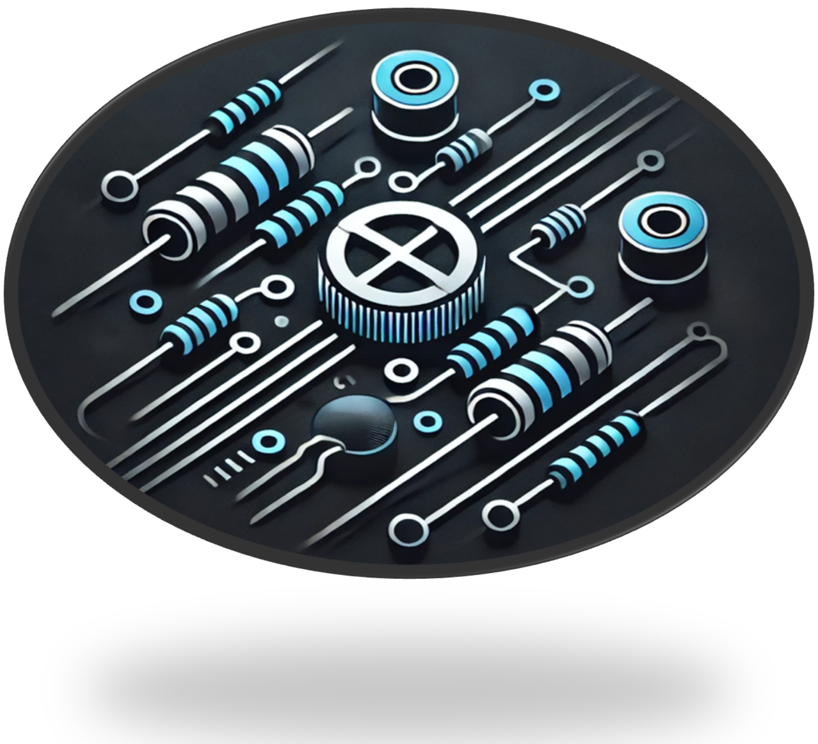
Ministry of Higher Education and Scientific Research
Al-Mustaqbal University

Computer Engineering Technologies Department



Lecturer: Zahraa Hazim

Electrical Engineering Fundamentals

Supplementary books:

1. **Fundamentals of Electric Circuits – Charles K. Alexander & Matthew N. O. Sadiku**
2. **Electrical Engineering: Principles and Applications – Allan R. Hambley**

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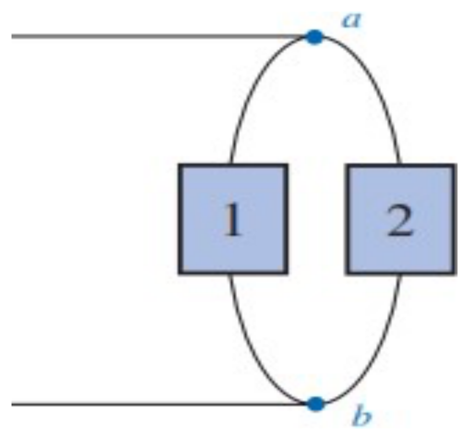
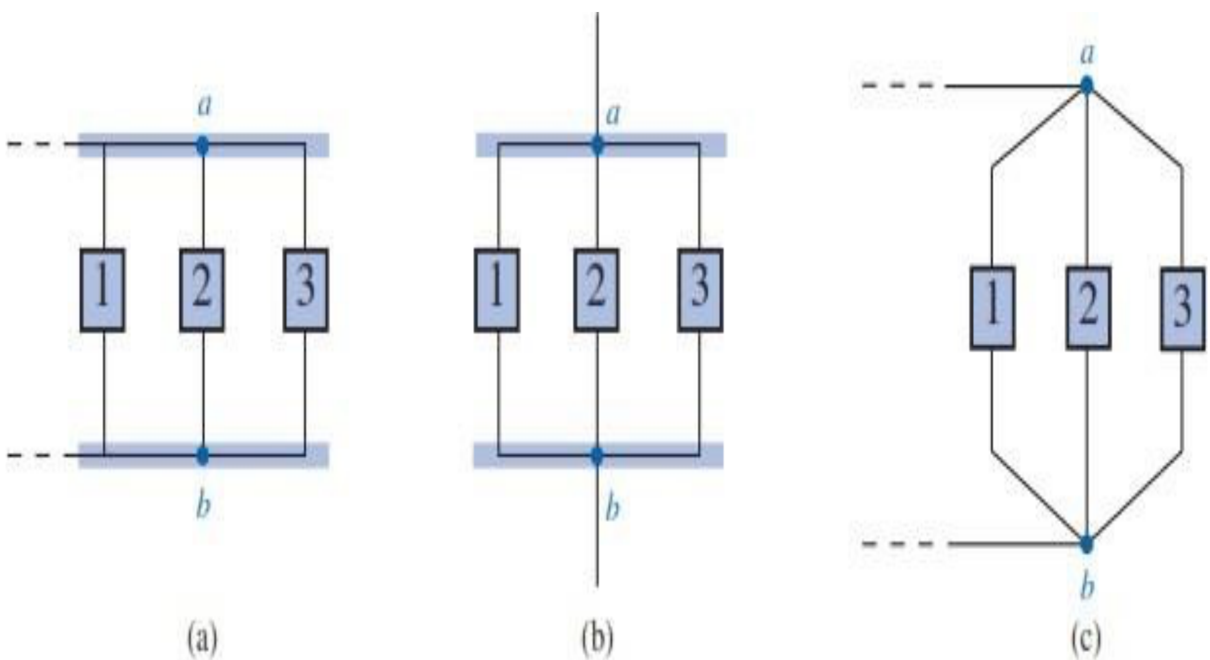
PARALLEL DC CIRCUITS

