Probability and statistics

Ass.Prof. Dr. Abdullah Jabar Hussain

Probability is a measure of how likely an event is to happen. Probability is represented as a fraction and always lies between 0 and 1. An event can be defined as a subset of sample space. The sample of throwing a coin is {head, tail} and the sample space of throwing dice is {1, 2, 3, 4, 5, 6}.

So, probability is simply how likely something is to happen. Whenever we're unsure about the outcome of an event, we can talk about the probabilities of certain outcomes—how likely they are. The analysis of events governed by probability is called statistics.

The basic concepts of probabilit is the possibility of an event to happen is equal to the ratio of the number of favorable outcomes and the total number of outcomes. Sometimes students get mistaken for "favorable outcome" with "desirable outcome". This is the basic formula.

The importance of probability cannot be ignored by any organization. It is used in economics, finance, healthcare, and market forecasting. Finance: In finance, probability and statistics could help shape monetary policies, developing price models for equity, bonds, and currencies.

The fundamental probability formula is:

P(A) = Number of Favorable Outcomes / Total Number of Possible Outcomes, where P(A) represents the probability of event A. For a sample space (S) and an event (E), the formula can be written as P(E) = n(E) / n(S), with n(E) being the number of outcomes favorable to E and n(S) being the total number of outcomes in the sample space.

Accordingly, Probability is the branch of mathematics concerning the occurrence of a random event, and four main types of probability exist: classical, empirical, subjective and axiomatic. Probability is synonymous with possibility, so you could say it's the possibility that a particular event will happen.

The relation ship between probability and statistics is as follow:

Probability is a theoretical tool that predicts the likelihood of future events based on known models, while statistics is an applied field that analyzes past data to infer underlying models or distributions. They are inverse problems: probability moves from a model to predict outcomes, whereas statistics moves from data to infer the model. Probability theory provides the mathematical foundation for statistical analysis, helping to interpret data and draw conclusions in the face of uncertainty.

الاحتماليه أداة نظرية تتنبأ باحتمالية الأحداث المستقبلية بناءً على نماذج معروفة، بينما الإحصاء مجال تطبيقي يُحلل البيانات السابقة لاستنتاج النماذج أو التوزيعات الأساسية. وهما مسألتان عكسيتان: ينتقل الاحتمال من نموذج للتنبؤ بالنتائج، بينما ينتقل الإحصاء من البيانات لاستنتاج النموذج. تُوفر نظرية الاحتمالات الأساس الرياضي للتحليل الإحصائي، مما يُساعد على تفسير البيانات واستخلاص النتائج في ظل عدم اليقين.

Random Experiment:

Consider an action which is repeated under essentially identical conditions. If it results in any one of the several possible outcomes, but it is not possible to predict which outcome will appear. Such an action is called as a Random Experiment. One performance of such an experiment is called as a Trial.

لنفترض وجود إجراء متكرر في ظل ظروف متطابقة تقريبًا. إذا نتج عن ذلك أيٌّ من النتائج المحتملة، ولكن من غير الممكن التنبؤ بالنتيجة التي ستظهر، يُسمى هذا الإجراء تجربة عشوائية. ويُسمى إجراء واحد من هذه التجربة محاولة.

In probability, a random variable is a function that assigns a numerical value to each outcome of a random experiment, allowing for mathematical analysis of chance events. For example, if you toss a coin, a random variable could assign '1' to heads and '0' to tails. These variables can be discrete (taking on distinct, countable values, like the number of heads in coin flips) or continuous (taking on any value within a given range, like the height of a person).

في علم الاحتمالات، المتغير العشوائي هو دالة تُعطي قيمة عددية لكل نتيجة من نتائج تجربة عشوائية، مما يسمح بالتحليل الرياضي للأحداث العشوائية. على سبيل المثال، إذا رميت عملة معدنية، يُمكن لمتغير عشوائي أن يُعطي القيمة "1" للصورة و"0" للكتابة. يمكن أن تكون هذه المتغيرات منفصلة (تأخذ قيمًا مميزة وقابلة للعد، مثل عدد الصور في رميات العملة المعدنية) أو متصلة (تأخذ أي قيمة ضمن نطاق معين، مثل طول الشخص).

Samle space:

The set of all possible outcomes of a random experiment is called as the sample space. All the elements of the sample space together are called as

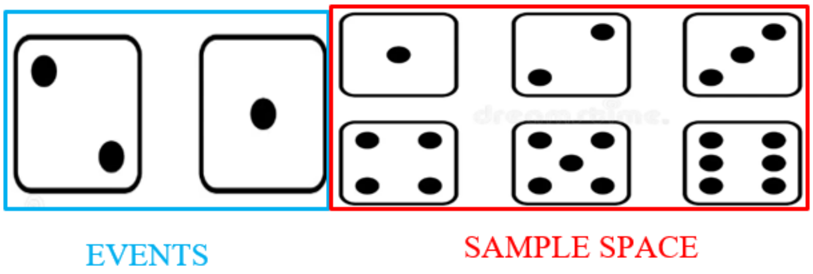
‘exhaustive cases’. The number of elements of the sample space i.e. the number of exhaustive cases is denoted by n(S) or N or n.

تُسمى مجموعة جميع النتائج الممكنة لتجربة عشوائية بفضاء العينة. وتُسمى جميع عناصر فضاء العينة مجتمعةً "الحالات الشاملة". ويُرمز لعدد عناصر فضاء العينة، أي عدد الحالات الشاملة، بالرمز n(S) أو N أو n.

Events:

Any subset of the sample space is called as an ‘Event’ and is denoted by some capital letter like A, B, C or A1, A2, A3,.. or B1, B2, ... etc.

