GRAPHICAL METHOD

Limitation of Graphical Method

Graphical solution is limited to linear programming models containing only two decision variables.

Procedure

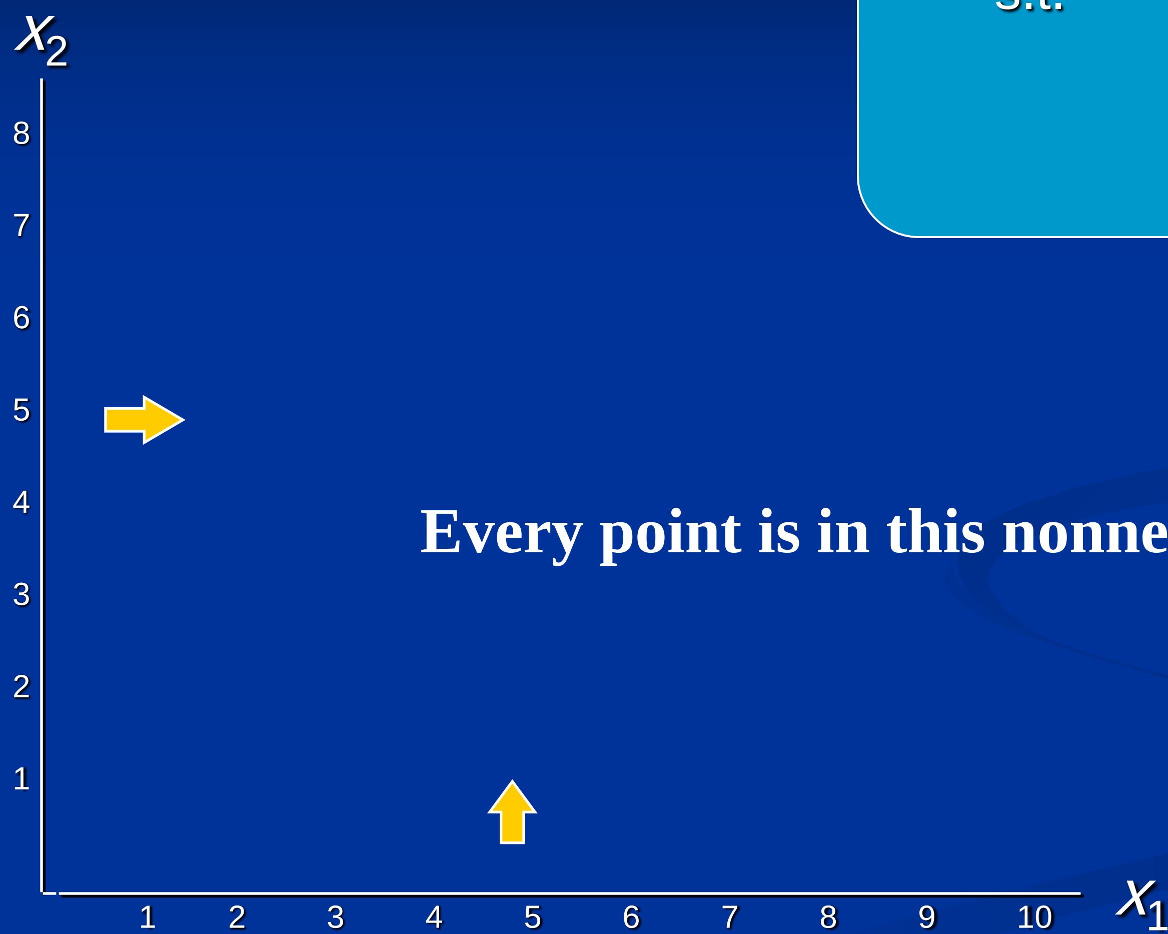
- Step I: Convert each inequality as equation
- Step II: Plot each equation on the graph
- Step III: Shade the 'Feasible Region'. Highlight the common Feasible region.
  - Feasible Region: Set of all possible solutions.
- Step IV: Compute the coordinates of the corner points (of the feasible region). These corner points will represent the 'Feasible Solution'.
  - Feasible Solution: If it satisfies all the constraints and non negativity restrictions.

Procedure (Cont…)

- Step V: Substitute the coordinates of the corner points into the objective function to see which gives the Optimal Value. That will be the 'Optimal Solution'.
  - Optimal Solution: If it optimizes (maximizes or minimizes) the objective function.
  - Unbounded Solution: If the value of the objective function can be increased or decreased indefinitely, Such solutions are called Unbounded solution.
  - Infeasible (Inconsistent) Solution: It means the solution of problem does not exist. This is possible when there is no common feasible region.

Example

$$
\begin{array}{lrcl}
\text{Max} & z = 5x_1 + 7x_2 & & \\
\text{s.t.} & x_1 & \le & 6 \\
 & 2x_1 + 3x_2 & \le & 19 \\
 & x_1 + x_2 & \le & 8 \\
 & x_1,\ x_2 \ge 0 & &
\end{array}
$$



Every point is in this nonnegative quadrant

Example (Cont…)

