range is $[0, \infty)$.

x	0	1	2		∞
y	0	1	4		∞

H W , find the domain and range of functions ($y=\sqrt{4-x}$, $y=\sqrt{1-x^2}$)

Graphs of Functions

If f is a function with domain D , its graph يتكون of the points نقاط in the Cartesian plane. The coordinates الاحداثيات are input-outputs مدخلات-مخرجات pairs for f , express in :

$$\{(x, f(x)) \mid x \in D\}$$

Example 6 : Draw the function $f(x) = x + 2$?

Solution : The graph of the function $f(x) = x + 2$ is the set of points with coordinates (x, y) , its graph is the straight line sketched موضح in the following Figure:

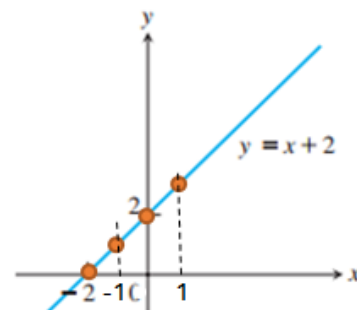
$$f(x)=y=x+2$$

$$\text{at } x=0 \quad y=2$$

$$\text{at } x=1 \quad y=3$$

$$\text{at } x=-1 \quad y=1$$

$$\text{at } x=-2 \quad y=0$$



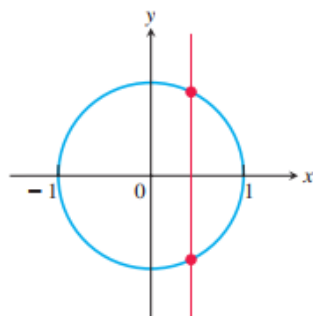
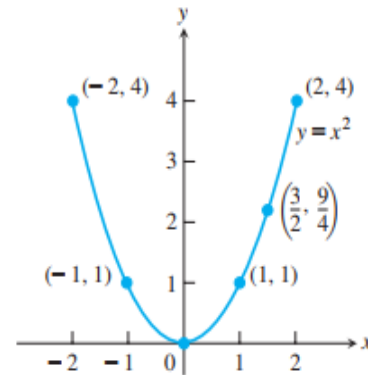


Example 7 : Draw the function $y = x^2$ over the interval $[-2,2]$?

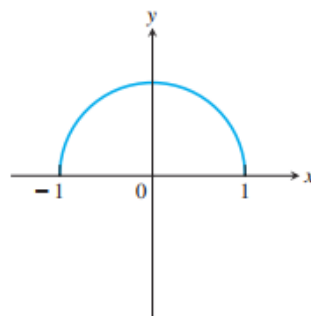
Solution:

$$y = x^2$$

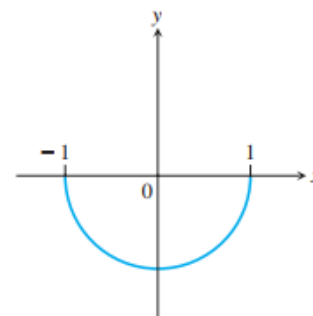
x	$y = x^2$
-2	4
-1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4



(a) $x^2 + y^2 = 1$



(b) $y = \sqrt{1 - x^2}$



(c) $y = -\sqrt{1 - x^2}$

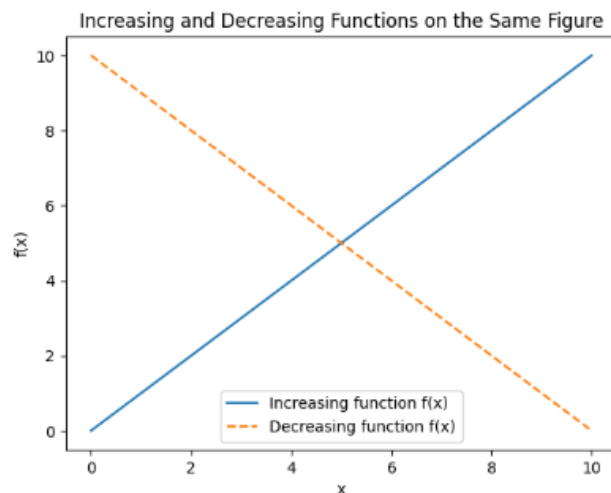
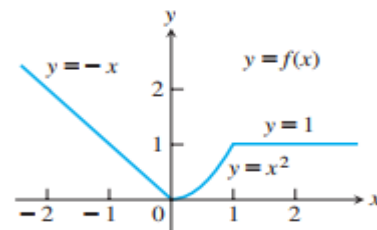
Figure 1 illustrate different graphs for different functions



Example 8 : draw the function :

$$f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

First formula
Second formula
Third formula



1. If $f(x_2) > f(x_1)$ whenever $x_1 < x_2$, then f is said to be **increasing** on I .
2. If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$, then f is said to be **decreasing** on I .



