(Mathematic 1)

Basic Fundamentals in Mathematics

The Rate of Change of a Function

Cartesian Coordinate إحداثيات ديكارتية has two number lines:

Horizontal line called x-axis has real numbers (.., -3,-2, -1 , 0 , 1 , 2 , 3, …)+ rational number اعداد نسبية

Vertical line called y-axis , has real numbers numbers (.., -3,-2, -1 , 0 , 1 , 2 , 3, …)+ rational number اعداد نسبية

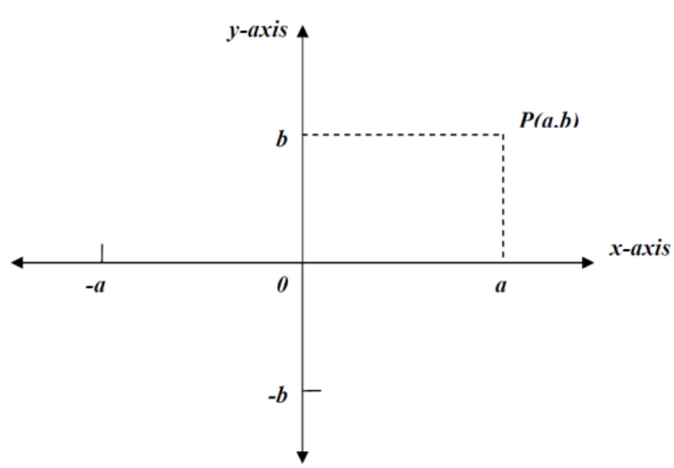
Real numbers included : الطبيعية Natural Numbers(N):{1, 2, 3, ...}

الكاملة Whole Numbers (W): {0, 1, 2, 3, ...}

الصحيحة Integers (Z): {..., -2, -1, 0, 1, 2, ...}

الأعداد النسبية Rational Numbers (Q): (1/2, -0.75, 0.333...).

Intersection of two lines called origin point (0,0).

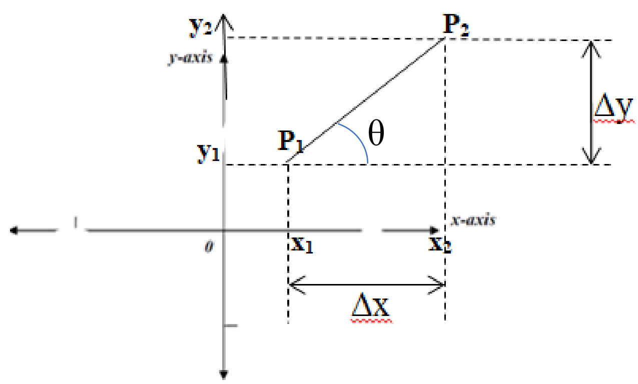


The Slope ميل of a line الخط

Increments مقدار الزيادة – When a particle moves from one position(x_1 , y_1) in the plane to another point (x_2 , y_2) , the net changes التغير in the particle's coordinates احداثيات are calculated تحسب by:

$\Delta x = x_2 - x_1$

$\Delta y = y_2 - y_1$



Slope of the line :

Let L the line in the plane has $P_1(x_1 , y_1)$ and $P_2(x_2 , y_2)$, then the slope (m) is:

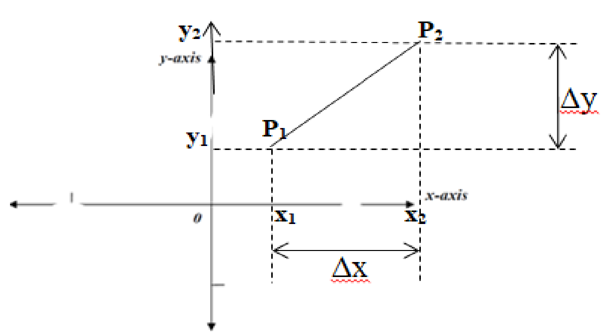
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad where \quad \Delta x \neq 0$$

- For the two perpendicular متعامدان lines L_1 and L_2: $m_1 . m_2 = -1$.
- For the parallel two lines: $m_1 = m_2$.

Angles of Inclination: زوايا الميل

The angle of inclination of a line measure تقاس counter clockwise عكس عقرب الساعة from the *x-axis* around the point of intersection.

θ = inclination angle



m=tan θ   prove that?

Ans. : $m=\frac{\Delta y}{\Delta x}=\frac{\text{مقابل}}{\text{مجاور}}=\tan\theta$

Example: 1

Find اوجد the slope ميل of the line determined المحدد by two points بنقطتين A(2,1) and B(-1,3) and find the slope of the line perpendicularالعمودي to AB.