$$
\begin{aligned}
\text{Max} \quad & z = 5x_1 + 7x_2 \\
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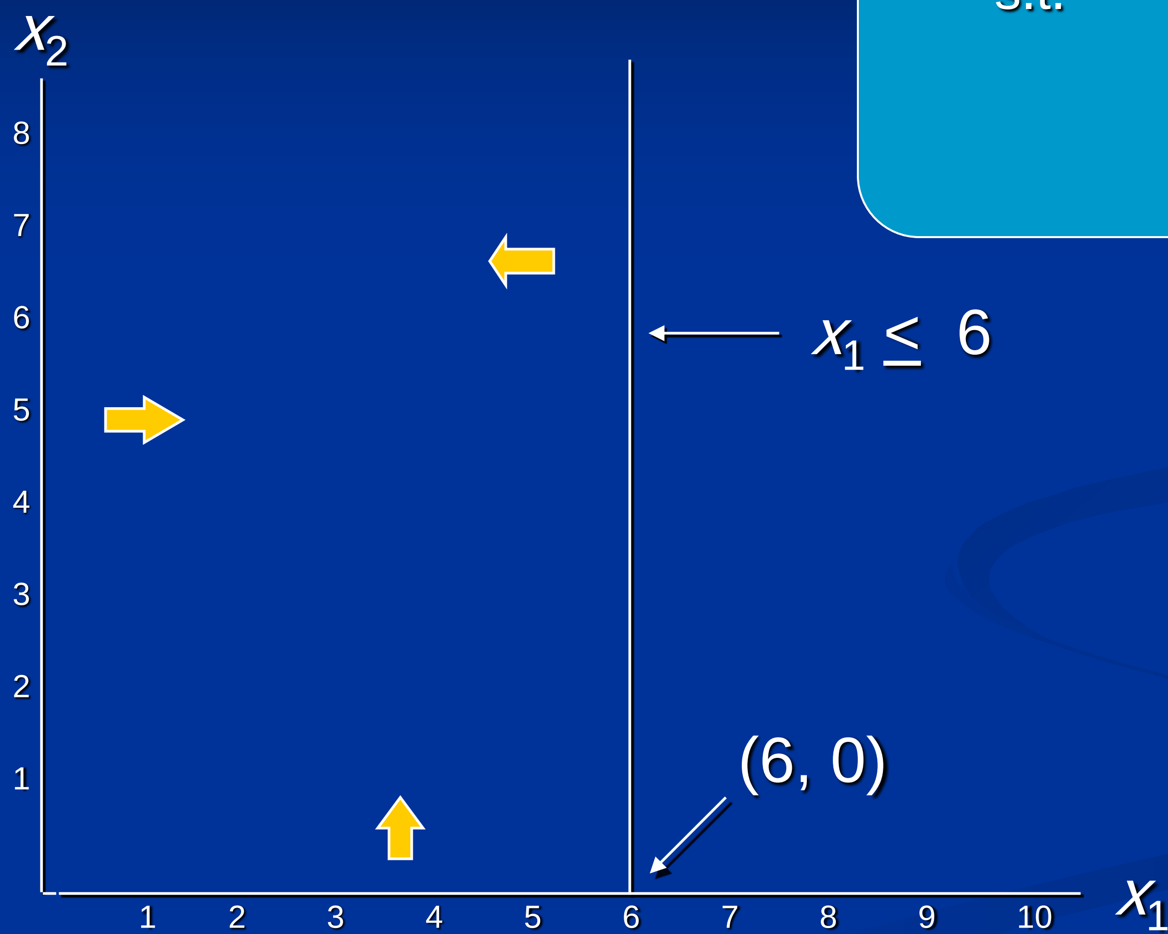
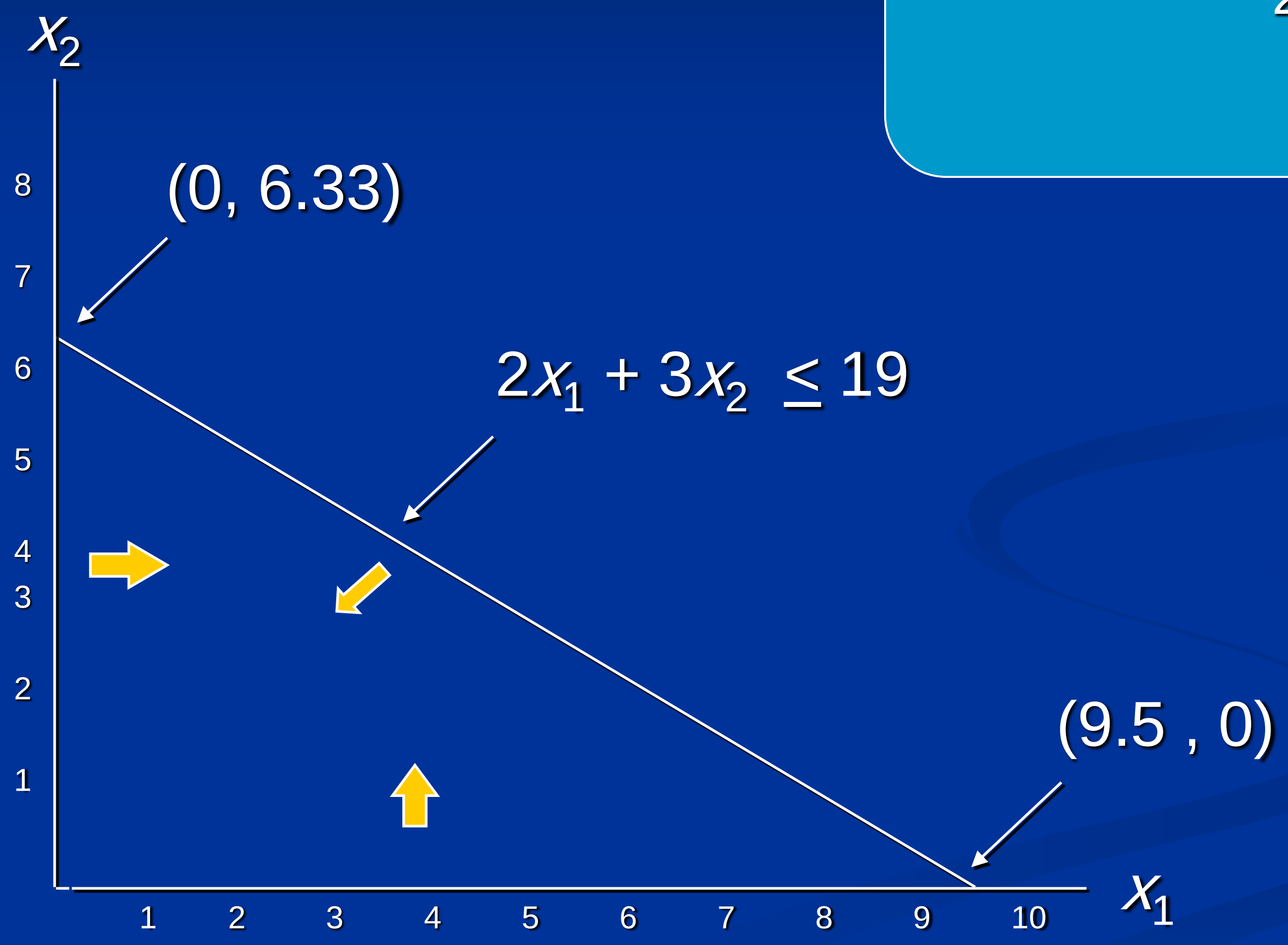


Example (Cont…)

$$\begin{array}{lrl} \text{Max} & z = 5x_1 + 7x_2 & \\ \text{s.t.} & x_1 & \leq 6 \\ & 2x_1 + 3x_2 & \leq 19 \\ & x_1 + x_2 & \leq 8 \\ & x_1, x_2 \geq 0 & \end{array}$$



Example (Cont…)

