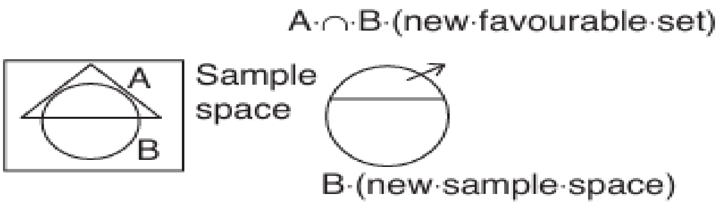
Explanation of conditional probability:

Let S be a finite sample space of a random experiment and A, B are events, such that P(A) > 0, P(B) > 0. If it is known that the event B has occurred, in light of this we wish to compute the probability of A, we mean conditional proba bility of A given B. The occurrence of event B would reduce the sample space to B, and the favourable cases would now be $A \cap B$.

ليكن S فضاء عينة محدودًا لتجربة عشوائية، وA وB حدثين، بحيث $P(A) > 0$، $P(B) > 0$. إذا علمنا أن الحدث B قد وقع، ففي ضوء ذلك نرغب في حساب احتمال A، أي الاحتمال الشرطي لـ A مع العلم أن B معطى. سيؤدي وقوع الحدث B إلى تقليل فضاء العينة إلى B، وستكون الحالات المواتية الآن $A \cap B$.



Notation The conditional probability of A given B is denoted by $P\left(\frac{A}{B}\right)$.

$$\therefore \quad P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{P(A \cap B)}{P(B)}.$$

NOTES

1. This definition is also valid for infinite sample spaces.
2. The conditional probability of B given A is denoted by $P\left(\frac{B}{A}\right)$ and $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$.

Multiplication Theorem

Let A and B be two events of certain random experiment such that A occurs only when B has already occurred. Then, for the conditional event $\frac{A}{B}$, the total possible outcomes are the outcomes favourable to the event B and its favourable outcomes are the outcomes favourable to both A and B.

So, $P\left(\frac{A}{B}\right) = \frac{n(A \cap B)}{n(B)}$

$$= \frac{n(A \cap B)}{n(S)} \times \frac{n(S)}{n(B)} = P(A \cap B) \times \frac{1}{P(B)}$$

That is, $P\left(\frac{A}{B}\right) \cdot P(B) = P(A \cap B)$

SOLVED EXAMPLES:

Example 1

When a cubical dice is rolled, find the probability of getting an even integer.

Solution

When a dice is rolled, the number of possible out comes is 6. The number of favourable outcomes of getting an even integer is 3.

The required probability $= \frac{3}{6} = \frac{1}{2}$.

Example 2

If a card is drawn from a pack of cards, find the probability of getting a queen.

Solution

When a card is drawn, the number of possible outcomes is 52. The number of favourable outcomes of getting a queen card is 4.

The required probability $= \frac{4}{52} = \frac{1}{13}$.

Example 3

A bag contains 5 green balls and 4 red balls. If 3 balls are picked from it at random, then find the odds against the three balls being red.

Solution

The total number of balls in the bag = 9. Three balls can be selected from 9 balls in ${}^9C_3$ ways.

Three red balls can be selected from 4 red balls in ${}^4C_3$ ways.

Probability of picking three red balls

$$= \frac{{}^4C_3}{{}^9C_3} = \frac{4}{84} = \frac{1}{21};\ P(\bar{E}) = \frac{20}{21}$$

Odds against the three balls being red are $= P(\bar{E}) : P(E) = \frac{20}{21} : \frac{1}{21} = 20 : 1.$

Example 4

When two dice are rolled together, find the probability of getting at least one 4.

Solution

Let E be the event that at least one dice shows 4. $\bar{E}$ be the event that no dice shows 4. The number of favourable outcomes of $\bar{E}$ is $5 \times 5 = 25 \cdot P(\bar{E}) = \frac{25}{36}$

$$\therefore \; P(E) = 1 - P(\bar{E}) = 1 - \frac{25}{36} = \frac{11}{36}.$$

clarifications

To find the probability of getting **at least one 4** when two dice are rolled, let's break it down:

🎲 Total Possible Outcomes

Each die has 6 faces, so:

- Total outcomes when rolling two dice = 6×6=36

Outcomes with *no* 4

We calculate the number of outcomes where **neither die shows a 4**:

- Die 1 has 5 options (1, 2, 3, 5, 6)
- Die 2 also has 5 options (1, 2, 3, 5, 6)
- So, outcomes with no 4 = 5×5=25

Outcomes with *at least one* 4

- These are the remaining outcomes: 36−25=1136 - 25 = 11

Final Probability

P(at least one 4)=11/36

or approximately **0.3056** (about **30.56%**).

Example 5

When two dice are rolled together find the probability that total score on the two dice will be 8 or 9.

Solution

When two dice are rolled, the total number of outcomes $= 6 \times 6 = 36$.

Favourable outcomes for getting the sum 8 or 9 are {(2, 6), (6, 2), (3, 5), (5, 3), (4, 4), (3, 6), (6, 3), (4, 5), (5, 4)}, i.e., the total number of favourable outcomes = 9.

The required probability $= \dfrac{9}{36} = \dfrac{1}{4}$.

Example 6

If two cards are drawn simultaneously from a pack of cards, what is the probability that both will be jacks or both are queens?

