



Matrix operation

Addition

If A and B are m x n matrices, then the **sum** $A + B$ is the m x n matrix whose columns are the sums of the corresponding columns in A and B. The sum $A + B$ is **defined** only when A and B are the **same size**. يجب ان تكون المصفوفتان متساويتان بعدد الصفوف والاعمدة

Theorem: Let A, B, and C be matrices of the same size, and let r and s be scalars.

- | | |
|--------------------------------|-------------------------|
| a. $A + B = B + A$ | d. $r(A + B) = rA + rB$ |
| b. $(A + B) + C = A + (B + C)$ | e. $(r + s)A = rA + sA$ |
| c. $A + 0 = A$ | f. $r(sA) = (rs)A$ |

Example 3 : Find $A + B$, and $A + C$ if:

$$A = \begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

Solution :

$$A + B = \begin{bmatrix} 5 & 1 & 6 \\ 2 & 8 & 9 \end{bmatrix}$$

but $A + C$ is not defined غير معرفة because A and C different sizes

Scalar multiplication

If r is a scalar and A is a matrix, then the **scalar multiple** rA .

Example 4 : If $A = \begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix}$ are the matrices , find $2B$, $A-2B$

Solution :

$$2B = 2 \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 6 & 10 & 14 \end{bmatrix}$$

$$A - 2B = \begin{bmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 2 \\ 6 & 10 & 14 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 3 \\ -7 & -7 & -12 \end{bmatrix}$$



Multiplication

Row-Column Rule for Computing AB

$$(AB)_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj}$$

صف عمود

$$A = \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} \quad B = \begin{bmatrix} \boxed{} \\ \boxed{} \end{bmatrix} \quad , \quad AB = \begin{bmatrix} \boxed{} \\ \boxed{} \end{bmatrix}$$

ضرب الارقام داخل المستطيل الافقي للمصفوفة الاولى بالأرقام داخل المستطيل العمودي في المصفوفة الثانية فيعطي الرقم الموجود داخل المربع الاحمر في المصفوفة الناتجة (المصفوفة الثالثة)

Example 5: if $A = \begin{bmatrix} 1 & 2 \\ 0 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 4 \\ 7 & 5 \end{bmatrix}$ find AB ?

Solution : $A = \begin{bmatrix} 1 & 2 \\ 0 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 4 \\ 7 & 5 \end{bmatrix}$, $AB = \begin{bmatrix} C & D \\ E & F \end{bmatrix}$

$$C = 1*2 + 2*7 = 16 \quad , \quad D = 1*4 + 2*5 = 14 \quad , \quad E = 0*2 + 6*7 = 42 \quad , \quad F = 0*4 + 6*5 = 30$$

$$\rightarrow AB = \begin{bmatrix} 16 & 14 \\ 42 & 30 \end{bmatrix}$$

Example 6: Compute AB , where:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$$

Solution :

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix} \quad , \quad AB = \begin{bmatrix} C & D & G \\ E & F & H \end{bmatrix}$$

$$C = 2*4 + 3*1 = 11 \quad , \quad D = 2*3 + 3*(-2) = 0 \quad , \quad G = 2*6 + 3*3 = 21 \quad , \quad E = 1*4 + (-5)*1 = -1$$

$$F = 1*3 + (-5)*(-2) = 13 \quad , \quad H = 1*6 + (-5)*3 = 9 \quad \rightarrow \quad AB = \begin{bmatrix} 11 & 0 & 21 \\ -1 & 13 & 9 \end{bmatrix}$$



Example 7: Find AB, where: $A = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ -3 & 0 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix}$

Solution :

هناك اربع صفوف في المصفوفة الاولى يعني اربع صفوف في المصفوفة الناتجة , وهناك عمودان (اللان سيتم الضرب بهما) في المصفوفة الثانية يعني سيكون هناك عمودان في المصفوفة الناتجة.
 يعني تختار عدد الصفوف من المصفوفة الاولى وعدد الاعمدة من المصفوفة الثانية.

