



## Complex Numbers. الأعداد المركبة.

The transition from “real calculus” to “complex calculus” starts with a discussion of complex numbers and their geometric representation in the complex plane

يبدأ الانتقال من "حساب التفاضل والتكامل الحقيقي" إلى "حساب التفاضل والتكامل المركب" بمناقشة الأعداد المركبة وتمثيلها الهندسي في المستوى المركب.

Complex analysis has many applications in heat conduction, fluid flow, electrostatics, and in other areas.

للتحليل المركب تطبيقات عديدة في توصيل الحرارة، وتدفق السوائل، والإلكتروستاتيك، وفي مجالات أخرى.

It is important for three main reasons:

1. Two-dimensional potential problems can be solved by analytic functions because satisfy Laplace's equation in two real variables.  
يمكن حل مسائل الجهد ثنائية الأبعاد بواسطة الدوال التحليلية لأنها تحقق معادلة لابلاس في متغيرين حقيقيين.
2. Many difficult integrals can be solved by complex integration.  
يمكن حل العديد من التكاملات الصعبة عن طريق التكامل المعقد .
3. Most functions in engineering mathematics are analytic functions, and their study as functions of a complex variable leads to a deeper understanding of their properties and to interrelations in complex

معظم الدوال في الرياضيات الهندسية هي دوال تحليلية، ودراستها كدوال لمتغير مركب تؤدي إلى فهم أعمق لخصائصها وللعلاقات المتبادلة في الأعداد المركبة

Equations without real solutions, such as:

$$x^2 = -1 \quad \text{or} \quad x^2 - 10x + 40 = 0$$

Complex number  $z$  is a pair of real numbers  $x$  and  $y$ , written as :

$$Z = (x, y)$$

$x$  =real part      and  $y$  =imaginary part of  $z$ , written:  $x = \text{Re } z$ ,  $y = \text{Im } z$



If Two complex numbers have:

If : equal real parts

equal imaginary parts

} → Two complex numbers are equal

(0, 1) is called the imaginary unit and is denoted by i,

$$i (0, 1)$$

**Addition:** of two complex numbers  $z_1(x_1, y_1)$  and  $z_2(x_2, y_2)$  is defined by

$$z_1 + z_2 = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2).$$

**Multiplication :** is defined by:

$$z_1 z_2 = (x_1, y_1)(x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1).$$

These imply that: ويشيران الى

$$(x_1, 0) + (x_2, 0) = (x_1 + x_2, 0)$$

$$(x_1, 0)(x_2, 0) = (x_1 x_2, 0)$$

$$(x, 0) = x. \quad \text{Similarly,} \quad (0, y) = iy$$

And complex number is:  $z = x + iy$  such as  $z=17+4i$

Note :  $i^2 = -1$

Proof:

$$z_1 z_2 = (x_1, y_1)(x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1).$$

$$i=(0,1) \text{ , من تعريف الضرب } i^2=i*i=(0,1)(0,1)= (0*0-1*1, 0*1+0*1)=-1$$



الجمع والضرب بصيغة المعادلة:

$$(x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2).$$

$$(x_1 + iy_1)(x_2 + iy_2) = x_1x_2 + ix_1y_2 + iy_1x_2 + i^2y_1y_2 \\ = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1).$$

Example 1: find  $z_1+z_2$  and  $z_1*z_2$  if  $z_1=8+3i$  , and  $z_2=9-2i$

Solution :  $z_1+z_2=(8+9)+(3-2)i=17+i$

$z_1*z_2=(8*9-3*(-2))+ (8*-2+9*3)i=(72+6)+(-16+27)i=78+11i$

Example 2: find  $z_1+z_2$  and  $z_1*z_2$  if  $z_1=4+5i$ , and  $z_2=2-3i$

Solution :  $z_1+z_2=(4+2)+(5i+(-3i))=6+2i$

$z_1z_2=(4+5i)(2-3i)=8-12i+10i-15i^2=8-2i-15(-1)=23-2i$

### Subtraction

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

Example 3: if  $z_1=8+3i$  and  $z_2=9-2i$  , find  $z_1-z_2$

Solution  $z_1 - z_2 = (8 + 3i) - (9 - 2i) = -1 + 5i$  :

**Division:**  $z = \frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}$

$$x_2 + iy_2$$

ضرب البسط والمقام بـ

$$z = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i \frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}.$$



Example 4: if  $z_1=8+3i$  and  $z_2=9-2i$  , find  $z_1/z_2$

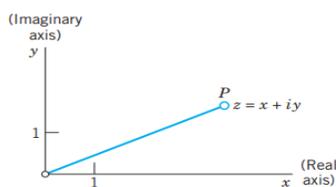
Solution 
$$\frac{z_1}{z_2} = \frac{8 + 3i}{9 - 2i} = \frac{(8 + 3i)(9 + 2i)}{(9 - 2i)(9 + 2i)} = \frac{66 + 43i}{81 + 4} = \frac{66}{85} + \frac{43}{85}i :$$

### Polar Form of Complex Numbers

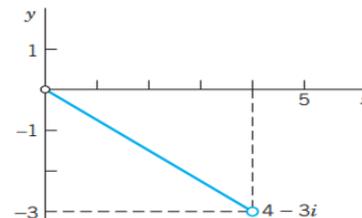
In complex plane:

The horizontal **x-axis**, called → **real axis**

The vertical **y-axis**, called → **imaginary axis**.