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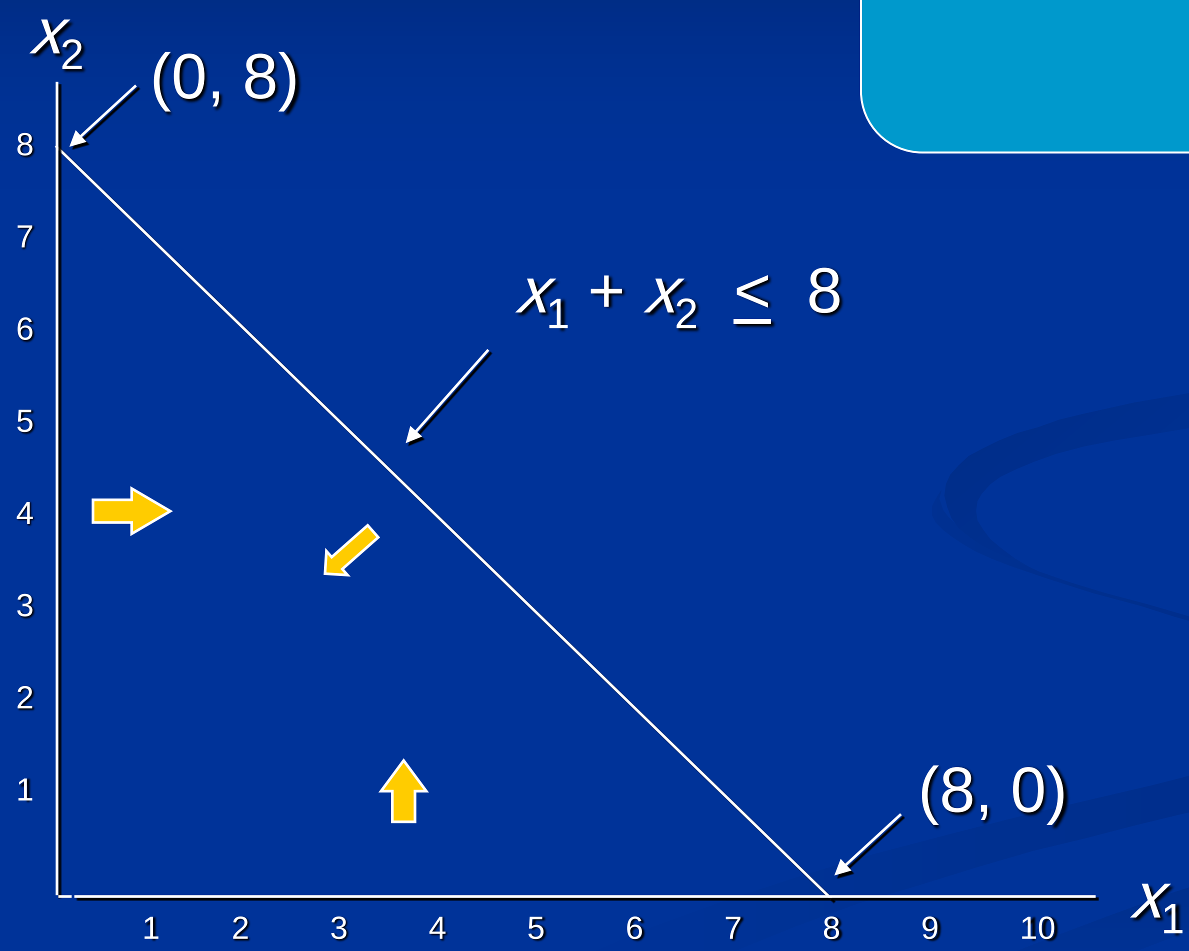

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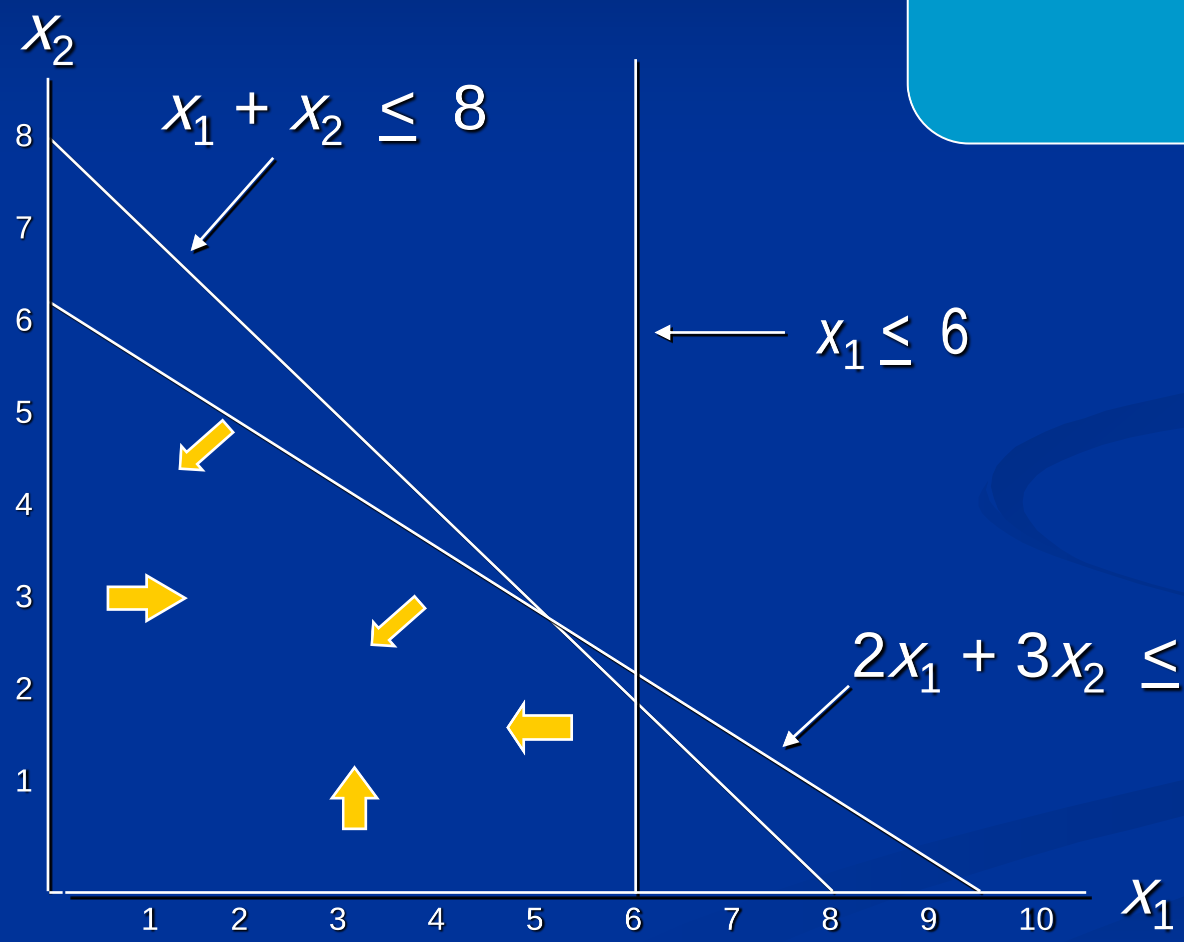
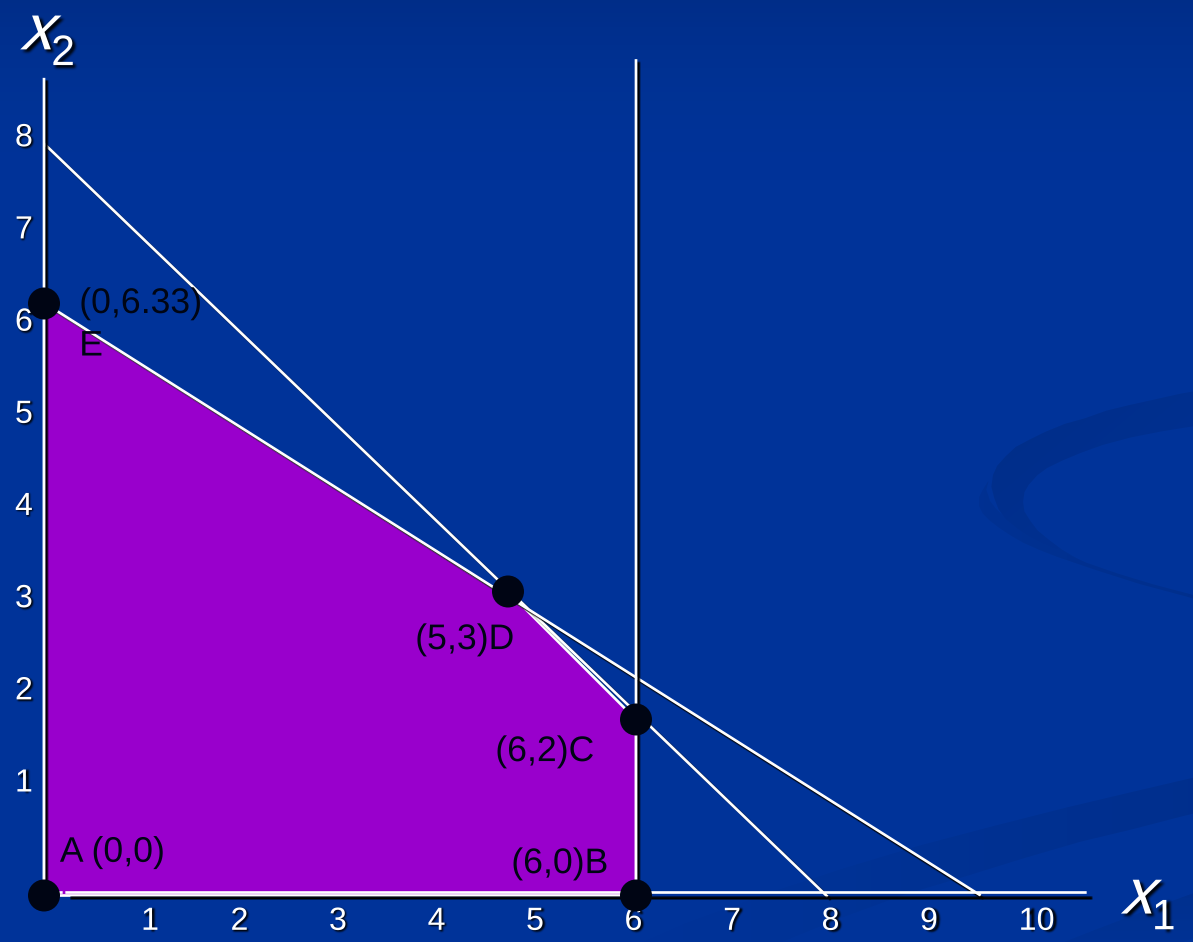