$$AB = \begin{bmatrix} c & g \\ d & h \\ e & i \\ f & j \end{bmatrix},$$

$$c = 2(4) + (-5)(7) + 0(3) = -27, \quad g = 2(-6) + (-5)(1) + 0(2) = -17, \quad d = (-1)(4) + 3(7) + (-4)(3) = 5$$

$$h = (-1)(-6) + 3(1) + (-4)(2) = 1, \quad e = 6(4) + (-8)(7) + (-7)(3) = -53, \quad i = 6(-6) + (-8)(1) + (-7)(2) = -58$$

$$f = (-3)(4) + 0(7) + 9(3) = 15, \quad j = (-3)(-6) + 0(1) + 9(2) = 36 \quad \rightarrow \quad AB = \begin{bmatrix} -27 & -17 \\ 5 & 1 \\ -53 & -58 \\ 15 & 36 \end{bmatrix}$$

Theorem

Let A be an $m \times n$ matrix, and let B and C have sizes for which the indicated sums and products are defined.

- $A(BC) = (AB)C$ (associative law of multiplication)
- $A(B + C) = AB + AC$ (left distributive law)
- $(B + C)A = BA + CA$ (right distributive law)
- $r(AB) = (rA)B = A(rB)$
for any scalar r
- $I_m A = A = A I_n$ (identity for matrix multiplication)

HW / find A*B

$$\text{Let } A = \begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$$



Determinant of Matrices محدد المصفوفات

Determinant of the matrix can be calculate by the following manner:

For matrix 2 x 2 : $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

المحدد = حاصل ضرب الطرفين – حاصل ضرب الوسطين

$$\det A = a_{11}a_{22} - a_{12}a_{21}$$

Example 8: find the determinant of the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$?

Solution : $\det A = 2*(-5) - 3*1 = -13$

For matrix 3 x 3 : $A = \begin{bmatrix} +a_{11} & -a_{12} & +a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$,

$$\det A = a_{11} \det \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \det \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \det \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$= a_{11}(a_{22}*a_{33} - a_{23}*a_{32}) - a_{12}(a_{21}*a_{33} - a_{23}*a_{31}) + a_{13}(a_{21}*a_{32} - a_{22}*a_{31})$$

Example 9: Compute the determinant of the matrix 3x3:

$$A = \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & 2 & 0 \end{bmatrix}$$

Solution : $A = \begin{bmatrix} + & - & + \\ 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix}$

$$\det A = 1 * \det \begin{bmatrix} 4 & -1 \\ -2 & 0 \end{bmatrix} - 5 * \det \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} + 0 * \det \begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix} = 1*(0-2) - 5*(0-0) + 0 = -2$$

$$\det A = a_{11}C_{11} + a_{12}C_{12} + \dots + a_{1n}C_{1n}$$

صيغة اخرى عند استخدام الاعمدة

$$\det A = a_{1j}C_{1j} + a_{2j}C_{2j} + \dots + a_{nj}C_{nj}$$

صيغة اخرى عند استخدام الصفوف



HW: Compute the determinants:

$$1. \begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix}$$

$$2. \begin{vmatrix} 0 & 4 & 1 \\ 5 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix}$$

$$3. \begin{vmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{vmatrix}$$

$$4. \begin{vmatrix} 1 & 2 & 4 \\ 3 & 1 & 1 \\ 2 & 4 & 2 \end{vmatrix}$$

If A is a triangular matrix مصفوفة مثلثية, then det A is the product of the entries on the main diagonal of A.