Solution

Here two events are mutually exclusive, $P(J \cup Q) = P(J) + P(Q)$. Probability of drawing two jacks is $P(J) = \dfrac{{}^{4}C_2}{{}^{52}C_2}$

Probability of drawing two queens is $P(Q) = \dfrac{{}^{4}C_2}{{}^{52}C_2}$

$$P(J \cup Q) = P(J) + P(Q)$$

$$= \frac{{}^{4}C_2}{{}^{52}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2} = 2 \cdot \frac{{}^{4}C_2}{{}^{52}C_2} = \frac{2}{221}.$$

Explantion

To solve this, let's break it down step by step:

🃏 Total Cards in a Deck

- A standard deck has **52 cards**
- There are **4 Jacks** and **4 Queens**

Desired Outcomes

We want the probability of:

- **Both cards being Jacks**, or
- **Both cards being Queens**

1. Probability of both being Jacks:

- Ways to choose 2 Jacks from 4:

$$\binom{4}{2} = 6$$

2. Probability of both being Queens:

- Ways to choose 2 Queens from 4:

$$\binom{4}{2} = 6$$

✅ Total favorable outcomes:

$$6(\text{Jacks}) + 6(\text{Queens}) = 12$$

🔢 Total possible outcomes:

- Ways to choose any 2 cards from 52:

$$\binom{52}{2} = \frac{52 \times 51}{2} = 1326$$

📊 Final Probability:

$$P(\text{both Jacks or both Queens}) = \frac{12}{1326} = \frac{2}{221}$$

So, the probability is $\frac{2}{221}$ or approximately **0.00905** (about **0.91%**).

Example 7

When two cards are drawn from a pack of cards, find the probability that the two cards will be kings or blacks.

Solution

The probability of drawing two kings $= \frac{^4C_2}{^{52}C_2}$

The probability of drawing two black cards is $= \frac{^{26}C_2}{^{52}C_2}$

The probability of drawing two black kings is $\frac{^2C_2}{^{52}C_2}$

∴ The required probability

$$= \frac{^4C_2}{^{52}C_2} + \frac{^{26}C_2}{^{52}C_2} - \frac{^2C_2}{^{52}C_2} = \frac{55}{221}.$$

Explanations:

🃏 Total Cards in a Deck

- 52 cards in total
- 4 Kings (♠, ♥, ♦, ♣)
- 26 black cards (♠ and ♣)

⚠️ Overlap: both cards are black Kings

- There are 2 black Kings: King of Spades and King of Clubs
- Ways to choose both = $\binom{2}{2} = 1$

So, we must subtract this overlap to avoid double-counting.

Example 8

A letter is selected at random from the set of English alphabet and it is found to be a vowel. What is the probability that it is '*e*'?

Solution

Let A be the event that the letter selected is '*e*' and B be the event that the letter is a vowel. Then, $A \cap B = \{e\}$ and $B = \{a, e, i\ o, u\}$

$$\text{So, } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\left(\frac{1}{26}\right)}{\left(\frac{5}{26}\right)} = \frac{1}{5}.$$

Examples about independent event:

As we explain before the idependent event is as follows:

Independent Events In a random experiment, if A, B are events such that $P(A) > 0$, $P(B) > 0$ and if $P\left(\frac{A}{B}\right) = P(A)$ or $P\left(\frac{B}{A}\right) = P(B)$ (conditional probability equals to unconditional probability) then we say A, B are independent events. If A, B are independent, $P(A \cap B) = P(A)\,P(B)$.

Example 9

Two coins are tossed one after the other and let A be the event of getting tail on second coin and B be the event of getting head on first coin, then find P (A/B)

Solution

Sample space = {HH, HT, TH, TT}, A = {HT, TT} and B = {HH, HT}, $(A \cap B)$ = {HT}

$$\therefore\ P(A) = \frac{2}{4} = \frac{1}{2} \quad \text{and} \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Thus $P\left(\frac{A}{B}\right) = P(A)$

$\therefore$ Logically too we understand that occurrence or non-occurrence of tail in 2nd coin.

Baye's Rule

Suppose $A_1, A_2, \ldots, A_n$ are n mutually exclusive and exhaustive events such that $P(A_i) \neq 0$. Then for $i = 1, 2, 3, \ldots, n$,

$$P\left(\frac{A_i}{A}\right) = \frac{P(A_i) \cdot P\left(\frac{A}{A_i}\right)}{\sum_{k=1}^{n} P(A_K) P\left(\frac{A}{A_K}\right)}$$

Where A is an arbitrary event of S.

شرح اضافي حول Bays theorem

نظرية بايز لها علاقه مباشره مع الاحتماليه المشروطه conditional probability . حيث اذ كانت A1, A2, A3 , ………………..An حوادث شامله وان B هو حدث معين واحتمال وقوعه لا يساوي صفر فأن

$$P(A1\backslash B) = \frac{P(A1).P(B\backslash A1)}{P(A1).P(B\backslash A1) + P(A2).P(B\backslash A2) + \cdots P(An).P(B\backslash An)}$$

مثال:

مصنع يتكون من ثلاثة اقسام لانتاج الحاسبات وكما يلي:

- **ينتج القسم A1 50% من الانتاج الكلي ومتوسط المعاب 3%**
- **ينتج القسم A2 30% من الانتاج الكلي ومتوسط المعاب 4%**
- **ينتج القسم A3 20% من الانتاج الكلي والمعاب 5%**
- **فاذا اخذنا حاسبه واحده بصوره عشوائيه من انتاج المصنع الكلي ووجدت معابه فما هو احتمال ان تكون من انتاج القسم A1**

A factory consists of three departments for the production of computers, as follows:

- **Department A1 produces 50% of the total production, with an average defect rate of 3%**
- **Department A2 produces 30% of the total production, with an average defect rate of 4%**
- **Department A3 produces 20% of the total production, with an average defect rate of 5%**