Permutation and combination:

Permutations involve arrangements where order matters (e.g., ABC is different from ACB), while combinations involve selections where order does not matter (e.g., ABC is the same as ACB). You use the permutation formula, P(n, r) = n! / (n-r)!, when the sequence of selected items is important, and the combination formula, C(n, r) = n! / (r! * (n-r)!), when the sequence is not important. **To illustrate this let us take the following example:**

If we take the letters AB then the arragment will be , AB and BA so 2 permutation and 1 combination. Another example let us take ABC then the arrangment wii be (ABC,ACB,BAC,BCA,CAB.CBA) hence 6 permutation and 1 combination, Another example if we take ABC and arrange it as follows:

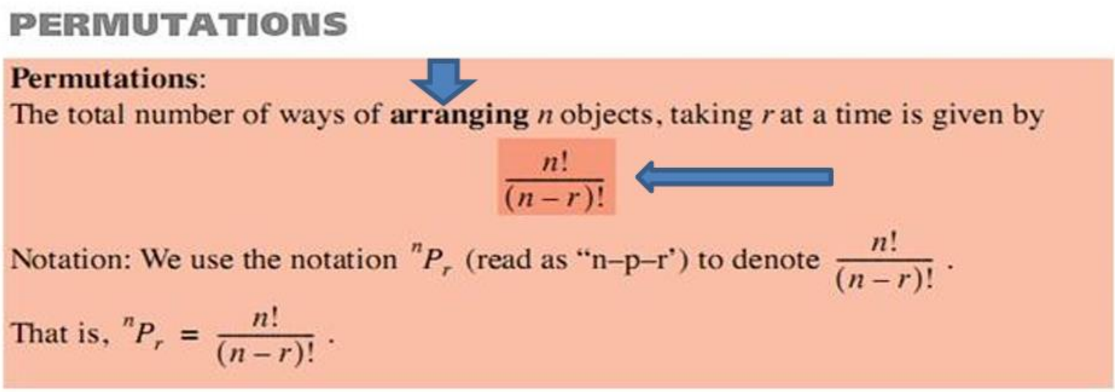
AB, AC, BA, BC ,CA, CB , this means n equal 3 and r equal 2. Permutation equal 6 and combination equal 3 so, the formula of permutation is as follows:

$$p^n_r = \frac{3!}{(3-2)!} = 6$$

While in case of combination is as follows:

$$c^n_r = \frac{3!}{(n-r)!r!} = \frac{6}{2} = 3$$

Additional examples and explaniations:



PERMUTATIONS

Permutations:
The total number of ways of **arranging** n objects, taking r at a time is given by

$$\frac{n!}{(n-r)!}$$

Notation: We use the notation ${}^nP_r$ (read as "n–p–r") to denote $\frac{n!}{(n-r)!}$.

That is, ${}^nP_r = \frac{n!}{(n-r)!}$.

example

the total number of arrangements of 8 books on a bookshelf if only 5 are used

solution

$$^{8}P_{5} = \frac{8!}{(8-5)!} = \frac{8!}{3!} = 6720\ .$$

example In how many ways can 5 boys be arranged in a row

(a) using three boys at a time?

(b) using 5 boys at a time?

We have 5 boys to be arranged in a row with certain constraints.

(a) The constraint is that we can only use 3 boys at a time. In other words, we want the number of arrangements (permutations) of 5 objects taken 3 at a time.

From rule 4: $n = 5, r = 3,$

$$\text{Therefore, number of arrangements} = {}^{5}P_{3} = \frac{5!}{(5-3)!} = \frac{120}{2} = 60$$

(b) This time we want the number of arrangements of 5 boys taking all 5 at a time.

From rule 4: $n = 5, r = 5,$

$$\text{Therefore, number of arrangements} = {}^{5}P_{5} = \frac{5!}{(5-5)!} = \frac{120}{0!} = 120$$

permutations with repetitions:

The number of permutations of n objects of which n_1 are identical, n_2 are identical, . . . , n_k are identical is given by $\dfrac{n!}{n_1! \times n_2! \times \ldots \times n_k!}$.

Example : How many different arrangements of the letters of the word HIPPOPOTAMUS are there?