Solution:

Slope of AB is:

$$m_{AB}=\frac{y_2-y_1}{x_2-x_1}=\frac{3-1}{-1-2}=-\frac{2}{3}$$

Slope of line perpendicular to AB is:

$$-\frac{1}{m_{AB}}=\frac{3}{2}$$

$(m_1 \,.\, m_2=-1)\rightarrow m_{AB} \,.\, m_{AB\perp}=-1$

Equations for lines معادلات الخطوط

An equation for a line is an equation that is satisfied تتحقق by the coordinates بإحداثيات of the points النقاط that lies تقعon the line and is not satisfied لا تتحقق by the coordinates of the points that lie elsewhere لنقاط خارج الخط .

Vertical lines الخطوط العمودية

Every vertical line *L* has to cross the x-axis at some point. x = a, y =0.

Horizontal lines الخطوط الافقية

The horizontal line cross the y-axis at point, x = 0, y =b.

Non vertical and non-horizontal lines

Point – slope equation of the line through the **point (x_1 , y_1)** with slope **m** is :

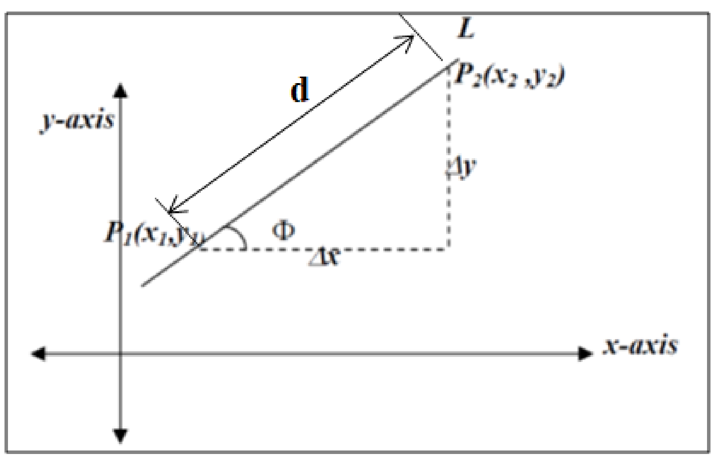
$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow \qquad y - y_1 = m(x - x_1)$$

اذا كانت نقطة واحدة معروفة

The distance from a point to a line : المسافة بين نقطة وخط او بين نقطتين

To calculate the distance d between the point $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 2: Write an equation for the line that passes through point :

a) P(-1 , 3) with slope m = -2 .

$y - y_1 = m(x - x_1) \rightarrow y - 3 = -2(x - (-1)) \rightarrow y + 2x = 1 \longrightarrow y = -2x + 1$

Example 3: Write the distance between the points $P_1(-2, 0)$ and $P_2(2, -2)$, and the equation for the line that passes through the points :

Solution : = $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\sqrt{(2-(-2))^2 + (-2-0)^2} \approx 4.47$

$$m = \frac{y_2 - y_1}{} = \frac{-2-0}{} = -\frac{1}{}$$

$$y - y_1 = m(x - x_1) \Rightarrow \quad y - 0 = -\frac{1}{2}(x - (-2)) \Rightarrow \quad 2y + x + 2 = 0$$

Example 4 : Find the slope of the line : 3x + 4y = 12

Solution:

$$y = -\frac{3}{4}x + 3 \Rightarrow \quad the \quad slope \quad is \quad m = -\frac{3}{4}$$

H.W.

Find :

a) an equation for the line through P(2 ,1) parallel to L: y = x + 2 .

b) an equation for the line through P(2 ,1) perpendicular to L: y = x + 2 .

Functions

Function is any rule that assigns to dependent and independent variables:

الدالة هي أي قاعدة تُسند إلى المتغيرات المستقلة والغير مستقلة:

$$\mathbf{y = f(x)}$$

Domain المجال and Range المدى

The symbol f represents the function, the letter x is the independent variable representing the input value to f, and y is the dependent variable or output value of f at x.

$$\underbrace{y}_{\text{output} \,\to\, \text{Range}} = f(\underbrace{x}_{\text{input} \,\to\, \text{Domain}})$$

Function is given by a formula to calculate the output value from the input variable, for example :

$y=x+2$, $x=0 \rightarrow y=2$, $x=1 \rightarrow y=3$

Domain = largest set of real x-values gives real y-values.

If we want to restrict تحديد the domain:

Q / find domain and range for the equation $y = x^2$? If:

1. $x \geq 0$ 2. $x \geq 2$ 3. Domain not restricted

Solution:

The function $y = x^2$

1. when $x \geq 0$, i.e $x = 0, 1, 2, 3, \ldots, \infty$
   substitute التعويض in function $y = x^2$, then $y = 0, 1, 4, 9, \ldots, \infty$
   domain is $x \geq 0$ and range is $[0, \infty)$.