Two elements, branches, or networks are in parallel if they have two points in common.





Different ways in which three parallel elements may appear.

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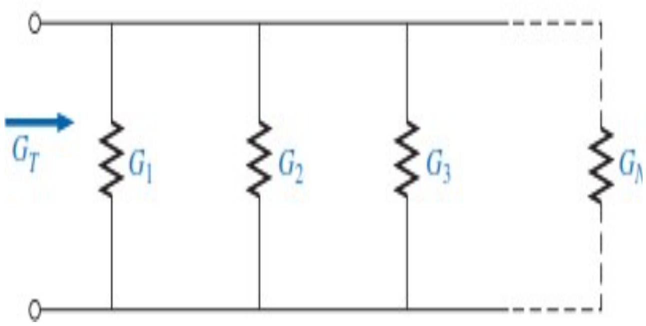
TOTAL CONDUCTANCE AND RESISTANCE

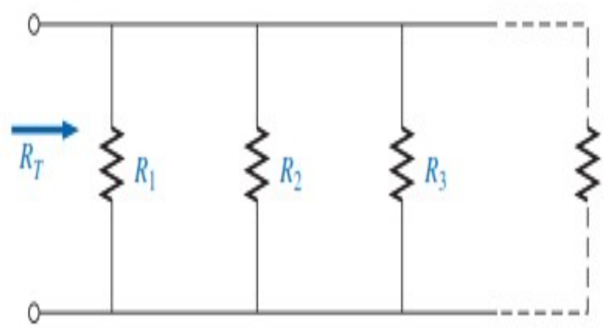
Recall that for series resistors, the total resistance is the sum of the resistor values.

For parallel elements, the total conductance is the sum of the individual conductance.

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

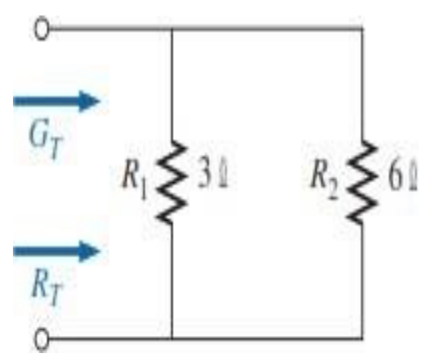




Example 3.1 Determine the total conductance and resistance for the parallel network of Fig. shown.

$$G_T = G_1 + G_2 = \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega} = 0.333\text{ S} + 0.167\text{ S} = \mathbf{0.5\text{ S}}$$

