Al-Mustaqbal University
Department of Computer Engineering Techniques
First stage
Subject: Mathematic 1 – UOMU
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1st term – Lect **5**

Applications of Differentiation

Curve Sketching:

Strategy for Graphing y = f(x)

1. Find f' and f''

2. Find the where f' is positive, negative, and zero (to find localmax. and min.)

- **a.** if changes from negative to positive, then f has a local minimum ;
- **b.** if changes from positive to negative, then f has a local maximum;
- **c.** if does not change sign (that is, is positive on both sides or negativeon both sides), then f has no local extremum.

3. Find the where f'' is positive, negative, and zero (to findinflection point.)

- **a.** If f'' < 0 , the graph of f is concave down
- **b.** If f'' > 0 , the graph of f is concave up
- **c.** at f'' = 0 inflection point.

4. make a summary table

5. Plot key points, such as the intercepts and the points found in Steps2–3, and sketch the curve.

استراتيجية رسم الدالة y = f(x)
1. أوجد المشتقة الأولى والمشتقة الثانية
2. أوجد القيم التي تكون فيها f' موجبة، سالبة، أو صفرًا (لإيجاد القيم العظمى والصغرى المحلية):
أ. إذا تغيّرت f' من سالبة إلى موجبة، فإن للدالة f قيمة صغرى محلية.
ب. إذا تغيّرت f' من موجبة إلى سالبة، فإن للدالة f قيمة عظمى محلية.
ج. إذا لم تتغير إشارة f' (أي بقيت موجبة على الجانبين أو سالبة على الجانبين)، فإن الدالة f لا تمتلك قيمة قصوى محلية.
3. أوجد القيم التي تكون فيها f'' موجبة، سالبة، أو صفرًا (لإيجاد نقطة الانعطاف).
أ. إذا كان $f'' < 0$ فإن منحنى الدالة f يكون مقعّرًا إلى الأسفل.
ب. إذا كان $f'' > 0$ فإن منحنى الدالة f يكون مقعّرًا إلى الأعلى.
ج. عند $f'' = 0$ تكون هناك نقطة انعطاف.
4. أنشئ جدولًا تلخيصيًا.
5. ارسم النقاط المهمة مثل نقاط التقاطع مع المحاور والنقاط التي تم إيجادها في الخطوتين (2–3)، ثم ارسم المنحنى

Example 1: sketch the graph of $y = x^3 - 3x^2 + 4$

Solution:

1) Find y', y'' : $\boldsymbol{y' = 3x^2 - 6x}$, $\boldsymbol{y'' = 6x - 6}$

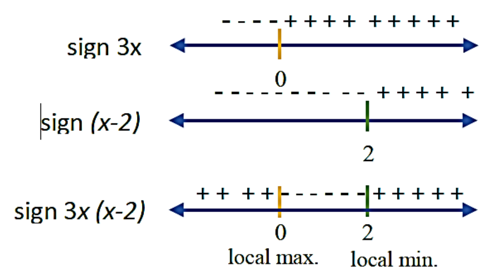
2) Find the where y' is positive, negative, and zero

$y' = 3x^2 - 6x \Longrightarrow 3x^2 - 6x = 0$

$3x\,(x-2) = 0$

$\boldsymbol{x = 0} \Longrightarrow$ **(0 , 4) local max**

$\boldsymbol{x = 2} \Longrightarrow$ **(2 , 0) local min.**

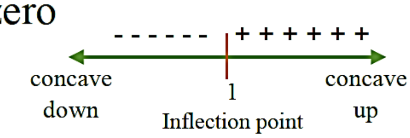


3) Find the where y'' is positive, negative, and zero

$\boldsymbol{y'' = 6x - 6}$

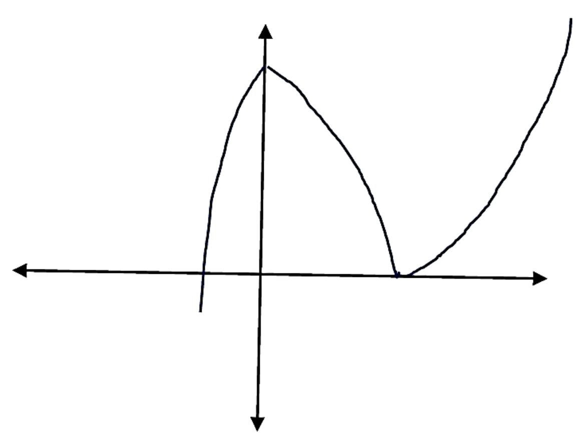
$6x - 6 = 0$

$\boldsymbol{x = 1}$, **(1,2) inflection point**



4) summary table

x	$y = x^3 - 3x^2 + 4$
-1	0
0	4 local max.
1	2 inflection point
2	0 local min.
3	4



L'Hospital's Rule:

Suppose that we have one of the following cases,

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0} \qquad \text{OR} \qquad \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\pm\infty}{\pm\infty}$$

where a can be any real number, infinity or negative infinity. In these cases we have,

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Example 2: Evaluate each of the following limits.

(a) $\lim_{x\to 0}\frac{\sin(x)}{x}$

(b) $\lim_{t\to 1}\frac{5t^4-4t^2-1}{10-t-9t^3}$

(c) $\lim_{x\to\infty}\frac{\mathbf{e}^x}{x^2}$

Solution

(a) $\lim_{x\to 0}\frac{\sin(x)}{x}$

So, we have already established that this is a $0/0$ indeterminate form so let's just apply L'Hospital's Rule.

$$\lim_{x\to 0}\frac{\sin(x)}{x}=\lim_{x\to 0}\frac{\cos(x)}{1}=\frac{1}{1}=1$$

(b) $\lim_{t\to 1}\frac{5t^4-4t^2-1}{10-t-9t^3}$

In this case we also have a $0/0$ indeterminate form and if we were really good at factoring we could factor the numerator and denominator, simplify and take the limit. However, that's going to be more work than just using L'Hospital's Rule.

$$\lim_{t\to 1}\frac{5t^4-4t^2-1}{10-t-9t^3}=\lim_{t\to 1}\frac{20t^3-8t}{-1-27t^2}=\frac{20-8}{-1-27}=-\frac{3}{7}$$

(c) $\lim\limits_{x \to \infty} \dfrac{\mathbf{e}^x}{x^2}$

This was the other limit that we started off looking at and we know that it's the indeterminate form ∞/∞ so let's apply L'Hospital's Rule.

$$\lim_{x \to \infty} \frac{\mathbf{e}^x}{x^2} = \lim_{x \to \infty} \frac{\mathbf{e}^x}{2x}$$

Now we have a small problem. This new limit is also a ∞/∞ indeterminate form. However, it's not really a problem. We know how to deal with these kinds of limits. Just apply L'Hospital's Rule.

$$\lim_{x \to \infty} \frac{\mathbf{e}^x}{x^2} = \lim_{x \to \infty} \frac{\mathbf{e}^x}{2x} = \lim_{x \to \infty} \frac{\mathbf{e}^x}{2} = \infty$$

Sometimes we will need to apply L'Hospital's Rule more than once.

Exercises:

Q1/ Evaluate each of the following limits

1. $\lim\limits_{x \to 0^+} x \ln x$

2. $\lim\limits_{x \to 2} \dfrac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$

3. $\lim\limits_{x \to \infty} x^{\frac{1}{x}}$