Al-Mustaqbal University / College of Engineering & Technology
Department Computer of engineering techniques

Class (1)
Assist. Lect. Saja Mohsen Abood
AC circuit Theory

AC circuit Theory

Lecture (3)

Series RLC Circuit

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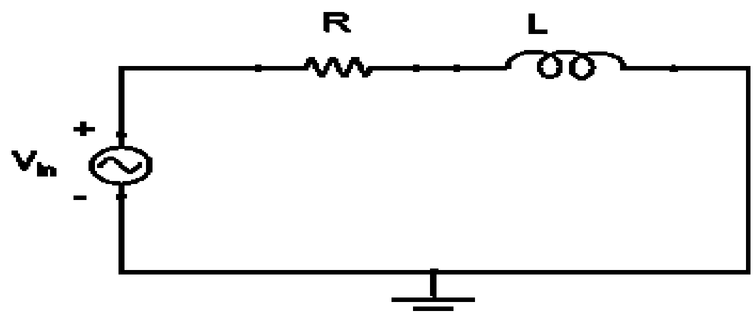
1. RL Series Circuit

Consider a simple RL circuit in which resistor, R, and inductor, L are connected in series with a voltage supply of Vin. The current flowing in the circuit is I and the current through resistor R and inductor L is IR and IL, respectively. However, the resistor and inductor are connected in series, that's why the current passing through both elements is the same. i.e.,

$$I_L = I_R = I \quad (1)$$

The voltages VR and VL are the voltage drop across the resistor and inductor



By applying the Kirchhoff voltage law (The summation of the drop voltages across R and L equal to the input voltage Vin) to this circuit, we get:

$$V_{in} = V_R + V_L \quad (2)$$

Like resistance, reactance is measured in Ohm's but is given the symbol X to distinguish it from a purely resistive "R" value and as the component in question is an inductor, the reactance of an inductor is called Inductive Reactance, XL and is measured in Ohms. Its value can be found from the formula.

$$XL = 2\pi f L \quad (3)$$

Where XL is inductive reactance in (Ω), π is the numeric constant of 3.142, f is the frequency in Hz, and **L** = inductance in H

sinusoidal voltage given by the expression:

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$$V_{in} = V_{max} sine\ wt$$

In an RL circuit, a phase shift occurs as well between the voltage across the inductor VL and the current I. As the circuit is a resistive-inductive load, the voltage V leads the current I, The phase shift can also be calculated using equation

$$\theta = \tan^{-1}\frac{V_L}{V_R}$$

Fig. below illustrate the voltage and current phase shift of a resistive- inductive load

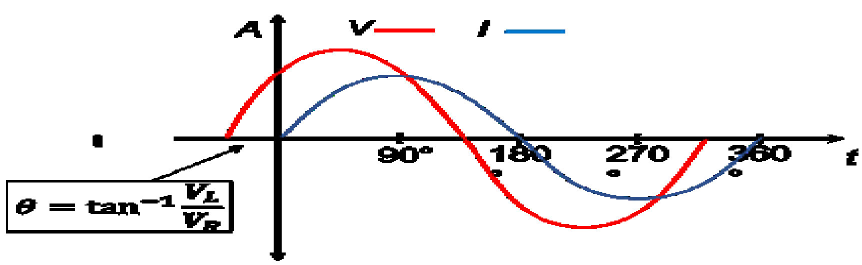

Table below , shows important equations required to theoretically calculate *VR, VL, Vs, R, XL,* and Z.

Table 1: Important equations

For voltages	For impedance
$V_R = V_S \times \cos(\theta)$	$R = Z \times \cos(\theta)$
$V_L = V_S \times \sin(\theta)$	$X_L = Z \times \sin(\theta)$
$Vs = \sqrt{VR^2 + VL^2}$	$Z = \sqrt{R^2 + xL^2}$