Even Functions and Odd Functions

The graphs of even زوجي and odd فردي functions have special symmetry properties,

Definitions : A function $y = f(x)$ is:

even function of x if $f(-x) = f(x)$,

odd function of x if $f(-x) = -f(x)$,

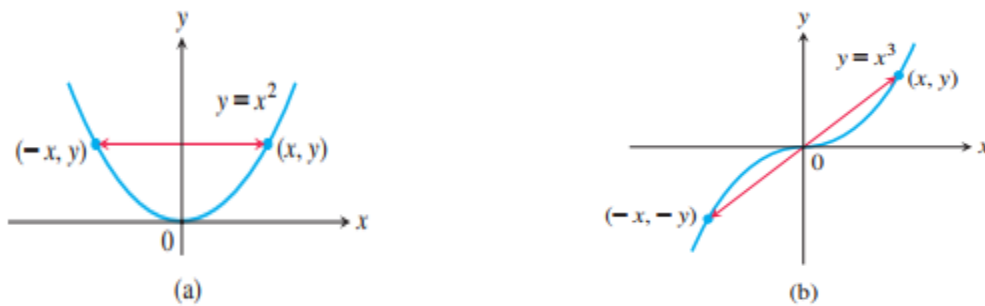


Figure 2 (a) The graph of $y = x^2$ (an even function) is symmetric about the y-axis.

(b) The graph of $y = x^3$ (an odd function) is symmetric about the origin.

Example 9 : is the function $f(x) = x^2$ even or odd ?

Let $x = -x$, then $(-x)^2 = x^2$ for all x ; symmetry about y-axis. So $f(-3) = 9 = f(3)$. Changing the sign of x does not change the value, then the function is an even.

HW is the functions even or odd?

$$f(x) = x^2 + 1$$

$$f(x) = x$$

$$f(x) = x + 1$$



Exponential Functions

$$f(x) = a^x \quad \text{where } a > 0 \text{ and } a \neq 1$$

All exponential functions have domain $(-\infty, \infty)$ and range $(0, \infty)$.

May be in the shape $y=e^x$, $y=e^{-x} \leftarrow 1/e^\infty = 1/\infty = 0$ $\nearrow$

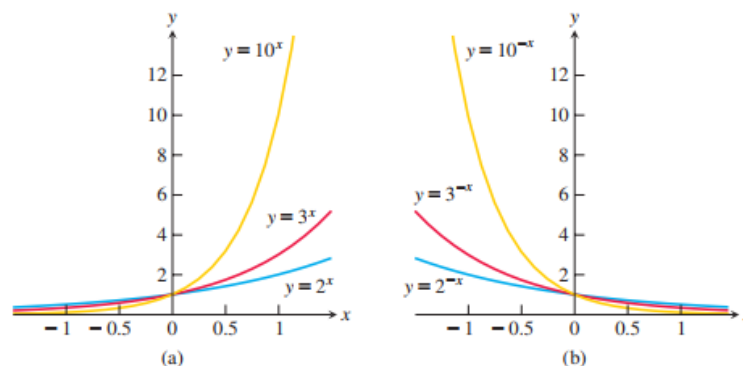


Figure 3
exponential

Graphs of
functions.

Logarithmic Functions

These are the functions $f(x) = \log_a x$, where the base $a \neq 1$ is a positive constant. They are the **inverse functions of the exponential functions**. In each case the domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.

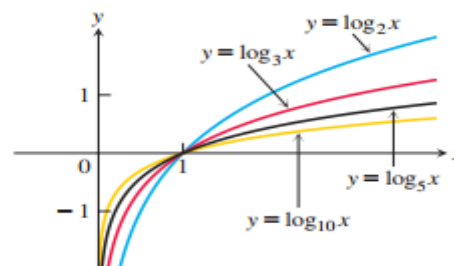


Figure 4 Graphs of four logarithmic functions