



Differentiation methods

The Chain Rule:

If $f(u)$ is differentiable at the point $u = g(x)$ and $g(x)$ is differentiable at x , then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x , and

إذا كانت الدالة $f(u)$ قابلة للتفاضل عند النقطة $u = g(x)$ وكانت الدالة $g(x)$ قابلة للتفاضل عند x ، فإن الدالة المركبة $(f \circ g)(x) = f(g(x))$ تكون قابلة للتفاضل عند x ، و

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

In Leibniz's notation, if $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx},$$

where dy/du is evaluated at $u = g(x)$.

Example 1: Derivative the function $y = (3x^2 + 1)^2$ by chain rule

Solution:

The function $y = f(u) = u^2$, $\frac{dy}{du} = 2u$

and function $u = g(x) = 3x^2 + 1$, $\frac{du}{dx} = 6x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot 6x \\ &= 2(3x^2 + 1) \cdot 6x \quad \text{Substitute for } u \\ &= 36x^3 + 12x. \end{aligned}$$

Calculating the derivative from the expanded formula

$$\frac{dy}{dx} = \frac{d}{dx}(9x^4 + 6x^2 + 1) = 36x^3 + 12x.$$



Implicit Differentiation

1. Differentiate both sides of the equation with respect to x , treating y as a differentiable function of x .

اشتق طرفي المعادلة بالنسبة إلى x ، مع اعتبار y دالة قابلة للاشتقاق لـ x .

2. Collect the terms with dy/dx on one side of the equation and solve for dy/dx .

اجمع الحدود التي تحتوي على dy/dx في أحد طرفي المعادلة، ثم حل المعادلة لإيجاد قيمة dy/dx .

Example 1: Find dy/dx if $y^2 = x^2 + \sin xy$

Solution:

$$y^2 = x^2 + \sin xy$$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^2) + \frac{d}{dx}(\sin xy)$$

$$2y \frac{dy}{dx} = 2x + (\cos xy) \frac{d}{dx}(xy)$$

$$2y \frac{dy}{dx} = 2x + (\cos xy) \left(y + x \frac{dy}{dx} \right)$$

$$2y \frac{dy}{dx} - (\cos xy) \left(x \frac{dy}{dx} \right) = 2x + (\cos xy)y$$

$$(2y - x \cos xy) \frac{dy}{dx} = 2x + y \cos xy$$

$$\frac{dy}{dx} = \frac{2x + y \cos xy}{2y - x \cos xy}$$



Derivative of Exponential and Logarithm Functions

$$1. \frac{d}{dx}(e^x) = e^x$$

$$2. \frac{d}{dx}(a^x) = a^x \ln(a)$$

$$3. \frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$$4. \frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln(a)}$$

Example 1: Differentiate each of the following functions

(a) $R(w) = 4^w - 5 \log_9(w)$

(b) $f(x) = 3e^x + 10x^3 \ln(x)$

Solution:

(a) $R(w) = 4^w - 5 \log_9$

This will be the only example that doesn't involve the natural exponential and natural logarithm functions.

$$R'(w) = 4^w \ln(4) - \frac{5}{w \ln(9)}$$

(b) $f(x) = 3e^x + 10x^3 \ln(x)$

Not much to this one. Just remember to use the product rule on the second term.

$$\begin{aligned} f'(x) &= 3e^x + 30x^2 \ln(x) + 10x^3 \left(\frac{1}{x}\right) \\ &= 3e^x + 30x^2 \ln(x) + 10x^2 \end{aligned}$$



Exercises:

Q1/ write the function in the form $y = f(u)$ and $u = g(x)$. Then find as $\frac{dy}{dx}$ a function of x:

1. $y(t) = \cos(t^2 + 1)$ by chain rule
2. $y = (2x + 1)^5$

Q2/ Find dy/dx

1. $x^2 + y^2 = 25$
2. $x^2(x - y)^2 = x^2 - y^2$
3. $x + \tan(xy) = 0$

Q3/ Differentiate the following function:

$$y = \frac{5e^x}{3e^x + 1}$$