and $$R_T = \frac{1}{G_T} = \frac{1}{0.5\text{ S}} = \mathbf{2\,\Omega}$$



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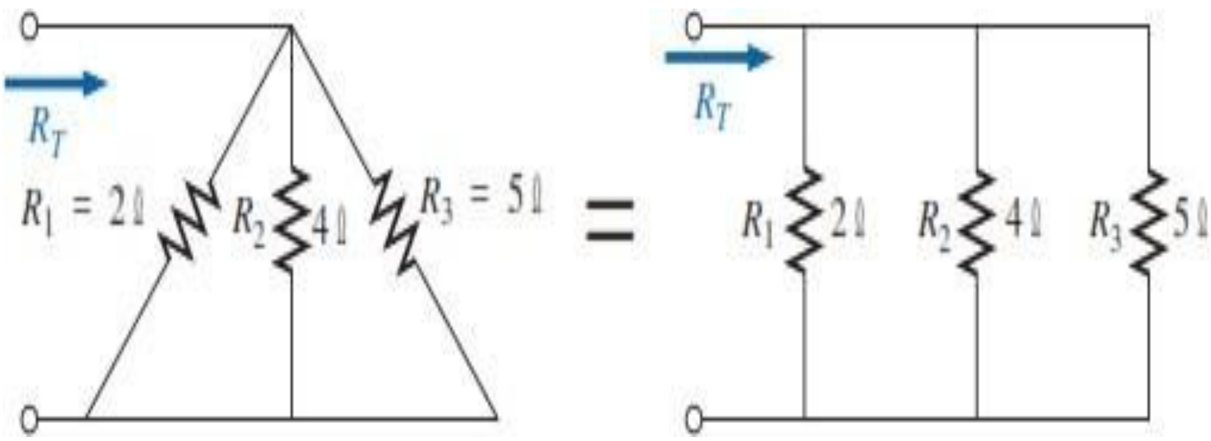
Example 3.2 Determine the total resistance for the network of Fig.shown.



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{2\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{5\,\Omega} = 0.5\text{ S} + 0.25\text{ S} + 0.2\text{ S}$$

$$= 0.95\text{ S}$$

and $$R_T = \frac{1}{0.95\text{ S}} = \mathbf{1.053\,\Omega}$$

The total resistance of parallel resistors is always less than the value of the smallest resistor.

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For *equal resistors in parallel, the equation becomes significantly* easier to apply. For *N equal resistors in parallel,*

$$\frac{1}{R_T} = \underbrace{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R}}_{N} = N\left(\frac{1}{R}\right)$$

and

$$\boxed{R_T = \frac{R}{N}}$$

and

$$\boxed{G_T = NG}$$

For two parallel resistors, we write

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2} = \frac{R_2 + R_1}{R_1R_2}$$

and

$$\boxed{R_T = \frac{R_1R_2}{R_1 + R_2}}$$

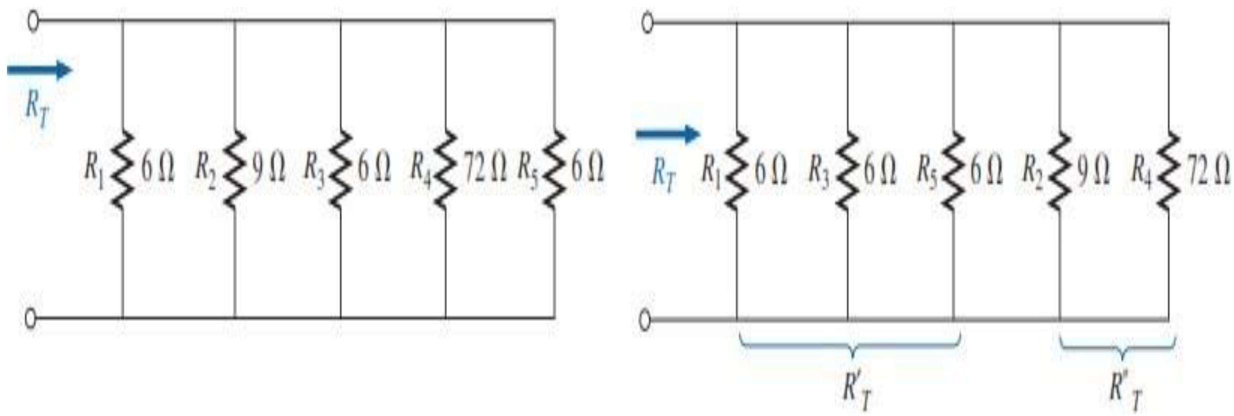
In words, *the total resistance of two parallel resistors is the product of the two divided by their sum.*

For three parallel resistors,

$$\boxed{R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}}$$

$$\boxed{R_T = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}}$$

Example 3.3 Calculate the total resistance of the parallel network of Fig. shown.