Example (Cont…)



Example (Cont…)

Objective Function : Max $Z= 5x_1+7x_2$

Corner Points	**Value of Z**
A – (0,0)	0
B – (6,0)	30
C – (6,2)	44
D – (5,3)	46
E – (0,6.33)	44.33

Optimal Point : (5,3)
Optimal Value : 46

Example

$$\begin{aligned}
\textbf{Max} \quad & Z = 3\,P1 + 5\,P2 \\
\textbf{s.t.} \quad & P1 \le 4 \\
& P2 \le 6 \\
& 3\,P1 + 2\,P2 \le 18 \\
& P1,\ P2 \ge 0
\end{aligned}$$

Example

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\text{Max} \quad & Z = 3\,P1 + 5\,P2 \\
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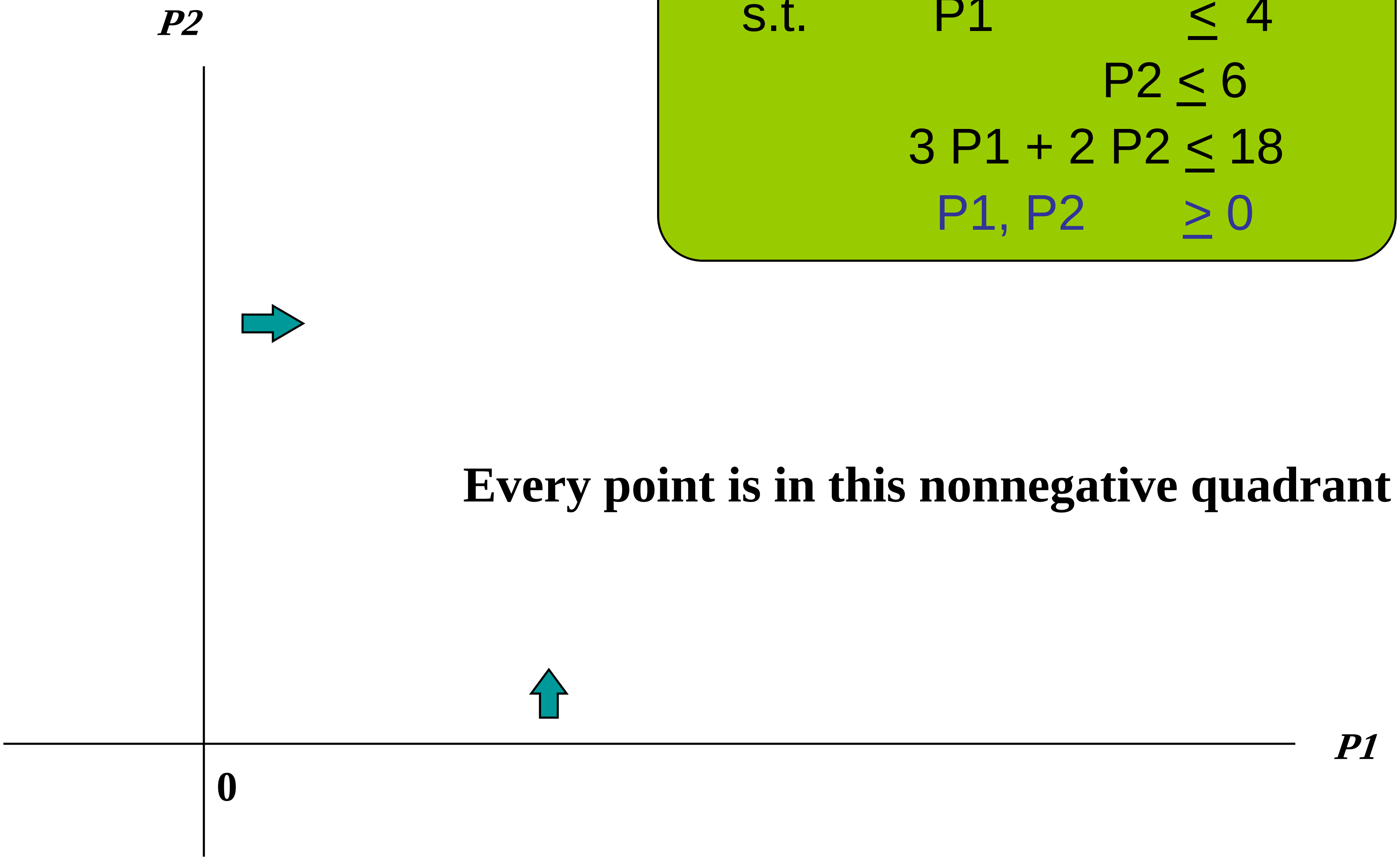


