Al-Mustaqbal University / College of Engineering & Technology
Department Computer of engineering techniques

Class (1)
Assist. Lect. Saja Mohsen Abood
AC circuit Theory

AC circuit Theory

Lecture (4)

Parallel RLC Circuit

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1. Parallel RL Circuit

In a parallel RL circuit, resistor R and inductor L are connected in parallel, and both are supplied by a voltage source, Vin. Since the resistor R and inductor L are connected in parallel, the voltage across them is equal. However, the currents I flowing in the resistor and inductor are different. The parallel RL circuit is not used as a filter for voltages because in this circuit, the output voltage is equal to the input voltage, and for this reason, it is not commonly used as compared to a series RL circuit.

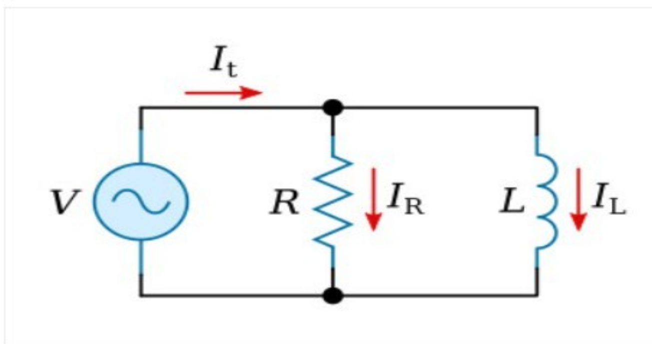

Figure 1: Parallel *RL* circuit

for an RL parallel circuit, we know that in a parallel circuit, the voltage across the inductor and resistor remains the same so,

$$VR = VL = VT$$

In parallel DC circuits, the simple arithmetic sum of the individual branch currents equals the total current. The same is true in an AC parallel circuit if only pure resistors or only pure inductors are connected in parallel.

However, when a resistor and inductor are connected in parallel, the two currents will be out of phase with each other. In this case, the total current is equal to the vector sum rather than the arithmetic sum of the currents.

Recall that the voltage and current through a resistor are in phase, but through a pure inductor, the current lags the voltage by precisely 90 degrees. This is still the case when the two are connected in parallel.

The relationship between the voltage and currents in a parallel RL circuit is illustrated in the vector (phasor) diagram in Fig. 2 and summarized as follows:

- The reference vector is labeled V and represents the voltage in the circuit, which is common to all elements.

- The angle theta (θ) represents the phase between the applied line voltage and current.

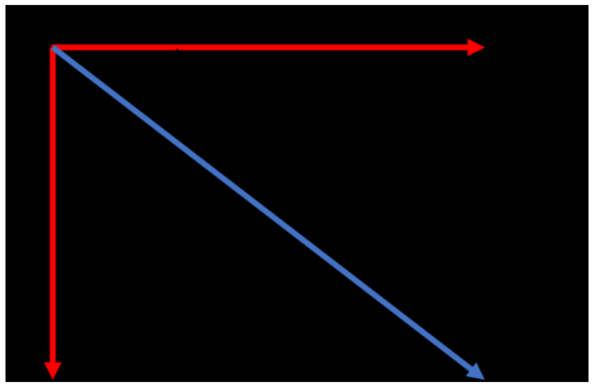

Figure 2: Parallel *RL* circuit vector (phasor) diagram.

As is the case in all parallel circuits, the current in each branch of a parallel RL circuit acts independently of the currents in the other branches. The current flow in each branch is determined by the voltage across that branch and the opposition to current flow, in the form of either resistance or inductive reactance, contained in the branch.

Ohm's law can then be used to find the individual branch currents as follows:

$$I_L = \frac{V}{X_L}$$

$$I_R = \frac{V}{R}$$

In the resistive branch, the current has the same phase as the applied voltage, but the inductive branch current lags the applied voltage by 90 degrees. As a result, the total line current (IT) consists of IR and IL with 90 degrees out of phase with each other.