$$R'_T = \frac{R}{N} = \frac{6\,\Omega}{3} = 2\,\Omega$$

$$R_T = R'_T \parallel R''_T$$

↑ In parallel with

$$R''_T = \frac{R_2R_4}{R_2+R_4} = \frac{(9\,\Omega)(72\,\Omega)}{9\,\Omega+72\,\Omega} = \frac{648\,\Omega}{81} = 8\,\Omega$$

$$= \frac{R'_TR''_T}{R'_T+R''_T} = \frac{(2\,\Omega)(8\,\Omega)}{2\,\Omega+8\,\Omega} = \frac{16\,\Omega}{10} = \mathbf{1.6\,\Omega}$$

PARALLEL ELEMENTS

$$V_1 = V_2 = E$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}$$

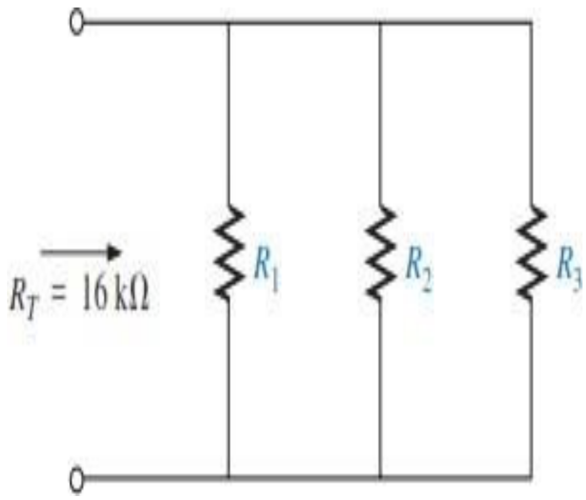
$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$

$$E\left(\frac{1}{R_T}\right) = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$$

$$I_s = I_1 + I_2$$

Example 3.4 Determine the values of R_1, R_2, and R_3 in Fig. shown if $R_2 = 2R_1$ and $R_3 = 2R_2$ and the total resistance is 16 kΩ.



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{16\,\text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

since $\quad R_3 = 2R_2 = 2(2R_1) = 4R_1$

and $\quad \frac{1}{16\,\text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2}\left(\frac{1}{R_1}\right) + \frac{1}{4}\left(\frac{1}{R_1}\right)$

$$\frac{1}{16\,\text{k}\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

with $\quad R_1 = 1.75(16\,\text{k}\Omega) = \mathbf{28\,k\Omega}$

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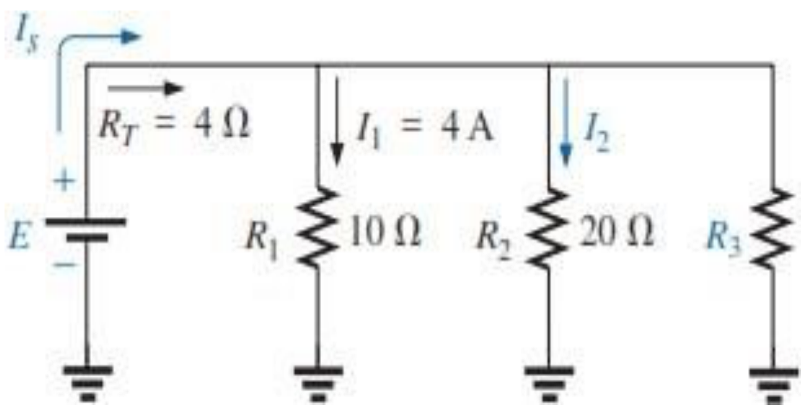
Example 3.5 Given the information provided in Fig. shown:

a. Determine R_3.
b. Calculate E.
c. Find I_s.
d. Find I_2.
e. Determine P_2.



Sol:

a. $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{4\,\Omega} = \frac{1}{10\,\Omega} + \frac{1}{20\,\Omega} + \frac{1}{R_3}$$