Example (Cont…)

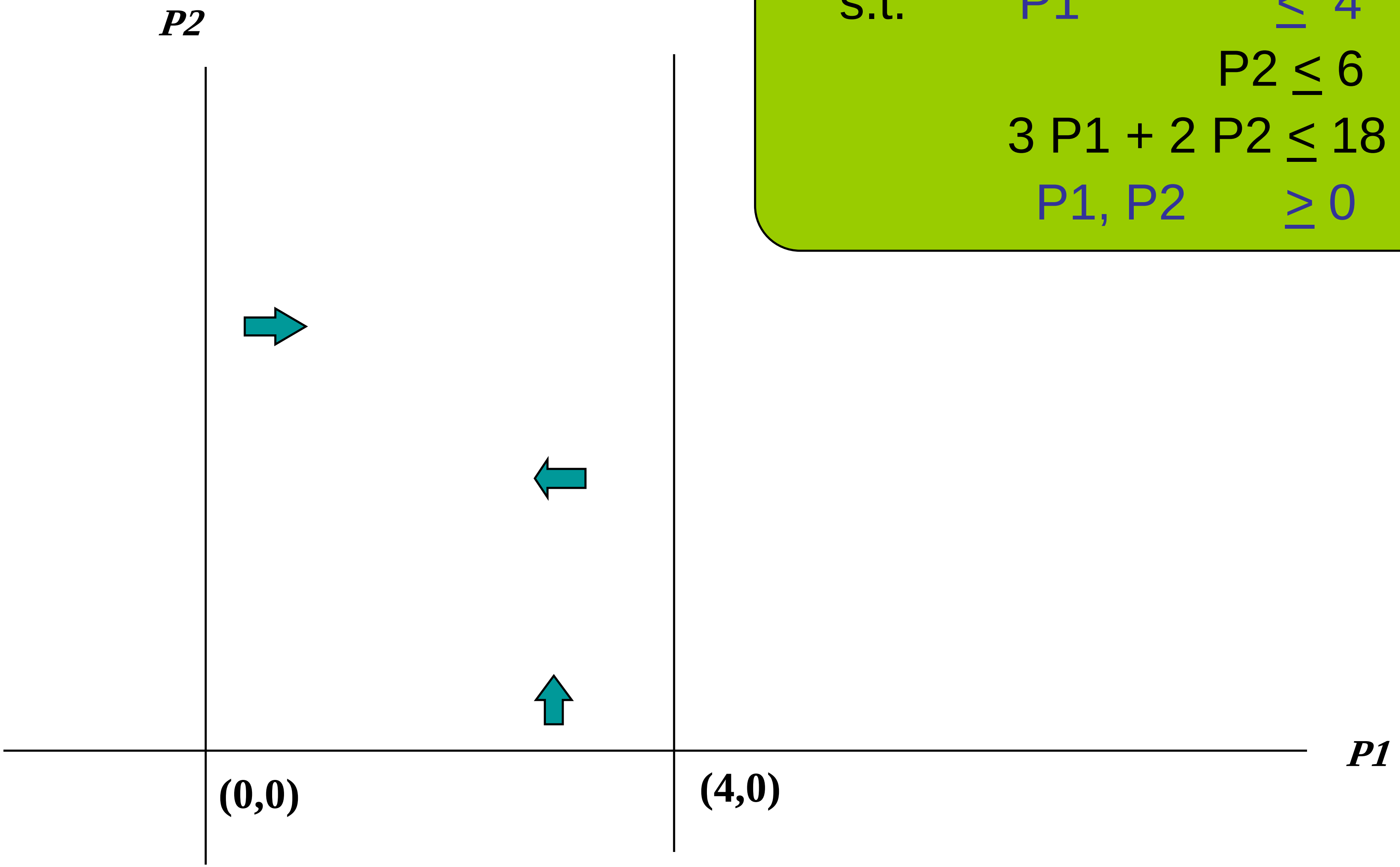
Max Z= 3 P1 + 5 P2

s.t. P1 $\leq$ 4

P2 $\leq$ 6

3 P1 + 2 P2 $\leq$ 18

P1, P2 $\geq$ 0



Example (Cont…)