2.RC Series Circuit

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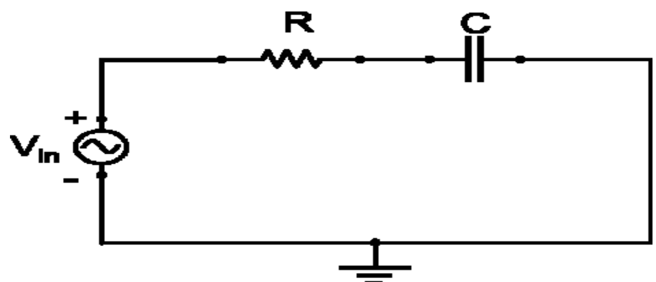
A resistor-capacitor circuit (RC circuit), or called RC filter / RC network, is an electrical circuit composed of resistors and capacitors driven by a voltage or current source. The RC circuit can be used as a low pass filter to remove the higher frequency signals. Allowing the low-frequency signals to pass. If a voltage source is connected across a capacitor, a charge will flow in the external circuit until the voltage across the capacitor is equal to the applied voltage. The charge that flows is proportional to the size of the capacitor its "capacitance" and to the applied voltage. The relationship is given by the equation

$$Q=CV \qquad (1)$$

Where **Q** is the charge in coulombs, **C** is the capacitance in farads, and **V** is the applied voltage in volts.

In an **RC** circuit, the capacitive impedance Xc decreases as the frequency of the input voltage increases, and current I flow through the circuit is proportionally increased. i.e., as that frequency increases, the capacitor will act as a short circuit to the high-frequency current in its path. At low frequencies, the capacitor tends to block current flow. However, the change of the input frequency will not change the value of the resistor R.



Reactance is a characteristic exhibited by capacitors and inductors in circuits with time-varying voltages and currents, such as common sinusoidal AC circuits. Like resistance, reactance opposes the flow of electric current and is measured in ohms. Capacitive reactance XC can be found by the equation

$$X_C = \frac{1}{2\pi f C}$$

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Where **f** is the frequency of the applied voltage or current and **C** is the capacitance in farads. As with resistance, the capacitor reactance obeys Ohm's law:

$$X_c = \frac{V_c}{I_c}$$

In an RC circuit, a phase shift occurs as well between the voltage across the capacitor VC and the current I. As the circuit is a resistive-capacitive load, the current leads the voltage, as shown in Fig. 4. The phase shift can be calculated using

$$\theta = \tan^{-1}\frac{-V_C}{V_R}$$

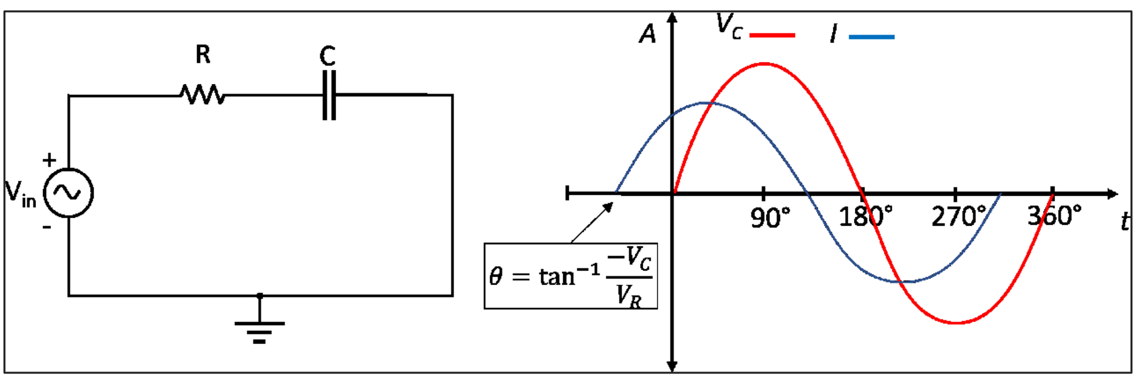

Fig. 4

Table 1 shows important equations required to theoretically calculate *VR, VC, Vs, R, XC,* and Z

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For voltages	For impedance
$V_R = V_S \times \cos(\theta)$	$R = Z \times \cos(\theta)$
$V_c = V_S \times \sin(\theta)$	$X_c = Z \times \sin(\theta)$
$\mathbf{Vs} = \sqrt{\mathbf{VR^2 + Vc^2}}$	$\mathbf{Z} = \sqrt{\mathbf{R^2 + xc^2}}$

