Al-Mustaqbal University
Department of Computer Engineering Techniques
First stage
Subject: Mathematic 1 – UOMU
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1st term – Lect **4**

Differentiation methods

The Chain Rule:

If $f(u)$ is differentiable at the point $u = g(x)$ and $g(x)$ is differentiable at x, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x, and

إذا كانت الدالة f(u) قابلة للتفاضل عند النقطة u = g(x) وكانت الدالة g(x) قابلة للتفاضل عند x، فإن الدالة المركبة (f ∘ g) (x) = f (g(x)) تكون قابلة للتفاضل عند x، و

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

In Leibniz's notation, if $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx},$$

where dy/du is evaluated at $u = g(x)$.

Example 1: Derivative the function $y = (3x^2 + 1)^2$ by chain rule

Solution:

The function $y = f(u) = u^2$, $\frac{dy}{du} = 2u$

and function $u = g(x) = 3x^2 + 1$, $\frac{du}{dx} = 6x$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot 6x$$

$$= 2(3x^2 + 1) \cdot 6x \qquad \text{Substitute for } u$$

$$= 36x^3 + 12x.$$

Calculating the derivative from the expanded formula

$$\frac{dy}{dx} = \frac{d}{dx}(9x^4 + 6x^2 + 1) = 36x^3 + 12x.$$

Implicit Differentiation

1. Differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.
   اشتق طرفي المعادلة بالنسبة إلى x، مع اعتبار y دالة قابلة للاشتقاق لـ x.

2. Collect the terms with dy/dx on one side of the equation and solve for dy/dx.
   اجمع الحدود التي تحتوي على dy/dx في أحد طرفي المعادلة، ثم حل المعادلة لإيجاد قيمة dy/dx.

Example 1: Find dy/dx if $y^2 = x^2 + \sin xy$

Solution:

$$y^2 = x^2 + \sin xy$$

$$\frac{d}{dx}\left(y^2\right) = \frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(\sin xy\right)$$

$$2y\frac{dy}{dx} = 2x + (\cos xy)\frac{d}{dx}(xy)$$

$$2y\frac{dy}{dx} = 2x + (\cos xy)\left(y + x\frac{dy}{dx}\right)$$

$$2y\frac{dy}{dx} - (\cos xy)\left(x\frac{dy}{dx}\right) = 2x + (\cos xy)y$$

$$(2y - x\cos xy)\frac{dy}{dx} = 2x + y\cos xy$$

$$\frac{dy}{dx} = \frac{2x + y\cos xy}{2y - x\cos xy}$$

Derivative of Exponential and Logarithm Functions

1. $\frac{d}{dx}(e^x) = e^x$

2. $\frac{d}{dx}(a^x) = a^x \ln(a)$

3. $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

4. $\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$

Example 1: Differentiate each of the following functions

(a) $R(w) = 4^w - 5\log_9(w)$

(b) $f(x) = 3\mathbf{e}^x + 10x^3 \ln(x)$

Solution:

(a) $R(w) = 4^w - 5\log_9$

This will be the only example that doesn't involve the natural exponential and natural logarithm functions.

$$R'(w) = 4^w \ln(4) - \frac{5}{w\ln(9)}$$

(b) $f(x) = 3\mathbf{e}^x + 10x^3 \ln(x)$

Not much to this one. Just remember to use the product rule on the second term.

$$f'(x) = 3\mathbf{e}^x + 30x^2 \ln(x) + 10x^3\left(\frac{1}{x}\right)$$
$$= 3\mathbf{e}^x + 30x^2 \ln(x) + 10x^2$$

Exercises:

Q1/ write the function in the form y = f(u) and u = g(x). Then find as $\frac{dy}{dx}$ a function of x:

1. $y(t) = \cos(t^2 + 1)$ by chain rule
2. $y = (2x + 1)^5$

Q2/ Find dy/dx

1. $x^2 + y2 = 25$
2. $x^2(x - y)^2 = x^2 - y^2$
3. $x + \tan(xy) = 0$

Q3/ Differentiate the following function:

$$y = \frac{5\mathbf{e}^x}{3\mathbf{e}^x + 1}$$