Max $Z = 3P1 + 5P2$

s.t. $P1 \leq 4$

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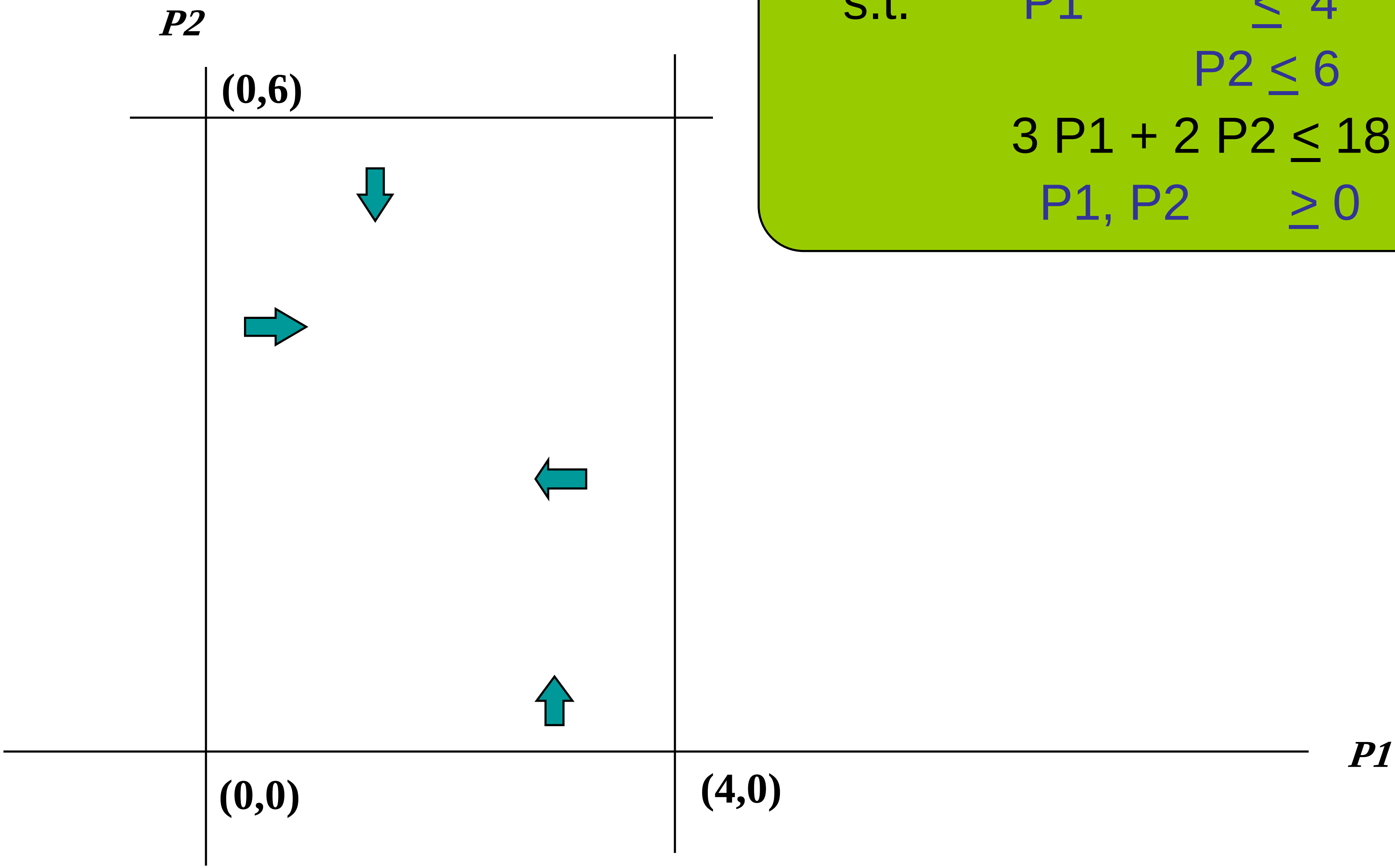
$3P1 + 2P2 \leq 18$

$P1, P2 \geq 0$



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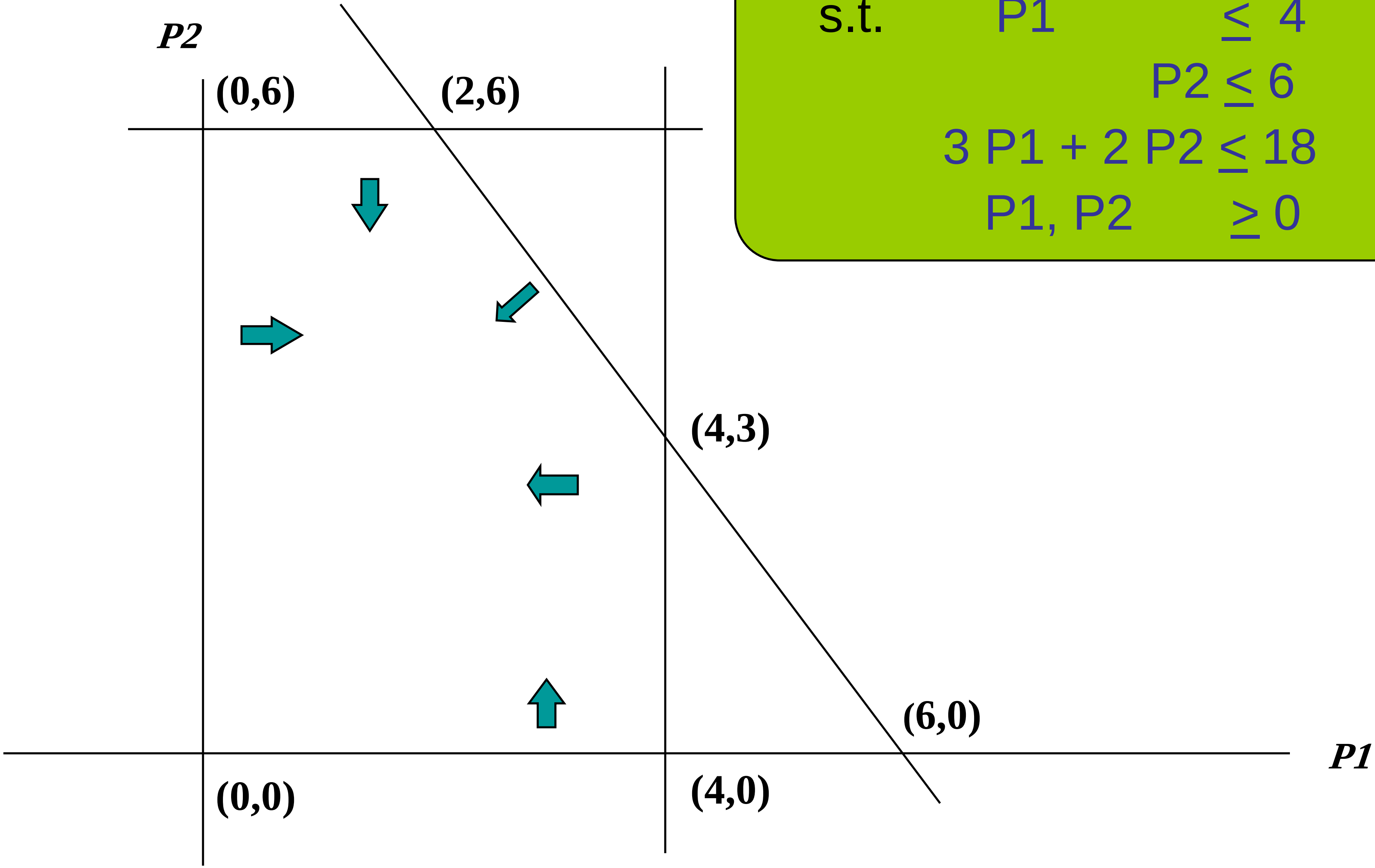
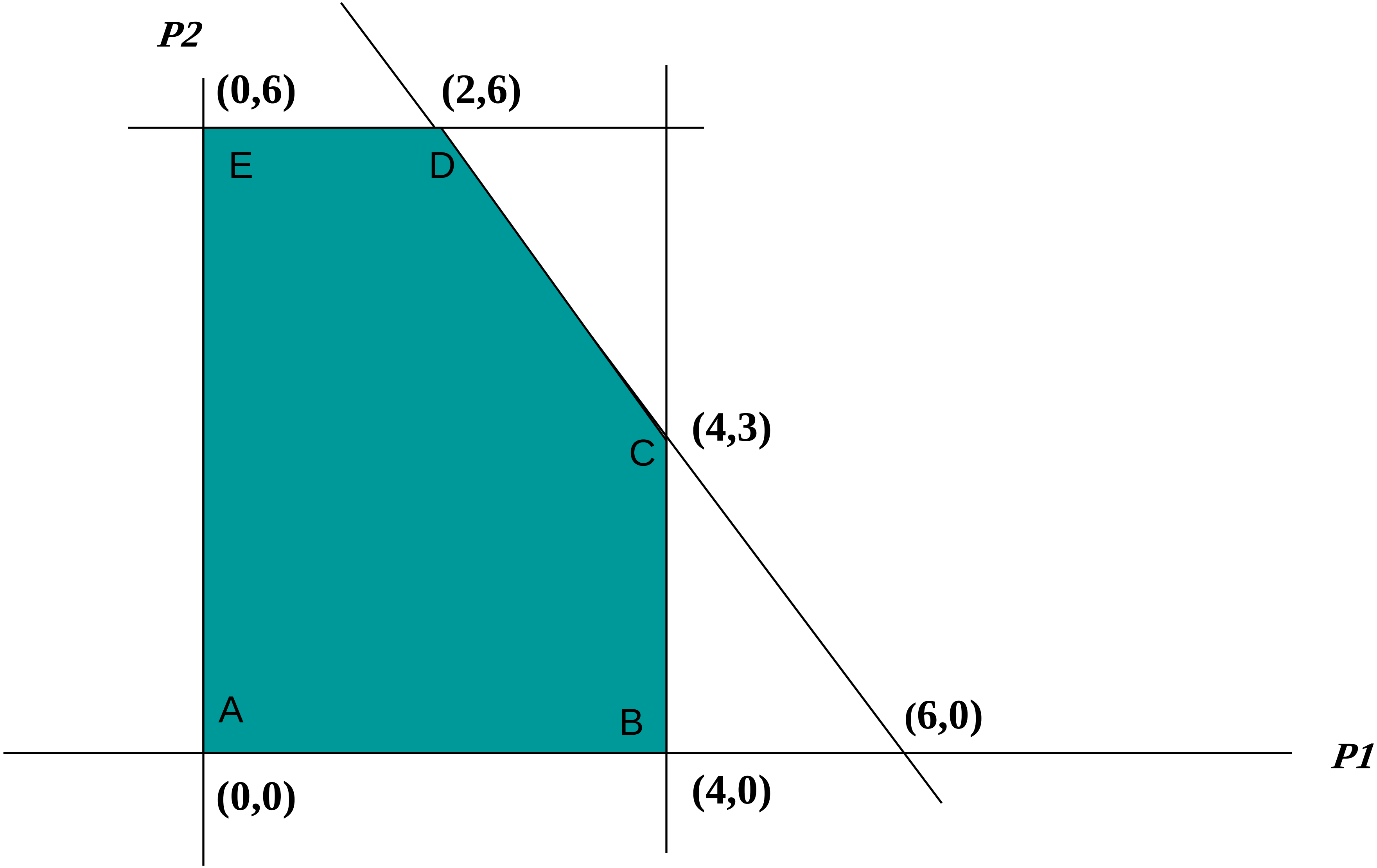