The current flow through the resistor and the inductor form the legs of a right triangle, and the total current is the hypotenuse.

$$I_T = \sqrt{I_R^2 + I_L^2}$$

The impedance (Z) of a parallel RL circuit is the total opposition to the flow of current. It includes the opposition (R) offered by the resistive branch and the inductive reactance (XL) offered by the inductive branch.
The impedance of a parallel RL circuit is calculated similarly to a parallel resistive circuit. However, since XL and R are vector quantities, they must be added vectorially. As a result, the equation for the impedance of a parallel RL circuit consisting of a single resistor and inductor is:

$$Z = \frac{R \cdot X_L}{\sqrt{R^2 + X_L^2}}$$

$$Z = \frac{V_T}{I_T}$$

In all parallel RL circuits, the phase angle theta (θ) by which the total current lags the voltage is somewhere between 0 and 90 degrees. The size of the angle is determined by whether there is more inductive current or resistive current.

If there is more inductive current, the phase angle will be closer to 90 degrees. It will be closer to 0 degrees if there is more resistive current. From the circuit vector diagram, you can see that the value of the phase angle can be calculated from the equation:

$$\theta = \tan^{-1}\left(\frac{I_L}{I_R}\right)$$

2.Parallel R-C Circuit

The conditions that exist in RC parallel circuits and the methods used for solving them are quite similar to those used for RL parallel circuits. The voltage is the same value across each parallel branch and provides the basis for expressing any phase differences The principal difference is one of phase relationship. In a pure capacitor the current leads the voltage by90 degrees, while in a pure inductor the current lags the voltage by 90 degrees.

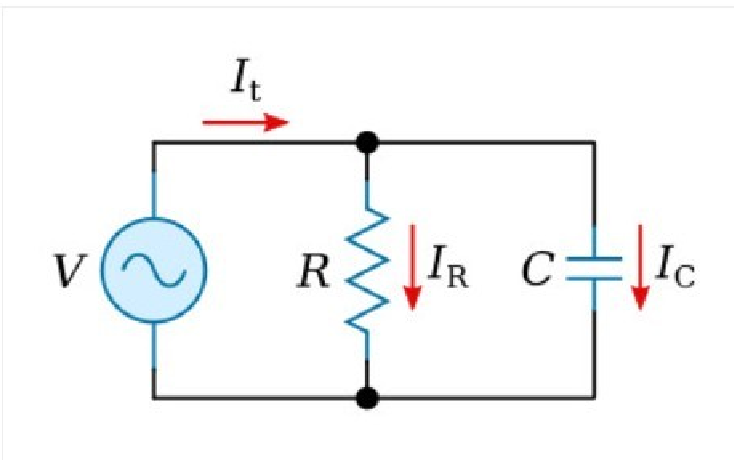

Figure 3: Parallel *RC* circuit

The **current through the resistor (I_R)** is:

$$I_R = \frac{V}{R}$$

The **Current through the capacitor (I_C)** is:

$$I_C = \frac{V}{X_C}$$

The vector addition of I_R and I_C gives a resultant that represents the total (I_T) use individual branch currents:

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$$I_T = \sqrt{{I_R}^2 + {I_C}^2}$$

The **impedance (*Z*)** of a parallel *RC* circuit is similar to that of a parallel *RL* circuit and is summarized as follows:

- Impedance can be calculated directly from the resistance and capacitive reactance values using the equation:

$$z = \frac{R\,X_C}{\sqrt{R^2 + X_C^2}}$$

- Impedance can be calculated using the Ohm's law equation

$$Z = V_T/I_T$$

- The impedance of a parallel RC circuit is always less than the resistance or capacitive reactance of the individual branches.