3. RLC Series Circuit

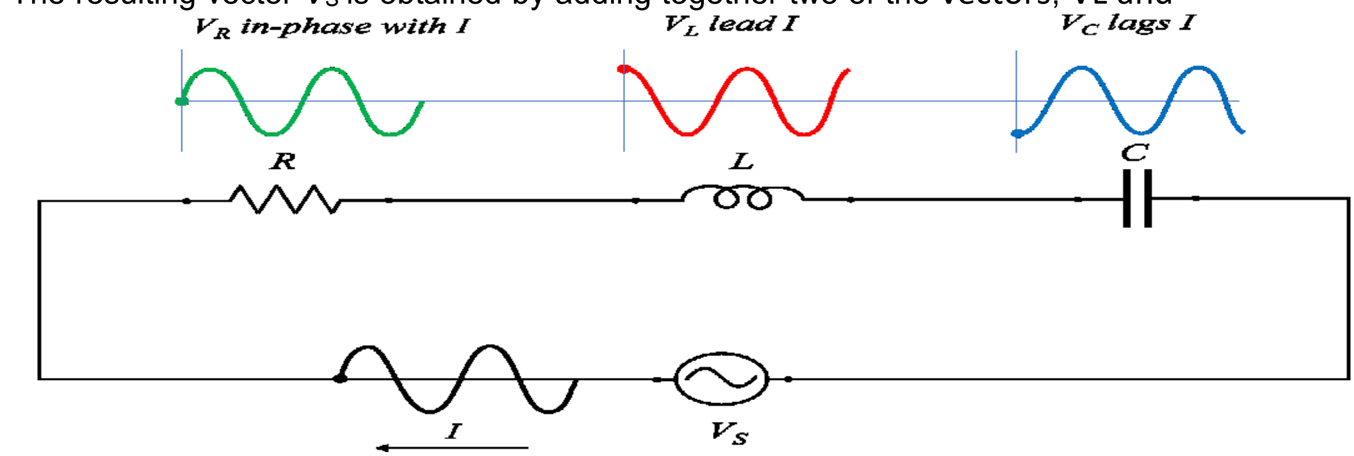
RLC circuits have many applications as oscillator circuits. Radio receivers and television sets use them for tuning to select a narrow frequency range from ambient radio waves. In this role, the circuit is often referred to as a tuned circuit. An RLC circuit can be used as a band-pass filter, band-stop filter, low-pass filter or high-pass filter. The tuning application, for instance, is an example of band-pass filtering. The RLC filter is described as a second-order circuit, meaning that any voltage or current in the circuit can be described by a second-order differential equation in circuit analysis

Table 1: Shows the resistivity, reactance, and Theta of each element in the circuit.

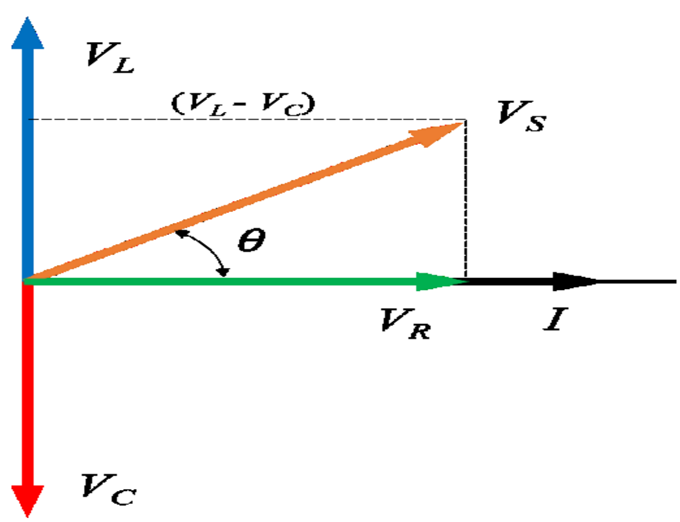
Circuit element	Resistance, R	Reactance, X	Theta θ
Resistor	R	0	0
Inductor	0	$2\pi f L$	+ 90°
Capacitor	0	$\frac{1}{2\pi f C}$	- 90°

Instead of analyzing each element individually, we can combine the three together elements into a series RLC circuit

The resulting vector V_S is obtained by adding together two of the vectors, VL and



VC and then adding this sum to the remaining vector VR. The resulting angle obtained between VS and I will be the circuits phase angle as shown below.