أي مجموعة فرعية من مساحة العينة تسمى "حدثًا" ويتم الإشارة إليها بحرف كبير مثل A، B، C أو A1، A2، A3 .. أو B1، B2....الخ



Sample space …………..ALL

Sample point…………..ONE

Event ……………………ONE OR MORE

How can data have obtained?

Data are obtained by observing either uncontrolled events in nature or by observing events in controlled situations. We use the term experiment to describe either method of data collection.

Some Important Terms:

1. Experiment: is the process by which an observation (or measurement) is obtained.

2.Outcome: A possible result of one trial of a probability experiment. Sample Point : is the one of each outcome.

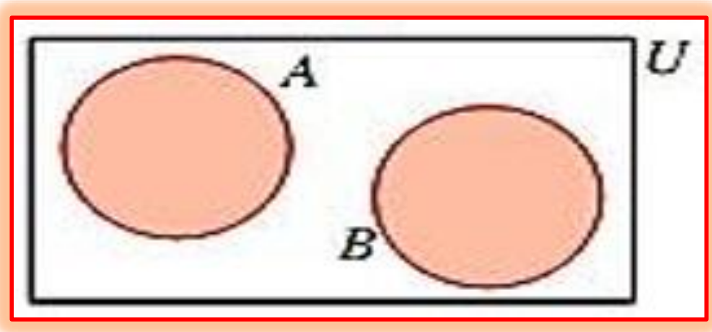
3. Event: is the outcome that is observed on a single repetition of the experiment. Sample space: is a collection of events. Or, the set of all events.

Favorable cases:

The cases which ensure the happening of an event A, are called as the cases favorable to the event A. The number of cases favorable to event A is denoted by n(A) or NA or nA.

Mutually Exclusive Events or Disjoint Events:

Two events A and B are said to be mutually exclusive or disjoint if $A \cap B = \phi$ i.e. if there is no element common to A & B.



Equally Likely Cases:

are said to be equally likely if they all have the same chance of occurrence i.e. no case is preferred to any other case.

If an experiment has equally likely outcomes and of these the event A is defined, then the **theoretical probability of event A** occuring is given by

$$P(A) = \frac{n(A)}{n(U)} = \frac{\text{Number of outcomes in which A occurs}}{\text{Total number of outcomes in the sample space}}$$

Where $n(U)$ is the total number of possible outcomes in the sample space, U, (i.e., $n(U) = N$).
As a consequence of this definition we have what are known as the **axioms of probability:**

1. $0 \le P(A) \le 1$
2. $P(\varnothing) = 0$ and $P(\varepsilon) = 1$
   That is, if $A = \varnothing$, then the event A can never occur.
   $A = U$ implies that the event A is a certainty.
3. If A and B are both subsets of U and are mutually exclusive, then
   $P(A \cup B) = P(A) + P(B)$.

Permutation and combination:

Permutations involve arrangements where order matters (e.g., ABC is different from ACB), while combinations involve selections where order does not matter (e.g., ABC is the same as ACB). You use the permutation formula, P(n, r) = n! / (n-r)!, when the sequence of selected items is important, and the combination formula, C(n, r) = n! / (r! * (n-r)!), when the sequence is not important**. To illustrate this let us take the following example:**

If we take the letters AB then the arragment will be , AB and BA so 2 permutation and 1 combination. Another example let us take ABC then the arrangment wii be (ABC,ACB,BAC,BCA,CAB.CBA) hence 6 permutation and 1 combination, Another example if we take ABC and arrange it as follows:

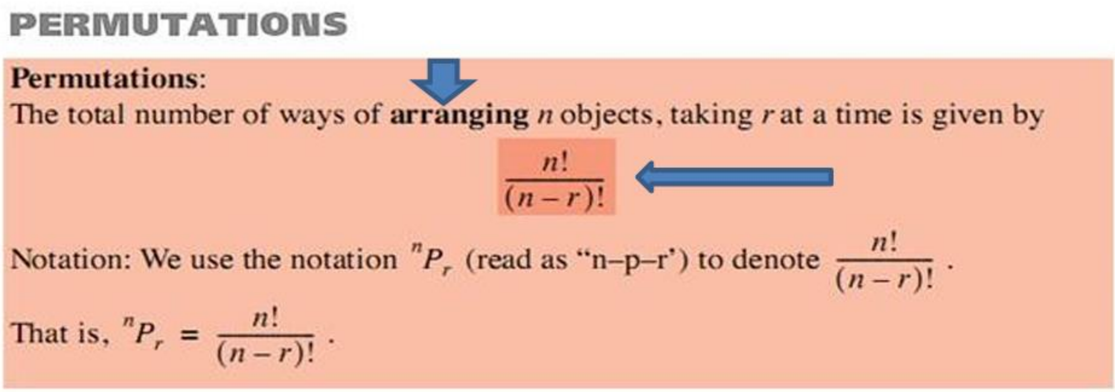
AB, AC, BA, BC ,CA, CB , this means n equal 3 and r equal 2. Permutation equal 6 and combination equal 3 so, the formula of permutation is as follows:

$$p^{n}{}_{r} = \frac{3!}{(3-2)!} = 6$$

While in case of combination is as follows:

$$c_r^n = \frac{3!}{(n-r)!r!} = \frac{6}{2} = 3$$

Additional examples and explaniations:



PERMUTATIONS

Permutations:
The total number of ways of **arranging** n objects, taking r at a time is given by

$$\frac{n!}{(n-r)!}$$

Notation: We use the notation ${}^nP_r$ (read as "n–p–r") to denote $\frac{n!}{(n-r)!}$.

That is, ${}^nP_r = \frac{n!}{(n-r)!}$.