The relationship between the voltage and currents in a parallel *RC* circuit (θ) is illustrated in the vector (phasor) diagram of Fig. **2** and summarized as follows:

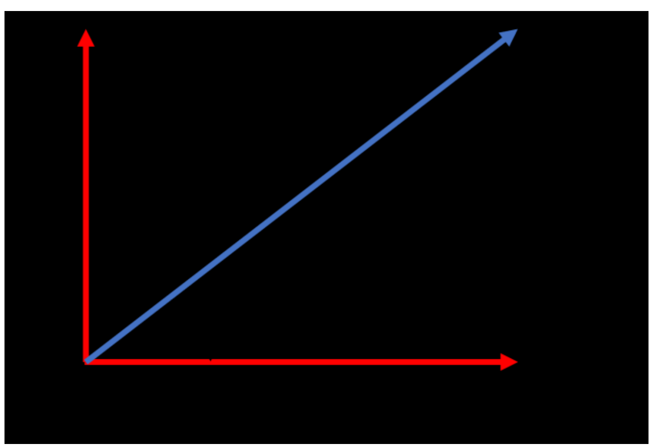

Figure 4. Parallel *RC* circuit (phasor) diagram

closer to 90 degrees, while if the resistive current is greater, the angle is closer to 0 degrees. The value of the phase angle can be calculated from the values of the two branch currents using the following equation:

$$\theta = \tan^{-1}\frac{I_C}{I_R}$$

2. Parallel RLC circuit

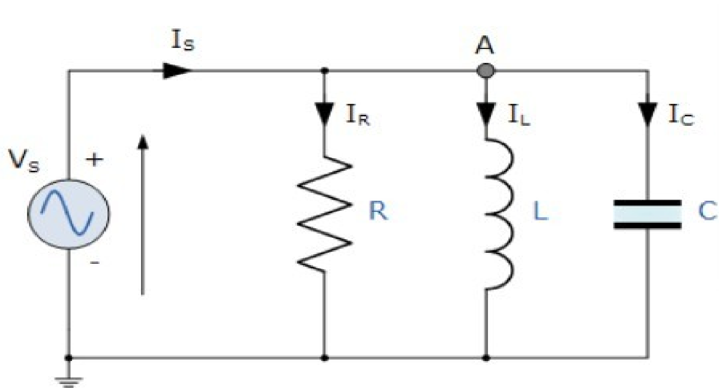

Fig.5 Parallel RLC Circuit

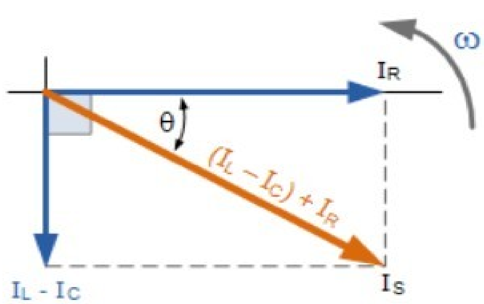

Fig.6 Phasor Diagram for a Parallel RLC Circuit

$$I_S = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

$$I_S = \frac{V}{Z}$$

From Kirchhoff's Current Law (KCL)

The first equation represents the sum of currents at a node in a parallel circuit:

$$I_S - I_R - I_L - I_C = 0$$

Total Impedance (Z)
In a parallel circuit, the reciprocal of the total impedance (**called Admittance)** is the phasor sum of the reciprocals of the individual impedances. The magnitude of the total impedance is:

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

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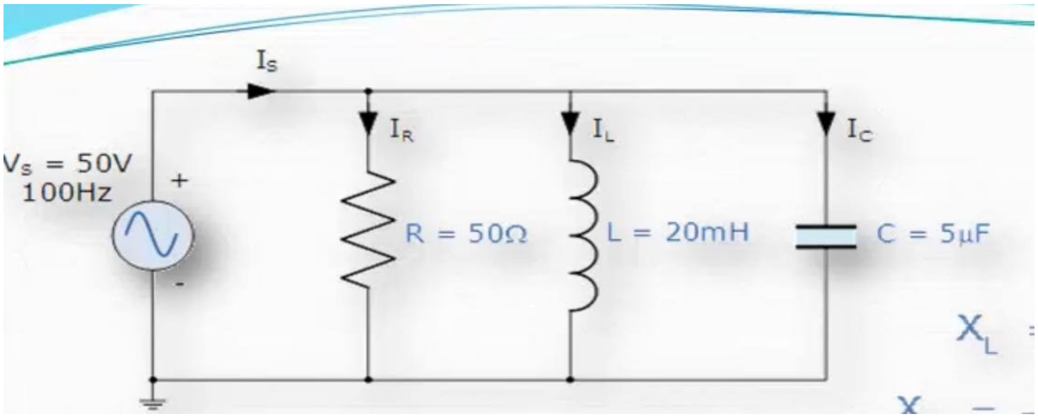
example A parallel RLC circuit containing a resistance of 50Ω, an inductance of 20 mH and a capacitor of 5μF across a 100V, 50Hz supply. Calculate the circuits current, the total circuit impedance, ,