$$0.25\text{ S} = 0.1\text{ S} + 0.05\text{ S} + \frac{1}{R_3}$$

$$0.25\text{ S} = 0.15\text{ S} + \frac{1}{R_3}$$

$$\frac{1}{R_3} = 0.1\text{ S}$$

$$R_3 = \frac{1}{0.1\text{ S}} = \mathbf{10\,\Omega}$$

b. $E = V_1 = I_1 R_1 = (4\text{ A})(10\,\Omega) = \mathbf{40\text{ V}}$

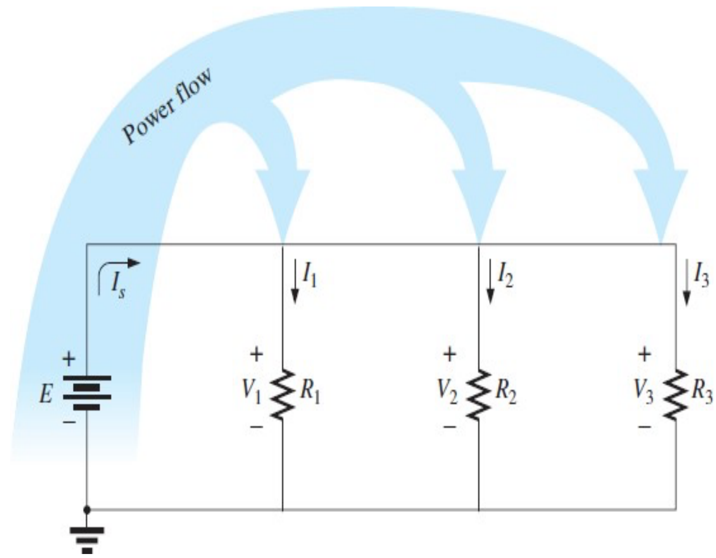
c. $I_s = \dfrac{E}{R_T} = \dfrac{40\text{ V}}{4\,\Omega} = \mathbf{10\text{ A}}$

d. $I_2 = \dfrac{V_2}{R_2} = \dfrac{E}{R_2} = \dfrac{40\text{ V}}{20\,\Omega} = \mathbf{2\text{ A}}$

e. $P_2 = I_2^2 R_2 = (2\text{ A})^2(20\,\Omega) = \mathbf{80\text{ W}}$

POWER DISTRIBUTION IN A PARALLEL CIRCUIT

$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$



The power delivered by the source

$$P_E = EI_s \qquad \text{(watts, W)}$$

as is the equation for the power to each resistor (shown for R_1 only):

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \qquad \text{(watts, W)}$$

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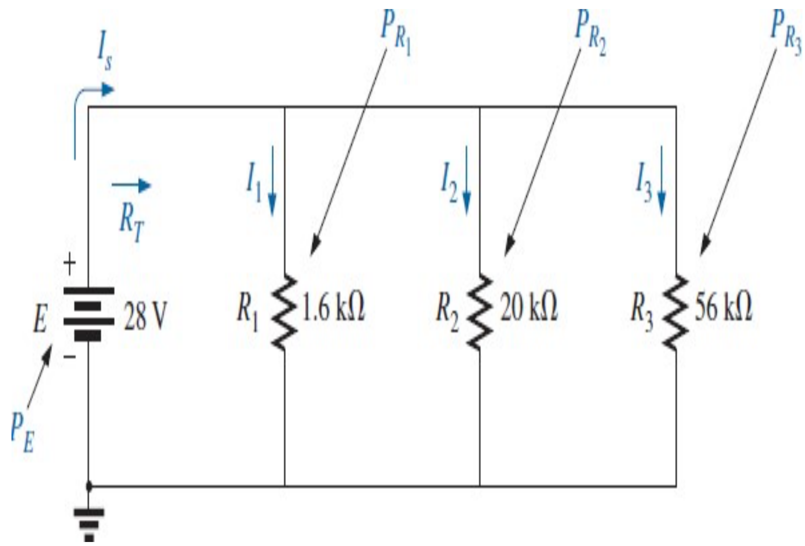
Example 3.7 For the parallel network shown:

a. Determine the total resistance R_T.

b. Find the source current and the current through each resistor.

c. Calculate the power delivered by the source.

d. Determine the power absorbed by each parallel resistor.



It should now be apparent from previous examples that the total resistance is less than 1.6 kΩ.

$$R_T = \frac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}} = \frac{1}{\dfrac{1}{1.6\text{ k}\Omega}+\dfrac{1}{20\text{ k}\Omega}+\dfrac{1}{56\text{ k}\Omega}}$$

$$= \frac{1}{625\times10^{-6}+50\times10^{-6}+17.867\times10^{-6}} = \frac{1}{692.867\times10^{-6}}$$

and $R_T = \mathbf{1.44\text{ k}\Omega}$

b. Applying Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{28\ \text{V}}{1.44\ \text{k}\Omega} = \mathbf{19.44\ mA}$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{28\ \text{V}}{1.6\ \text{k}\Omega} = \mathbf{17.5\ mA}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{28\ \text{V}}{20\ \text{k}\Omega} = \mathbf{1.4\ mA}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{28\ \text{V}}{56\ \text{k}\Omega} = \mathbf{0.5\ mA}$$

c. $$P_E = EI_s = (28\ \text{V})(19.4\ \text{mA}) = \mathbf{543.2\ mW}$$

d. Applying each form of the power equation:

$$P_1 = V_1 I_1 = EI_1 = (28\ \text{V})(17.5\ \text{mA}) = \mathbf{490\ mW}$$

A review of the results clearly shows the fact that the larger the resistor, the less the power absorbed.