Solution : $\dfrac{12!}{3! \times 2!} = 39916800$ arrangements.

COMBINATIONS

On the otherhand, combinations represent a counting process where the order has no importance. For example, the number of combinations of the letters A, B, C and D, if only two are taken at a time, can be enumerated as:

AB, AC, AD, BC, BD, CD,

That is, the combination of the letters A and B, whether written as AB or BA, is considered as being the same.

Instead of **combination** the term **selection** is often used.

Combinations:

The total number of ways of **selecting** n objects, taking r at a time is given by

$$\frac{n!}{(n-r)!r!}$$

Notation: We use the notation $\binom{n}{r}$ (read as "n–c–r') to denote $\frac{n!}{(n-r)!r!}$.

That is, $\binom{n}{r} = \frac{n!}{(n-r)!r!}$. Note:Sometimes ${}^{n}C_{r}$ is used instead of $\binom{n}{r}$.

Example : in how many ways can 5 books be selected from 8 different books?

Solution In this instance, we are talking about selections and therefore, we are looking at combinations. Therefore we have. the selection of 8 books taking 5 at a time is equal to

$$\binom{8}{5} = \frac{8!}{(8-5)!5!} = \frac{8!}{3!5!} = 56$$

Example : A sports committee at the local hospital consists of 5 members. A new committee is to be elected, of which 3 members must be women and 2 members must be men. How many different committees can be formed if there were originally 5 women and 4 men to select from?

First we look at the number of ways we can select the women members

We have to select 3 from a possible 5, therefore, this can be done in ${}^{5}C_{3} = 10$ ways.

Similarly, the men can be selected in ${}^{4}C_{2} = 6$ ways.

we have that the total number of possible committees = ${}^{5}C_{3} \times {}^{4}C_{2} = 60$.

Therefore,

Definition of Probability Consider a random experiment which results in a sample space containing n(S) cases which are exhaustive, mutually exclusive and equally likely. Suppose, out of n(S) cases, n(A) cases are favorable to an event A. Then the probability of event A is denoted by P(A) and is defined as follows.

$$P(A) = \frac{n(A)}{n(S)} = \frac{\text{number of cases favourable to event A}}{\text{number of cases in the sample space S}}$$

Complement of an event:

The complement of an event A is denoted by $\bar{A}$ and it contains all the elements of the sample space which do not belong to A. For example: Random experiment: an unbiased die is rolled. S = {1, 2, 3, 4, 5, 6} (i) Let A: number on the die is a perfect square $\therefore$ A = {1, 4} $\therefore$ $\bar{A}$ = {2, 3, 5, 6}

(ii) Let B: number on the die is a prime number

$\therefore$ B = {2, 3, 5} $\therefore$ $\bar{B}$ = {1, 4, 6}

Note: $P(A) + P(\bar{A}) = 1$ i.e. $P(A) = 1 - P(\bar{A})$

For any events A and B, $P(A) = P(A \cap B) + P(A \cap \bar{B})$

Independent Events:

Two events A & B are said to be independent if

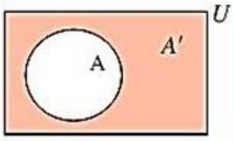
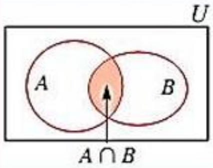
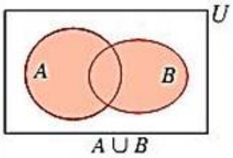
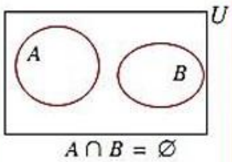
$$P(A \cap B) = P(A).P(B)$$