2. when $x \geq 2$, i.e $x = 2, 3, \ldots, \infty$
   substitute التعويض in function $y = x^2$, then $y = 4, 9, \ldots, \infty$
   domain is $x \geq 2$ and range is $[4, \infty)$ or $\{x^2: x \geq 2\}$ or $\{y: y \geq 4\}$.

3. Domain not restricted غير محددة قيمها , i.e domain of x is $(-\infty, \infty)$

x	$-\infty$	…….	-2	-1	0	1	2	…..	∞
y	∞	…….	4	1	0	1	4	……	∞

Then , the range of (y) is $[0, \infty)$.

Example 5 : find domain and range for equation y=√x , if domain not restricted?

Solution : domain not restricted for x , but must be positive value inside root then: domain is x≥0 i.e [0 , ∞) and y ≥0 i.e range is [0 , ∞) .

x	0	1	2	…..	∞
y	0	1	4	……	∞

H W , find the domain and range of functions (y=$\sqrt{4-x}$, y=$\sqrt{1-x^2}$)

Graphs of Functions

If f is a function with domain D, its graph الشكلconsists يتكون of the points نقاط in the Cartesian plane. The coordinates الاحداثياتare inputمدخلات-output مخرجات pairs for f, express in :

$$\{(x, f(x)) \mid x \in D\}$$

Example 6 : Draw the function f(x) = x + 2 ?

Solution : The graph of the function f(x) = x + 2 is the set of points with coordinates (x, y), its graph is the straight line sketched موضحin the following Figure:

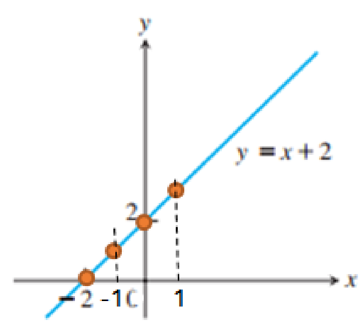
f(x)= y =x+2

at x=0 y=2

at x=1 y=3

at x=-1 y= 1

at x=-2 y=0

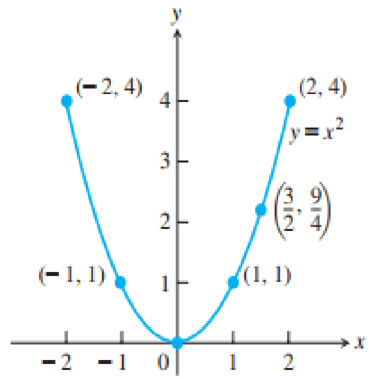


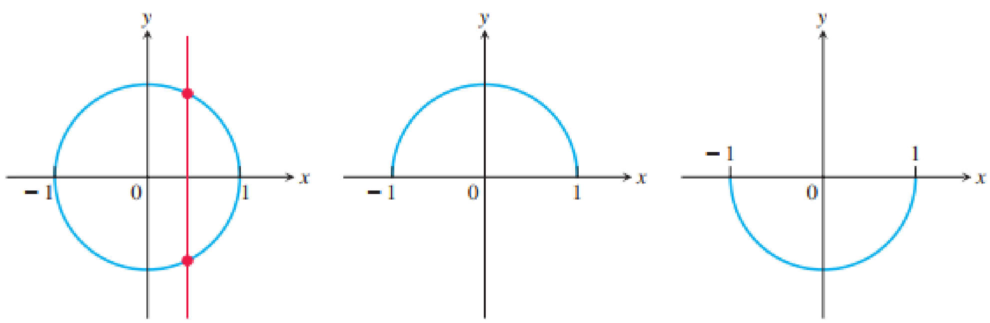
Example 7 : Draw the function $y = x^2$ over the interval [-2,2]?

Solution:

$y = x^2$

x	$y = x^2$
-2	4
-1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4



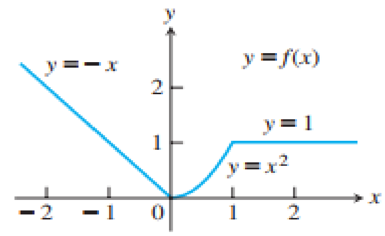


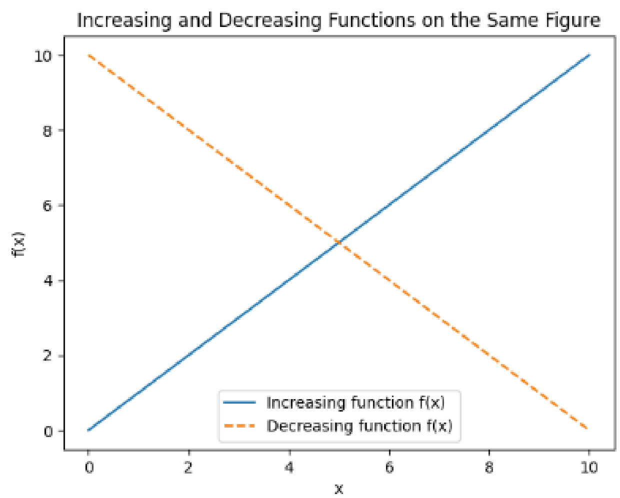
(a) $x^2 + y^2 = 1$ (b) $y = \sqrt{1 - x^2}$ (c) $y = -\sqrt{1 - x^2}$