KIRCHHOFF'S CURRENT LAW

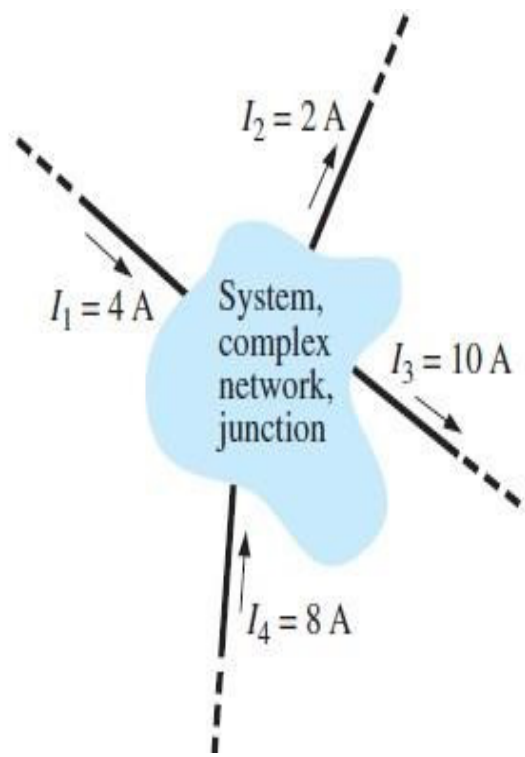
Kirchhoff's voltage law provides an important relationship among voltage levels around any closed loop of a network.

We now consider **Kirchhoff's current law (KCL),** which provides an equally important relationship among current levels at any junction.

Kirchhoff's current law (KCL) ***states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero.*** In other words, ***the sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction***

$$\Sigma I_i = \Sigma I_o$$

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$\mathbf{12\text{ A} = 12\text{ A}} \quad \text{(checks)}$$



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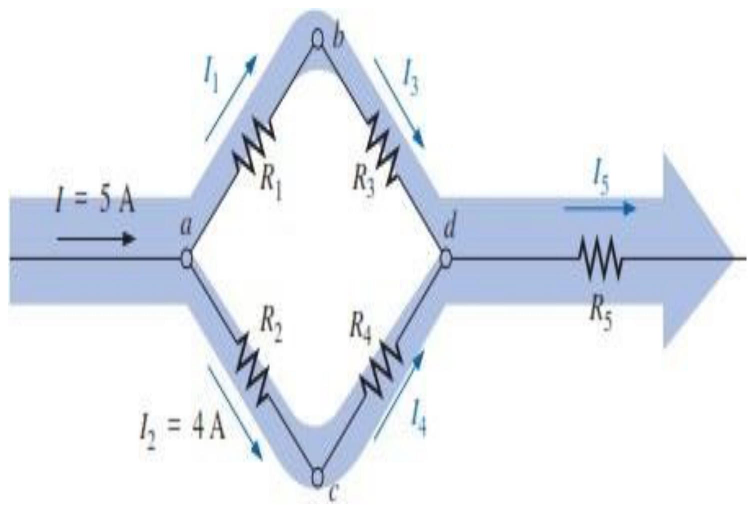
Example 3.6 Determine I_1, I_3, I_4, and I_5 for the network of Fig. shown.



At node *a*:

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$
$$I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\text{ A}}$$

At node *b*:

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$
$$I_3 = I_1 = \mathbf{1\text{ A}}$$

At node *d*:

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5 = \mathbf{5\text{ A}}$$

At node *c*:

$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$
$$I_4 = I_2 = \mathbf{4\text{ A}}$$

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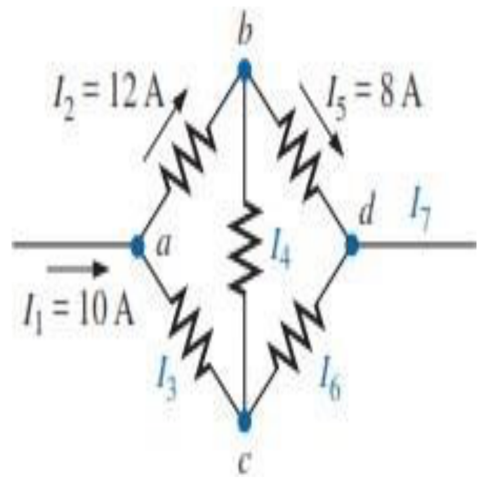
Example 3.7 **Find** the magnitude and direction of the currents I_3**,** I_4**,** I_6, and I_7 for the network of Fig. shown. Even though the elements are not in series or parallel, Kirchhoff's current law can be applied to determine all the unknown currents.