Given:

- $V_s = 50$ V
- $f = 100$ Hz
- $R = 50\Omega$
- $L = 20$ mH $= 0.02$
- $C = 5\mu F = 5 \times 10^{-}$

1. **Inductive Reactance**

$$X_L = 2\pi f L = 2 \times \pi \times 100 \times 0.02 \approx \mathbf{12.57\ \Omega}$$

2. **Capacitive Reactance**

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 100 \times 5 \times 10^{-6}} \approx \mathbf{318.31\ \Omega}$$

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:(I_R) Current through Resistor

$$I_R = \frac{V_s}{R} = \frac{50}{50} = \mathbf{1}\ \text{A}$$

:(I_L) Current through Inductor

$$I_L = \frac{V_s}{X_L} = \frac{50}{12.57} \approx \mathbf{3.98}\ \text{A}$$

:(I_C) Current through Capacitor

$$I_C = \frac{V_s}{X_C} = \frac{50}{318.31} \approx \mathbf{0.157}\ \text{A}$$

(I_s) Total Source Current .4

:In parallel AC circuits, the total current is the phasor sum of the branch currents

$$I_s = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$I_s = \sqrt{1^2 + (3.98 - 0.157)^2}$$

$$I_s = \sqrt{1 + (3.823)^2} \approx \mathbf{3.95}\ \text{A}$$

Observation: Since $I_L > I_C$, the circuit is **inductive**, and the total current .lags the voltage

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Total Impedance (Z)

There are two main ways to find the total impedance in this parallel circuit:
Method 1: Using Ohm's Law
Since we already have the total source voltage

$$Z = \frac{V_s}{I_s}$$

$$Z = \frac{50\ \text{V}}{3.95\ \text{A}}$$

$$\mathbf{Z \approx 12.66\ \Omega}$$

method 2

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$\frac{1}{Z} = \sqrt{(0.02)^2 + (0.07955 - 0.00314)^2}$$

$$\frac{1}{Z} = \sqrt{0.0004 + (0.07641)^2}$$

$$\frac{1}{Z} = \sqrt{0.0004 + 0.005838}$$

$$\frac{1}{Z} = \sqrt{0.006238} \approx 0.07898$$

$$Z = \frac{1}{0.07898} \approx \mathbf{12.66\ \Omega}$$

The **admittance Y** is the reciprocal of impedance, measured in siemens (S).

The admittance **Y** of an element (or a circuit) is the ratio of the phasor current through it to the phasor voltage across it, or

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}} \qquad (9.46)$$

The admittances of resistors, inductors, and capacitors can be obtained from Eq. (9.39). They are also summarized in Table 9.3.

As a complex quantity, we may write **Y** as

$$\mathbf{Y} = G + jB$$

where $G = \text{Re}\,\mathbf{Y}$ is called the *conductance* and $B = \text{Im}\,\mathbf{Y}$ is called the *susceptance*. Admittance, conductance, and susceptance are all expressed in the unit of siemens (or mhos). From Eqs. (9.41) and (9.47),

$$G + jB = \frac{1}{R + jX}$$

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By rationalization,

$$G + jB = \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

Equating the real and imaginary parts gives

$$G = \frac{R}{R^2 + X^2}, \qquad B = -\frac{X}{R^2 + X^2}$$

showing that $G \neq 1/R$ as it is in resistive circuits. Of course, if $X = 0$, then $G = 1/R$.