Example (Cont…)



Example (Cont…)

Objective Function : Max Z= 3 P1 + 5 P2

Corner Points	**Value of Z**
A – (0,0)	0
B – (4,0)	12
C – (4,3)	27
D – (2,6)	36
E – (0,6)	30

Optimal Point : (2,6)
Optimal Value : 36

Example

$$
\begin{aligned}
\textbf{Min} \quad & z = 5x_1 + 2x_2 \\
\textbf{s.t.} \quad & 2x_1 + 5x_2 \geq 10 \\
& 4x_1 - x_2 \geq 12 \\
& x_1 + x_2 \geq 4 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Example

$$
\begin{aligned}
\text{Min} \quad z &= 5x_1 + 2x_2 \\
\text{s.t.} \quad 2x_1 + 5x_2 &\geq 10 \\
4x_1 - x_2 &\geq 12 \\
x_1 + x_2 &\geq 4 \\
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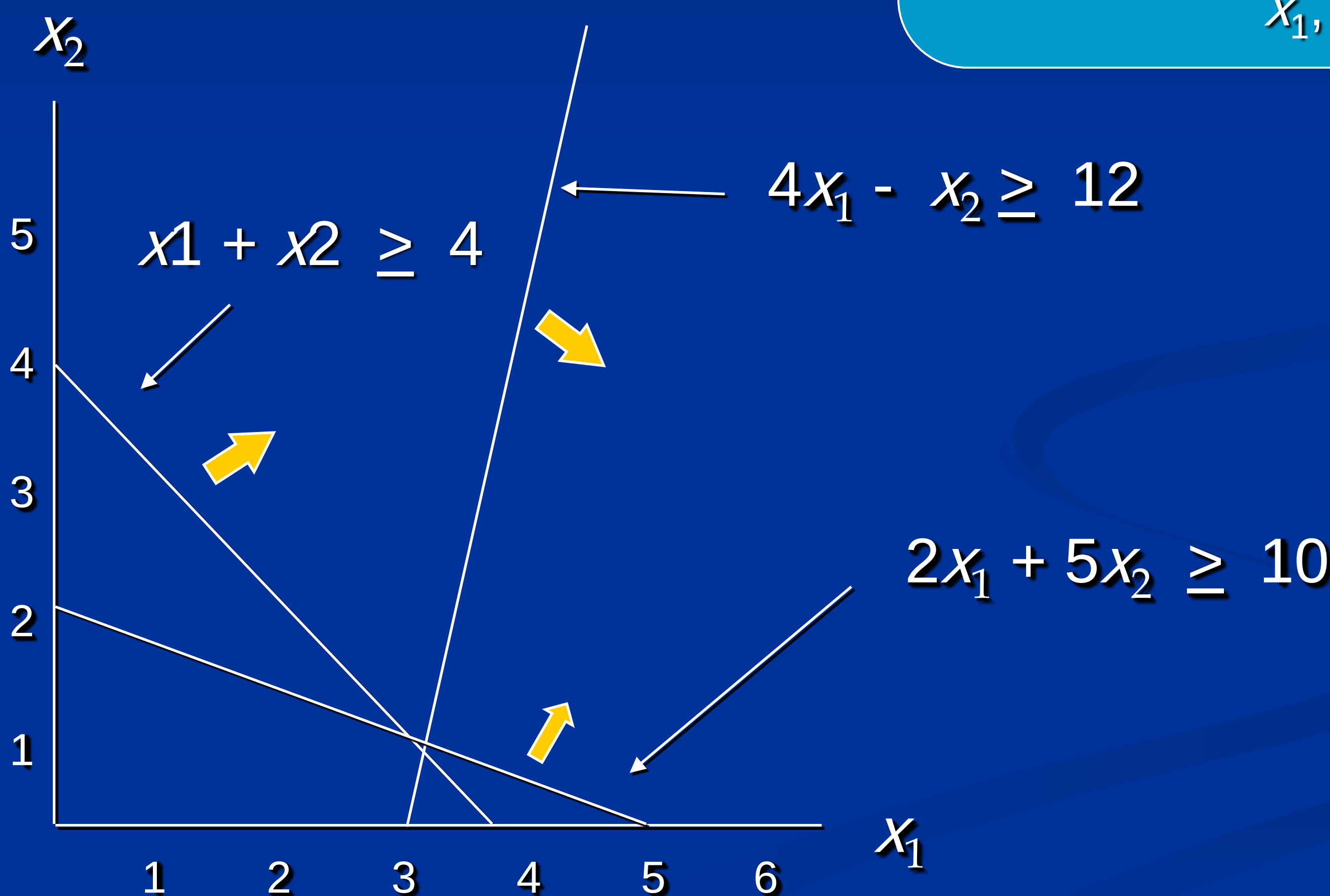

