Matrix

Linear equation is an equation that can be written in the form:

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

Where : b , a_1, …., a_n = real or complex numbers, for example:

$$3x_1 - 5x_2 = -2 \quad \text{and} \quad 2x_1 + x_2 - x_3 = 2\sqrt{6}$$ المعادلة الخطية

While : $4x_1 - 5x_2 = x_1x_2$ and $x_2 = 2\sqrt{x_1} - 6$ المعادلة الغير خطية

None linear because (x_1x_2, $\sqrt{x_1}$)

System of linear equations (or a **linear system**):

A collection مجموعةof one or more linear equations involving same variables, for example:

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 \qquad - 5x_3 &= 10 \end{aligned}$$

Matrix : The rectangular array contain information of a linear system.

يحتوي الترتيب المتعامد(المصفوفة) على معلومات النظام الخطي

For previous example , the coefficients معاملاتof each variable aligned in columns, as below:

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix}$$

Called the **coefficient matrix** (or **matrix of coefficients**) مصفوفة المعاملات

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{bmatrix}$$

Called the **augmented matrix**المصفوفة الموسعة

Square Matrix:

Number of rows صفوف= Number of columns الاعمدة. **Example :** $X = \begin{bmatrix} 2 & -7 & 7 \\ 2 & 5 & 6 \\ 5 & 4 & 3 \end{bmatrix}$

Diagonal Matrix

A **diagonal matrix** is a square matrix have:

Elements a_{ij} **≠ 0** when i=j such as (a_{11}, a_{22}, a_{33}) , (diagonal elements)عناصر قطر المصفوفة .

Diagonal matrix of order "3 × 3":

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \text{ for example } \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Elements $\mathbf{a_{ij} = 0}$ **when i ≠ j** , (a_{12} a_{21} , a_{13} , a_{31}). named (non-diagonal entries المدخلات)

Zero matrix: m x n matrix, all entries كل المدخلات(diagonal and non-diagonal) =0.

Other Example of a Diagonal Matrix

Diagonal Matrix of order (4 × 4)

$$D_{4,4} = \begin{bmatrix} 22 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 \\ 0 & 0 & 17 & 0 \\ 0 & 0 & 0 & -34 \end{bmatrix}$$

Sum and product of two diagonal matrices also diagonal.

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$$\text{If } A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \quad \text{and } B = \begin{bmatrix} 8 & 0 \\ 0 & 5 \end{bmatrix} \quad \text{, then } AB = \begin{bmatrix} -24 & 0 \\ 0 & -15 \end{bmatrix}$$

- The determinant value can be determined only for square matrices.
- **Rectangular matrix:** m equations(rows)≠ n unknowns المجاهيل (columns).

Examples of diagonal matrices:

❖ **Scalar عدد ثابت matrices**: $a_{ij}=0$ for $i \neq j$ and $a_{ij}=k$ for $i=j$, when $a_{ij}=k=$ scalar value.

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$ The scalar k=5

❖ **Identity matrix**(I or I_n): square matrix has **(1)** on main diagonal and 0s elsewhere.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

❖ **A null matrix** (or **zero matrix**), denoted by 0 with any size (m x n) where every entry is zero.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

❖ A **symmetric** matrix is a matrix A such that **transpose A^T = A**. (necessary square). **Equal** numbers on opposite sides of the main diagonal. تساوي الارقام لجانبي القطر

Example 1 : which of the following matrices are symmetric and non-symmetric?

$$\begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}, \quad \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & 8 \\ 0 & 8 & -7 \end{bmatrix}$$ Solution : **Symmetric**

$$\begin{bmatrix} 1 & -3 \\ 3 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & -4 & 0 \\ -6 & 1 & -4 \\ 0 & -6 & 1 \end{bmatrix}$$ Solution : **Non-symmetric**

Transpose of a Matrix منقول المصفوفة

Given **m x n** matrix A, transpose of A is the n x m matrix, denoted by A^T , columns are formed from the corresponding rows of A. الصف يصبح عمود والعمود يصبح صف

Let A and B denote matrices whose sizes are appropriate for the following sums and products.

a. $(A^T)^T = A$

b. $(A + B)^T = A^T + B^T$

c. For any scalar r, $(rA)^T = rA^T$

d. $(AB)^T = B^T A^T$

Example 2 : Find the transpose of the following matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad B = \begin{bmatrix} -5 & 2 \\ 1 & -3 \\ 0 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & 5 & -2 & 7 \end{bmatrix}$$

Solution :

$$A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}, \quad B^T = \begin{bmatrix} -5 & 1 & 0 \\ 2 & -3 & 4 \end{bmatrix}, \quad C^T = \begin{bmatrix} 1 & -3 \\ 1 & 5 \\ 1 & -2 \\ 1 & 7 \end{bmatrix}$$

Triangular

A triangular matrix is a square matrix (n x n) where all entries **above or below** the main diagonal are zero.

Upper triangular matrices have zeros below the diagonal. $\begin{pmatrix} 1 & 5 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 4 \end{pmatrix}$

Lower triangular matrices have zeros above the diagonal. $\begin{pmatrix} 1 & 0 & 0 \\ 4 & 5 & 0 \\ 2 & 6 & 3 \end{pmatrix}$

HW

1. Find **the transpose of the following matrix:**

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$, $C = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 6 & 9 \end{bmatrix}$, $E = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

2. Find **the transpose of the following upper triangular matrices:**

$A = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 7 & 2 & 9 \\ 0 & 5 & 3 & 4 \\ 0 & 0 & 6 & 8 \\ 0 & 0 & 0 & 2 \end{bmatrix}$