Figure 1 illustrate different graphs for different functions

Example 8 : draw the function :

$$f(x) = \begin{cases} -x, & x < 0 \quad \text{First formula} \\ x^2, & 0 \le x \le 1 \quad \text{Second formula} \\ 1, & x > 1 \quad \text{Third formula} \end{cases}$$





1. If $f(x_2) > f(x_1)$ whenever $x_1 < x_2$, then f is said to be **increasing** on I.
2. If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$, then f is said to be **decreasing** on I.

Even Functions and Odd Functions

The graphs of even زوجي and odd فردي functions have special symmetry properties,

Definitions : A function $y = f(x)$ is:

even function of x if $f(-x) = f(x)$,
odd function of x if $f(-x) = -f(x)$,

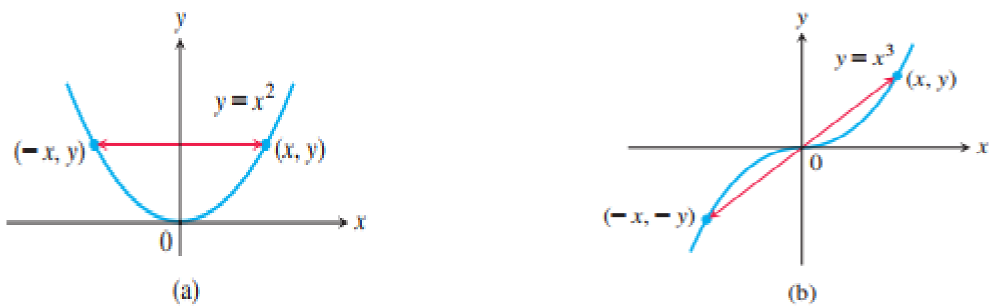


Figure 2 (a) The graph of $y = x^2$ (an even function) is symmetric about the y-axis. (b) The graph of $y = x^3$ (an odd function) is symmetric about the origin.

Example 9 : is the function $f(x) = x^2$ even or odd ?

Let x=-x , then $(-x)^2 = x^2$ for all x; symmetry about y-axis. So $f(-3) = 9 = f(3)$. Changing the sign of x does not change the value, then the function is an even.

HW is the functions even or odd?

$f(x) = x^2 + 1$

$f(x) = x$

$f(x) = x + 1$

Exponential Functions

$f(x) = a^x$ where $a > 0$ and $a \neq 1$

All exponential functions have domain $(-\infty, \infty)$ and range $(0, \infty)$.

May be in the shape $y=e^x$, $y=e^{-\infty}=1/e^{\infty}=1/\infty=0$

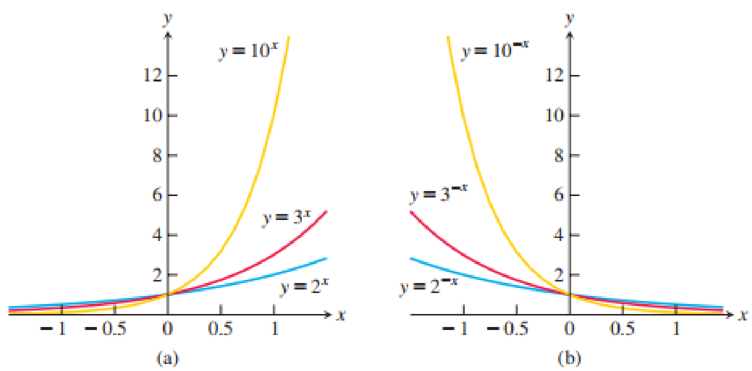


Figure 3 Graphs of exponential functions.

Logarithmic Functions

These are the functions $f(x) = \log_a x$, where the base $a \neq 1$ is a positive constant. They are the **inverse functions of the exponential functions**. In each case the domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.

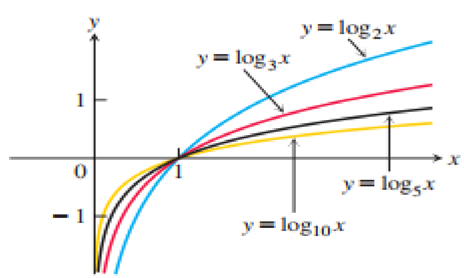


Figure 4 Graphs of four logarithmic functions