Determinant of triangle matrix = حاصل ضرب القيم الموجودة في المحور فقط لان عند استخدام الطريقة الاعتيادية فان الكثير من الحدود ستذهب مع الصفر

Example 10: find the determinant of upper triangle matrix

$$1. \text{ Upper triangle matrix } A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$2. \text{ Lower triangle matrix } A = \begin{bmatrix} 5 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 7 & 1 \end{bmatrix}$$

$$3. \text{ Diagonal matrix المصفوفة القطرية } A = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Solution :

$$1. \det A = 2 \det \begin{bmatrix} 4 & 5 \\ 0 & 6 \end{bmatrix} - 3 \det \begin{bmatrix} 0 & 5 \\ 0 & 6 \end{bmatrix} + 1 \det \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = 2*(4*6-5*0) - 3*(0*6-5*0) + 1(0*0-2*0) = 2*4*6 - 0 + 0 = 2*4*6 = 48 \text{ للتوضيح}$$

$$2. \det(A) = 5*3*1 = 15 \text{ حل مباشر}$$

$$3. \det(A) = 7*2*9 = 126$$



Inverse Matrices (Nonsingular) المصفوفات العكسية

An $n \times n$ matrix A is said to be invertible انعكاسية if there is an $n \times n$ matrix C such that:

$$CA = I \text{ and } AC = I$$

where $I = I_n = \mathbf{identity}$ matrix. In this case, C is an inverse of A .

$$A^{-1}A = I \text{ and } AA^{-1} = I$$

$$: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example 11: check if the C is inverse of A

$$A = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix} \text{ and } C = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}$$

Solution :

$$AC = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix} \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$CA = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \text{Thus } C = A^{-1}$$

Invertible matrix called **nonsingular matrix** مصفوفة غير منفردة.

Not invertible matrix called **singular matrix** مصفوفة منفردة.

القاعدة تنص على ان ارقام الطرفين تتبادلان وارقام الوسطين تعكس اشارتهما ثم تقسم على قيمة \det الخاص بالمصفوفة

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $ad - bc \neq 0$, then A is invertible and

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

If $ad - bc = 0$, then A is not invertible.



Example 12: Find the inverse of : $A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

Solution : Since $\det A = 3(6) - 4(5) = -2 \neq 0$, A is invertible, and

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 6/(-2) & -4/(-2) \\ -5/(-2) & 3/(-2) \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 5/2 & -3/2 \end{bmatrix}$$

حل المعادلات باستخدام معكوس المصفوفة **Solve system of equations by using inverse matrix**

If A is an invertible انعكاسية $n \times n$ matrix, then solve of the equation $Ax = b$ has the unique solution $x = A^{-1}b$, where $x = x_1, x_2$.

$$Ax = b \rightarrow x = A^{-1}b$$

Example 13: solve the system of equations by using inverse matrix

$$3x_1 + 4x_2 = 3$$

$$5x_1 + 6x_2 = 7$$

Solution :

$$\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} \rightarrow Ax = b \rightarrow x = A^{-1}b$$

Calculate A^{-1} (inverse of A)

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det A = ad - bc = 3 \cdot 6 - 4 \cdot 5 = 18 - 20 = -2 \rightarrow A^{-1} = \frac{1}{-2} \begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix} \text{ حاصل ضرب رقم مع مصفوفة}$$

$$A^{-1} = \begin{bmatrix} -3 & 2 \\ 5/2 & -3/2 \end{bmatrix}, \text{ then } x = A^{-1}b = \begin{bmatrix} -3 & 2 \\ 5/2 & -3/2 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} \text{ حاصل ضرب صف مع عمود}$$

$$x = \begin{bmatrix} -3 \cdot 3 + 2 \cdot 7 \\ 5/2 \cdot 3 + (-3/2) \cdot 7 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix} \rightarrow x_1 = 5, x_2 = -3$$



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HW

Find the inverses of the matrices

1. $\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 5 & 4 \\ 9 & 7 \end{bmatrix}$

3. $\begin{bmatrix} 8 & 3 \\ -7 & -3 \end{bmatrix}$

4. $\begin{bmatrix} 3 & -2 \\ 7 & -4 \end{bmatrix}$

Use the inverse matrix to solve the system

$$5x_1 + 4x_2 = -3$$

$$9x_1 + 7x_2 = -5$$