Example (Cont…)

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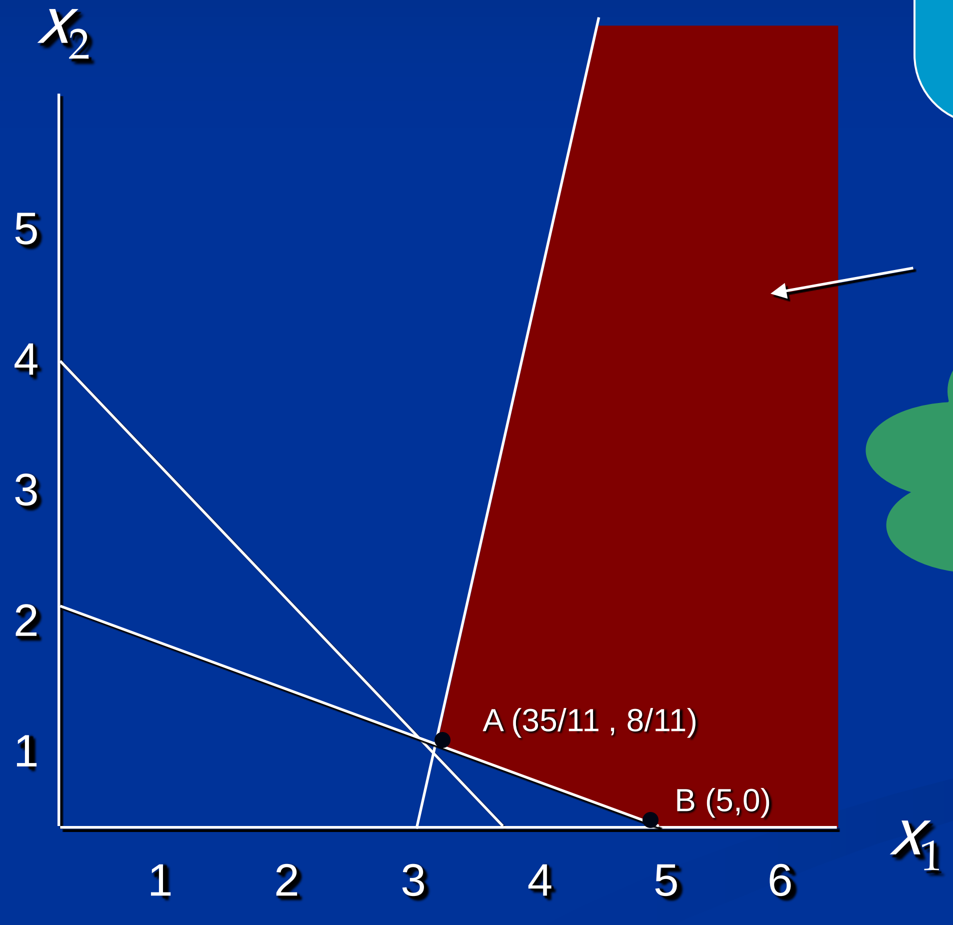


Feasible Region

This is the case of 'Unbounded Feasible Region'.

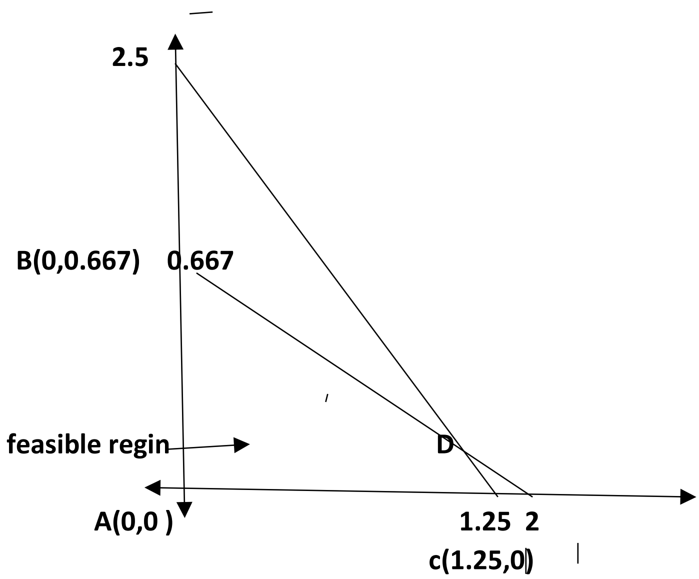
Q1: - by using the (Graphical method) find the optimum solution of X 1, X2.

Max Z=6X1+4X2 Sub in to

X1+3X2≤2

4X1+2X2≤5

X1,X2≥0



Solution: -

1- **In Eq . X1+3X2=2**
   at X1=0 ⟹ X2=0.667 (0,0.667)
   at X2=0 ⟹ X1=2 (2,0)

2- **In Eq. 4X1+X2=5**
   at X1=0 ⟹ X2=2.5 (0,2.5)
   at X2=0 ⟹ X1= 1.25 (1.25,0)

to find point D

(X1+3X2=2)*4 (1)
4 X1+2X2=5 (2)

+4 X1+12X2= 8
-4X1 - 2 X2 = -5

X2=0.3 ⟹ sub in any eq 1or 2 X1=1.1 D=(1.1,0.3)

point	X1	X2	Z=6X1+4X2	
A	0	0	Z=0	
B	0	0.667	Z=2.668	
C	1.25	0	Z=7.5	
D	1.1	0.3	Z=7.8	optimum solution in point D