Vectors

Important properties of linear systems can be described with the concept and notation of vectors. يمكن وصف الخصائص المهمة للأنظمة الخطية باستخدام مفهوم المتجهات ورموزها

A vector: quantity has magnitude and direction. كمية تمتلك قيمة واتجاه

Scalar : is a quantity that has magnitude alone (mass, time, or speed) كمية تمتلك قيمة

Zero vector: Vector has entries = 0 , no length and no direction.

$$\text{Zero vector} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The concept is connects vectors to ordinary systems of equations.

ربط المتجهات بأنظمة المعادلات الاعتيادية

The term vector appears in variety of mathematical and physical contexts, "vector spaces". يظهر مصطلح "المتجه" في سياقات رياضية وفيزيائية متنوعة "فضاءات المتجه"

Vectors in R (R^2 , R^3, R^n)

A matrix with only one column is called a **column vector**. For example of vectors are (u ,v, w) with two entries(x_1, x_2) مدخلان:

$u=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $v=\begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $w=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ **column vector** , $[1 \quad 4 \quad 2]$ **raw vector**

Where: x and y= real numbers, R^2=vector contains 2 entries,

Two vectors in R^2 are **equal** if and only if their **corresponding entries** are **equal**.

$$\begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix} \quad , \quad \begin{bmatrix} 4 \\ 7 \end{bmatrix} \neq \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

R^3=vector contains 3 entries. $\mathbf{u}=\begin{bmatrix} \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \end{bmatrix}$

R^n=vector contains n entries. $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$

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Geometric Descriptions of R^2

Consider a rectangular coordinate الاحداثيات المتعامدة system in the plane, each point determined by pair of numbers (a , b) with the column vector $\begin{bmatrix} a \\ b \end{bmatrix}$, and regard R^2 as set of all points.

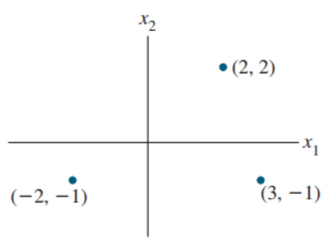


FIGURE 1 Vectors as points.

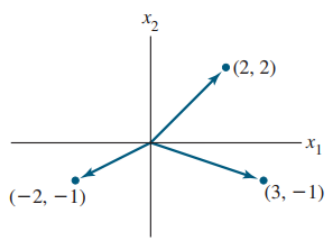


FIGURE 2 Vectors with arrows.

Operations on vectors (addition, subtraction, scalar multiplication)

Algebraic Properties of R^n

For all u; v; w in R^n and scalars قيم عددية c and d:

(i) $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$

(ii) $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$

(iii) $\mathbf{u}+\mathbf{0}=\mathbf{0}+\mathbf{u}=\mathbf{u}$

(iv) $\mathbf{u}+(-\mathbf{u})=-\mathbf{u}+\mathbf{u}=\mathbf{0}$,

(v) $c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$

(vi) $(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$

(vii) $c(d\mathbf{u})=(cd)\mathbf{u}$

(viii) $1\mathbf{u}=\mathbf{u}$

$\mathbf{u}\cdot\mathbf{v}=\mathbf{v}\cdot\mathbf{u}$

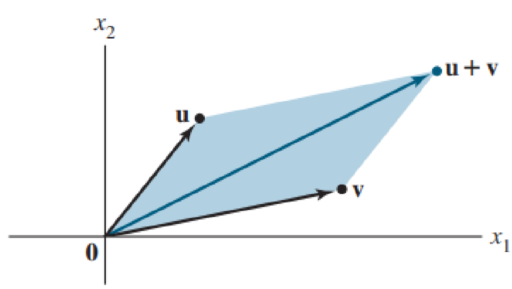
$(\mathbf{u}+\mathbf{v})\cdot\mathbf{w}=\mathbf{u}\cdot\mathbf{w}+\mathbf{v}\cdot\mathbf{w}$

$(c\mathbf{u})\cdot\mathbf{v}=c(\mathbf{u}\cdot\mathbf{v})=\mathbf{u}\cdot(c\mathbf{v})$

$\mathbf{u}\cdot\mathbf{u}\geq 0$, and $\mathbf{u}\cdot\mathbf{u}=0$ if and only if $\mathbf{u}=\mathbf{0}$

Parallelogram Rule for Addition

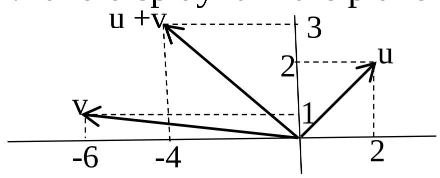
If u and v in R^2 are represented as points in the plane, then u + v as shown in figure.



Example 1 : Two vectors u = $\begin{bmatrix}2\\2\end{bmatrix}$ and v = $\begin{bmatrix}-6\\1\end{bmatrix}$, find u+v and display it in the plane ?