Note: If A & B are independent then

A & $\bar{B}$ are independent

$\bar{A}$ & B are independent

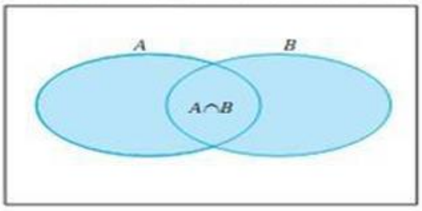
$\bar{A}$ & $\bar{B}$ are independent

Event	Set language	Venn diagram	Probability result
The **complement** of A is denoted by A'.	A' is the complement to the set A, i.e., the set of elements that do not belong to the set A.		$P(A') = 1 - P(A)$ $P(A')$ is the probability that event A does not occur.
The **intersection** of A **and** B : $A \cap B$	$A \cap B$ is the intersection of the sets A and B, i.e., the set of elements that belong to **both** the set A **and** the set B.		$P(A \cap B)$ is the probability that both A and B occur.
The **union** of events A and B : $A \cup B$	$A \cup B$ is the union of the sets A and B, i.e., the set of elements that belong to A **or** B or both A **and** B.		$P(A \cup B)$ is the probability that either event A or event B (or both) occur. From this we have what is known as the '**Addition rule**' for probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
If $A \cap B = \varnothing$ the events A and B are said to be **disjoint**. That is, they have no elements in common.	If $A \cap B = \varnothing$ the sets A and B are **mutually exclusive**.		If A and B are mutually exclusive events then event A and event B cannot occur simultaneously, i.e., $n(A \cap B) = 0$ $\Rightarrow P(A \cap B) = 0$ Therefore: $P(A \cup B) = P(A) + P(B)$

General addition rule

Given two events, A and B, the probability of their union, $A \cup B$, is equal to

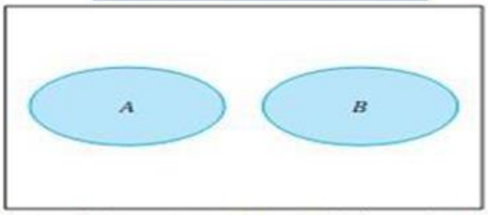
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Special case of additon rule (mutually exclusive)

When two events A and B are **mutually exclusive** or **disjoint,** it means that when A occurs, B cannot, and vice versa. This means that the probability that they both occur, $P(A \cap B)$, must be zero. Figure is a Venn diagram representation of two such events with no simple events in common.

$$P(A \cup B) = P(A) + P(B)$$



When two events A and B are **mutually exclusive,** then $P(A \cap B) = 0$ and the Addition Rule simplifies to

Multiplication Rule

Conditional Probability	$P(A \cap B) = P(A \mid B) \times P(B)$
Independence	$P(A \cap B) = P(A) \times P(B)$

⇒ *P(first choice and second choice) = P(second | first) × P(first choice)*

Conditional probability

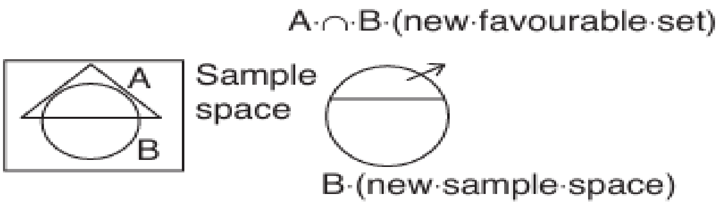
If the events are *not independent*, one event affects the probability for the other event. In this case *conditional probability* must be used. The conditional probability of B given that A occurs, or *on condition* that A occurs, is written **Pr [B|A]**. This is read as the probability of B given A, or the probability of B on condition that A occurs.

Independence : Two events, A and B, are said to be independent if and only if the probability of event B is not influenced or changed by the occurrence of event A, or vice versa

Explanation of conditional probability:

Let S be a finite sample space of a random experiment and A, B are events, such that P(A) > 0, P(B) > 0. If it is known that the event B has occurred, in light of this we wish to compute the probability of A, we mean conditional proba bility of A given B. The occurrence of event B would reduce the sample space to B, and the favourable cases would now be A ∩ B.

ليكن S فضاء عينة محدودًا لتجربة عشوائية، وA وB حدثين، بحيث $P(A) > 0$، $P(B) > 0$. إذا علمنا أن الحدث B قد وقع، ففي ضوء ذلك نرغب في حساب احتمال A، أي الاحتمال الشرطي لـ A مع العلم أن B معطى. سيؤدي وقوع الحدث B إلى تقليل فضاء العينة إلى B، وستكون الحالات المواتية الآن A ∩ B.



Notation The conditional probability of A given B is denoted by $P\left(\frac{A}{B}\right)$.