The complex plane



The number 4-3i in the complex plane

In polar coordinates  $r, \theta$  defined by  $x = r \cos \theta,$   $y = r \sin \theta$

$$z = x + iy \longrightarrow z = r(\cos \theta + i \sin \theta).$$

**r** is called the **absolute value** or **modulus** of  $z$  and is denoted by  $|z|$  Hence:

$$|z| = r = \sqrt{x^2 + y^2}$$

$\theta$  is called the **argument** of  $z$  and is denoted by  $\text{Arg. } z$

$$\tan \theta = \frac{y}{x}$$

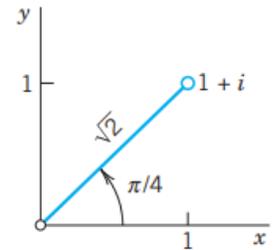
$$-\pi < \text{Arg } z \leq \pi$$



Example 5: if  $z=1+i$  , find absolute value and Arg.  $z$  (or  $\theta$ )

Solution : (1) absolute value=  $|z| = r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

(2) Arg.  $z$  ,  $\tan \theta = \frac{y}{x} = \frac{1}{1} = 1 \rightarrow \text{Arg. } z = \theta = \tan^{-1}(1) = 45 = \frac{\pi}{4}$



Example 6: if  $z=3+3\sqrt{3}i$  , find absolute value and Arg.  $z$  (or  $\theta$ )

Solution :

(1) absolute value=  $|z| = r = \sqrt{3^2 + (3\sqrt{3})^2} = \sqrt{9 + 27} = \sqrt{36} = 6$

(2) Arg.  $z$  ,  $\tan \theta = \frac{3\sqrt{3}}{3} = \sqrt{3} \rightarrow \text{Arg. } z = \theta = \tan^{-1}\sqrt{3} = 60 = \frac{\pi}{3}$

## Powers

$$z^n = r^n (\cos n\theta + i \sin n\theta)$$

## Roots

$$w = \sqrt[n]{z} , \quad z = r(\cos \theta + i \sin \theta) \quad \text{and} \quad w = R(\cos \phi + i \sin \phi)$$

$$w^n = R^n(\cos n\phi + i \sin n\phi) = z = r(\cos \theta + i \sin \theta)$$

## Complex variables المتغيرات المركبة

$f$  is Real function ,  $S =$  set of real numbers (usually an interval)

Every  $x$  (real number) in  $S$ ,  $f(x)$  called the value of  $f$  at  $x$ .

**Now in complex:**  $S =$  set of complex numbers.

Every  $z$  in  $S$  complex number,  $w$  called the value of  $f$  at  $z$ :

$$w = f(z) \quad z = \text{complex variable}$$

Domain **المجال** of  $f = S$

Range **المدى** of  $f =$  Values of a function  $f$



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المطلوب هو حساب  $u$  ,  $v$

$$u=f(x,y) \text{ and } v(x,y)$$

$$w=f(z) \text{ , and } w=u+vi \text{ , } z= x+yi$$

تعويض الدالة  $z$  في الدالة  $w$  فتصبح الدالة  $w$  دالة لـ  $x, y$  وطالما ان الدالة  $w$  هي ايضا دالة لـ  $u, v$  اذا بالمقارنة بين الدالتين لـ  $w$  نستخرج  $u$  دالة لـ  $x, y$  , والـ  $v$  دالة لـ  $x, y$

Example 7: if  $w=f(z)=z^2+3z$  , calculate  $u, v$  and value of  $f(z)$  at  $z=1+3i$

Solution :

$$Z=x+iy \text{ , } w=(x+iy)^2+3(x+iy)$$

$$w=(x^2+2iy+i^2y^2)+3x+3iy$$

$$w=(x^2+2ixy+(-1)y^2)+ 3x+3iy$$

$$w= \underline{x^2-y^2} + \underline{2ixy} + \underline{3x} + \underline{3iy} = x^2-y^2 + 2ixy + 3x + 3iy$$

but  $w = \underbrace{u+iv}$  then  $u = x^2 - y^2 + 3x$  and  $v = 2xy + 3y$

value of  $f(z)$  at  $z=1+3i$  ? from equation :  $x=1$  and  $y=3$

$$\text{then } w=f(z)=(1)^2-(3)^2+3(1)+2(1)(3)i + 3(3)i = 1-9+3+6i+9i = -5+15i$$

HW

1. Find  $z_1+z_2$  ,  $z_1-z_2$  ,  $\frac{z_1}{z_2}$  and  $z_1*z_2$  if  $z_1=7-i$  ,  $z_2=3+6i$
2. Find  $z_1+z_2$  ,  $z_1-z_2$  ,  $\frac{z_1}{z_2}$  and  $z_1*z_2$  if  $z_1=5+2i$  ,  $z_2=5-2i$
3. if  $z=2+2\sqrt{3}i$ , find absolute value and Arg.  $z$  (or  $\theta$ )
4. if  $z=3\sqrt{3}+3i$ , find absolute value and Arg.  $z$  (or  $\theta$ )
5. If  $w=f(z)=z^2+3z$  , calculate  $u, v$  and value of  $f(z)$  at  $z=2+i$
6. If  $w=f(z)=z^2+3z$  , calculate  $u, v$  and value of  $f(z)$  at  $z=1-2i$