Solution :

$$\mathbf{u}+\mathbf{v}=\begin{bmatrix}-4\\3\end{bmatrix}$$



Example 2: Let **u** = $\begin{bmatrix}3\\-1\end{bmatrix}$, display the vectors **u**, 2**u**, and $-\frac{2}{3}$**u** on a graph?

Solution : $\mathbf{u}=\begin{bmatrix}3\\-1\end{bmatrix}$, $\mathbf{2u}=\begin{bmatrix}6\\-2\end{bmatrix}$, $-\frac{2}{3}\mathbf{u}=\begin{bmatrix}-2\\\frac{2}{3}\end{bmatrix}$

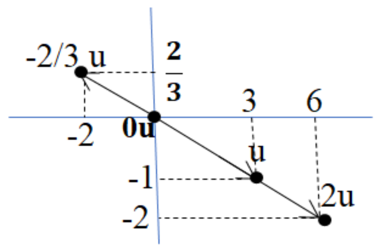


Figure 3 Typical multiples of u

Example 3 : Two vectors u = $\begin{bmatrix}-1\\2\end{bmatrix}$ and v = $\begin{bmatrix}-3\\3\end{bmatrix}$, compute **u** + **v** and **u**-2**v**?

Solution :

Compute u+v :

$$u+v=\begin{bmatrix}-1\\2\end{bmatrix}+\begin{bmatrix}-3\\3\end{bmatrix}=\begin{bmatrix}-1-3\\2+3\end{bmatrix}=\begin{bmatrix}-4\\5\end{bmatrix}$$

Compute u−2v:

$$2v=\begin{bmatrix}2(-3)\\2(3)\end{bmatrix}=\begin{bmatrix}-6\\6\end{bmatrix} \rightarrow u-2v=\begin{bmatrix}-1\\2\end{bmatrix}-\begin{bmatrix}-6\\6\end{bmatrix}=\begin{bmatrix}-1+6\\2-6\end{bmatrix}=\begin{bmatrix}5\\-4\end{bmatrix}$$

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ملاحظة: في بعض الاحيان يكتب المتجه $\begin{bmatrix}3\\-1\end{bmatrix}$ بالشكل (3 , -1) ويجب ان نميزه عن مصفوفة بصف وعمودين [3 -1]

$$\begin{bmatrix}3\\-1\end{bmatrix} \neq \begin{bmatrix}3 & -1\end{bmatrix}$$

The Length of a Vector

If **v** is in R^n, with entries v_1 , …., v_n, then **Length** of **v** = $\sqrt{\mathbf{v.v}}$ is the nonnegative scalar and equal to :

$$\|\mathbf{v}\| = \sqrt{\mathbf{v}\cdot\mathbf{v}} = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}, \quad \text{and} \quad \|\mathbf{v}\|^2 = \mathbf{v}\cdot\mathbf{v} \quad \text{……(1)}$$

Unit Vector has a magnitude of 1 i.e. $||u||=1$

Can be creating u from v by: $u= v \,.\, 1/||v||$

Example 4: Let **v** = (1, - 2, 2, 0) , Find a unit vector **u** in the same direction as **v**?

Solution : First, compute the length of **v**:

$$\|\mathbf{v}\|^2 = \mathbf{v}\cdot\mathbf{v} = (1)^2 + (-2)^2 + (2)^2 + (0)^2 = 9$$
$$\|\mathbf{v}\| = \sqrt{9} = 3$$

Then, multiply **v** by $1/||v||$ to obtain u:

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{3}\mathbf{v} = \frac{1}{3}\begin{bmatrix}1\\-2\\2\\0\end{bmatrix} = \begin{bmatrix}1/3\\-2/3\\2/3\\0\end{bmatrix}$$

To check that $||u||=1$: $||u||=\sqrt{(1/3)^2 + (-\frac{2}{3})^2 + (\frac{2}{3})^2 + 0^2} = \sqrt{\frac{1}{9} + \frac{4}{9} + \frac{4}{9} + 0}=1$

Example 5: Let **v** = (3, 4) , Find a unit vector **u** in the same direction as **v**?

Solution : First, compute the length of **v**:

$$|v| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$u = \frac{v}{|v|} = \left(\frac{3}{5}, \frac{4}{5}\right)$$

Example 6: Let **v** = (-2,1, 2) , Find a unit vector **u** in the same direction as **v**?

Solution : First, compute the length of **v**:

$$|v| = \sqrt{(-2)^2 + 1^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

$$u = \frac{1}{3}(-2, 1, 2)$$

Example 7: Let **v** = (0,-5, 12) , Find a unit vector **u** in the same direction as **v**?

Solution : First, compute the length of **v**:

$$|v| = \sqrt{0^2 + (-5)^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

$$u = \frac{1}{13}(0, -5, 12)$$

Example 8: Let **v** = (2,-1, 2,1) , Find a unit vector **u** in the same direction as **v**?

Solution : First, compute the length of **v**:

$$|v| = \sqrt{2^2 + (-1)^2 + 2^2 + 1^2} = \sqrt{4 + 1 + 4 + 1} = \sqrt{10}$$

$$u = \frac{1}{\sqrt{10}}(2, -1, 2, 1)$$