Considering the overall system, we know that the current entering must equal that leaving. Therefore,

$$I_7 = I_1 = \mathbf{10\ A}$$

Applying Kirchhoff's current law at node *a,*

$$I_1 + I_3 = I_2$$

$$10\text{ A} + I_3 = 12\text{ A}$$

$$I_3 = 12\text{ A} - 10\text{ A} = \mathbf{2\ A}$$

At node *b, since 12 A are entering and 8A are leaving,* I_4 must be leaving. Therefore,

$$I_2 = I_4 + I_5$$

$$12\text{ A} = I_4 + 8\text{ A}$$

$$I_4 = 12\text{ A} - 8\text{ A} = \mathbf{4\ A}$$

At node *c,* I_3 is leaving at 2A and I_4 is entering at 4A, requiring that I_6 be leaving. Applying Kirchhoff's current law at node *c,*

$$I_4 = I_3 + I_6$$

$$4\text{ A} = 2\text{ A} + I_6$$

$$I_6 = 4\text{ A} - 2\text{ A} = \mathbf{2\ A}$$

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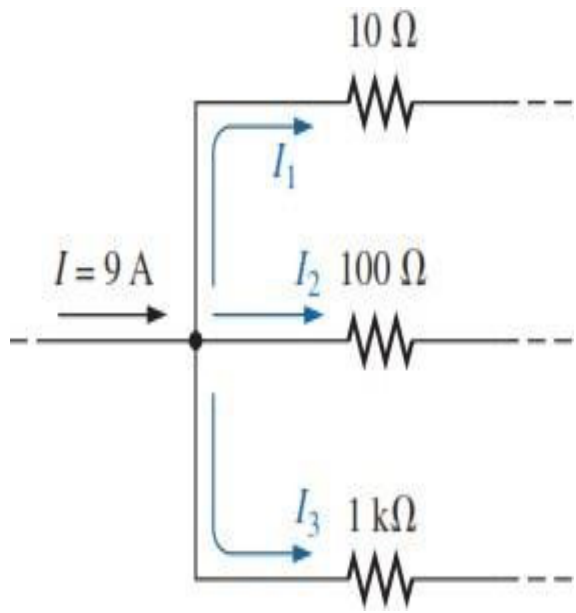
CURRENT DIVIDER RULE



-Most of the current will pass through the smallest resistor of 10 Ω, and the least current will pass through the 1 kΩ resistor.

-In fact, the current through the 100 Ω resistor will also exceed that through the 1 k Ω resistor.

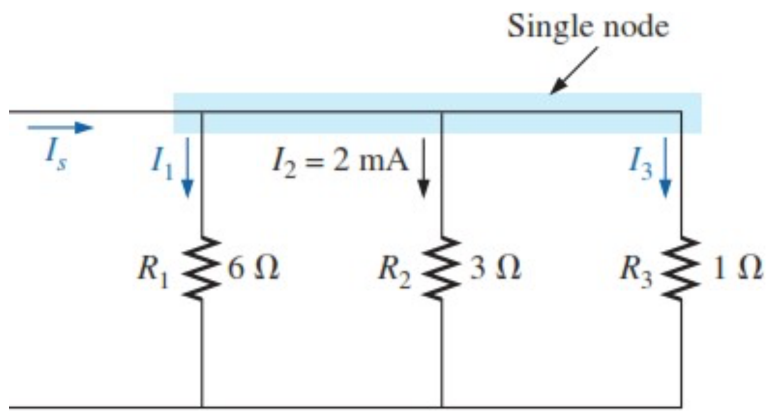
-By recognizing that the resistance of the 100 Ω is 10 times that of the 10 Ω resistor. The result is a current through the 10 Ω resistor that is 10 times that of the 100 Ω resistor.

-Similarly, the current through the 100 Ω resistor is 10 times that through the 1 kΩ resistor.