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We can see from the phasor diagram in Fig. 2 above that the voltage vectors produce a rectangular triangle, comprising of hypotenuse VS, horizontal axis VR and vertical axis VL

– VC. We notice that this forms our old favorite the Voltage Triangle and we can therefore use Pythagoras's theorem on this voltage triangle to mathematically obtain the value of VS as shown. The voltage triangle for a series RLC Circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (1)$$

From Fig. 3, to calculate the phase diffrence of the RLC circuit:

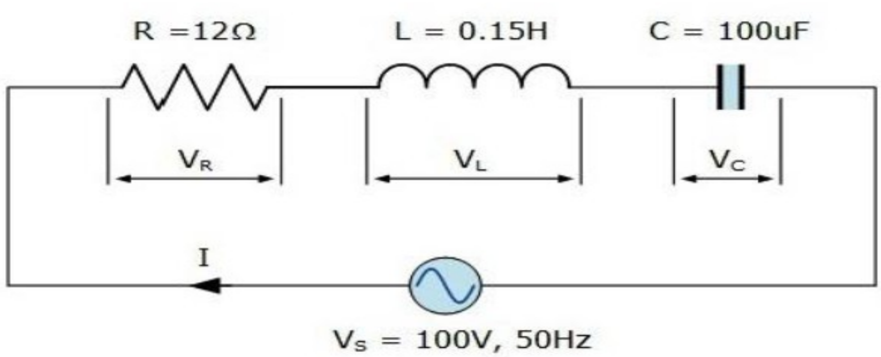
$$\theta = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

or

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Example

A series RLC circuit containing a resistance of 12Ω, an inductance of 0.15H and a capacitor of 100uF are connected in series across a 100V, 50Hz supply. Calculate the total circuit impedance, the circuits current, power factor and draw the voltage phasor diagram.



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S0l.

Data given

Resistance (R): 12 Ω

Inductance (L): 0.15 H

Capacitance (C): 100 μF = 100×10^{-6} F

Supply Voltage (V_s): 100 V

Frequency (f): 50 Hz

Inductive Reactance

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.15$$

$\approx 47.12\Omega$

Capacitive Reactance

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times \pi \times 50 \times 100 \times 10^{-6}}$$

$\approx 31.83\Omega$

1.Calculate Total Circuit Impedance (Z)

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.12 - 31.83)^2}$$

$$Z = \sqrt{144 + (15.29)^2} = \sqrt{144 + 233.78} \qquad Z \approx 19.44\Omega$$

2.Calculate Circuit Current (I)

$$I = \frac{V_s}{Z} = \frac{100}{19.44}$$

$$I \approx 5.14 \text{ A}$$

3.Calculate Power Factor

The power factor is the ratio of resistance to total impedance

$$\text{Power Factor} = \frac{R}{Z} = \frac{12}{19.44} \approx 0.617$$

.Since $X_L > X_C$, the circuit is **inductive**, meaning the power factor is **lagging**

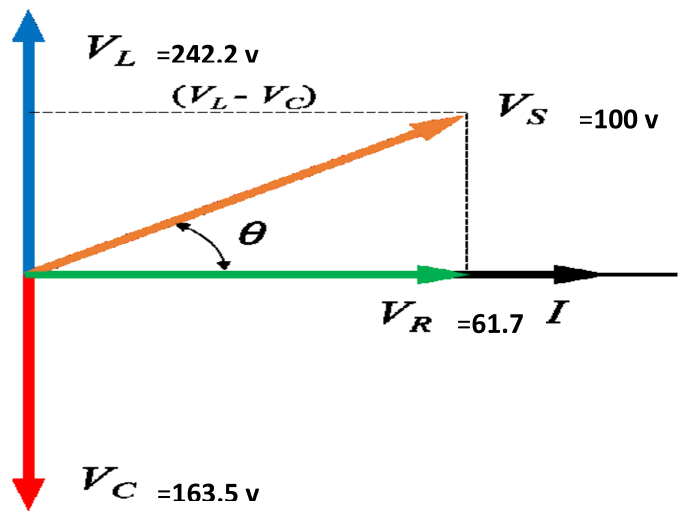
4. Voltage phasor diagram

$$V_R = I \times R = 5.14 \times 12 = 61.7 \text{ volts}$$

$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2 \text{ volts}$$

$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5 \text{ volts}$$

$$\cos^{-1} 0.617 = 51.8 \; lagging$$


V_L =242.2 v
$(V_L - V_C)$
V_S =100 v
θ
V_R =61.7
I
V_C =163.5 v