For two parallel elements of equal value, the current will divide equally.
For parallel elements with different values, the smaller the resistance, the greater the share of input current.
For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

Example 3.8

a. Determine currents I_1 and I_3 for the network in Fig.
b. Find the source current Is



SOL:

a. Since R_1 is twice R_2, the current I_1 must be one-half I_2, and

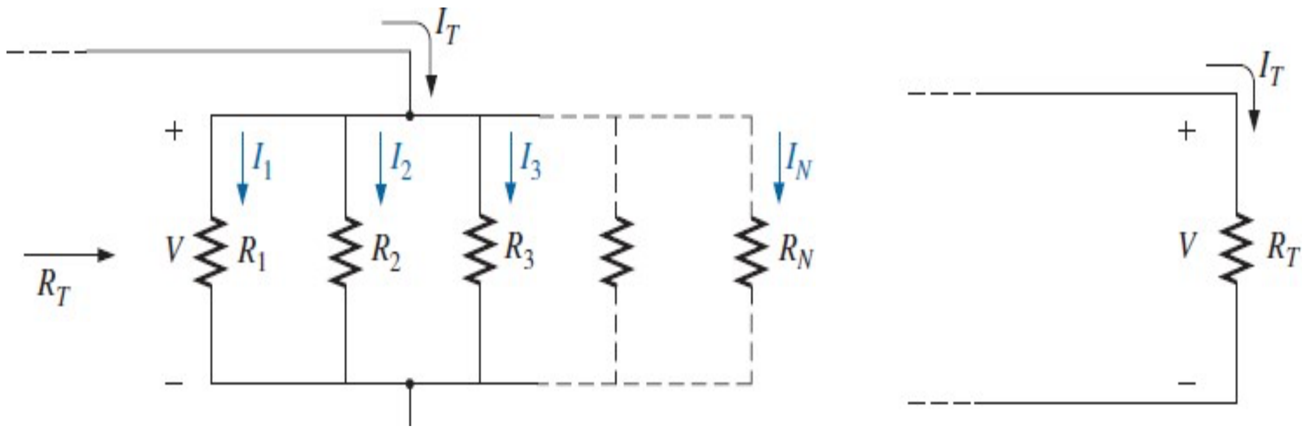
$$I_1 = \frac{I_2}{2} = \frac{2\text{ mA}}{2} = \mathbf{1\text{ mA}}$$

Since R_2 is three times R_3, the current I_3 must be three times I_2, and

$$I_3 = 3I_2 = 3(2\text{ mA}) = \mathbf{6\text{ mA}}$$

b. Applying Kirchhoff's current law:

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$
$$I_s = 1\text{ mA} + 2\text{ mA} + 6\text{ mA} = \mathbf{9\text{ mA}}$$



The current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistor of interest and multiplied by the total current entering the parallel configuration.

The current I_T *can then be determined using Ohm's law:* $$I_T = \frac{V}{R_T}$$

Since the voltage *V is the same across parallel elements, the following* is true:

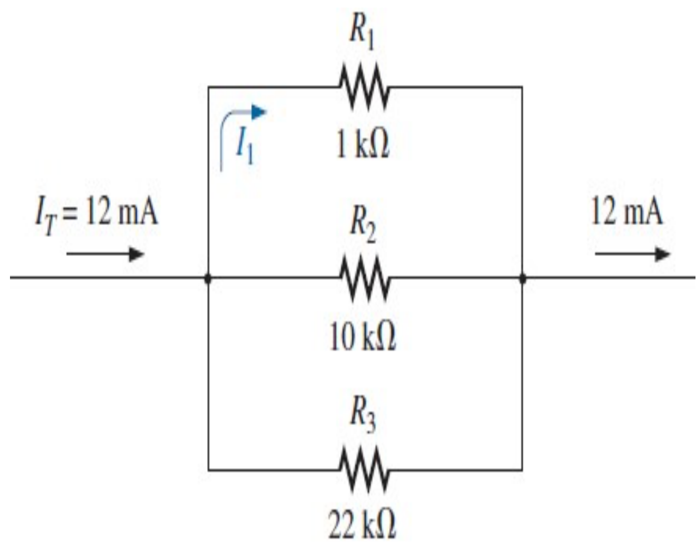
$$V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_xR_x$$

$$I_T = \frac{I_xR_x}{R_T}$$

Solving for Ix, the result is the **current divider rule:**

$$I_x = \frac{R_T}{R_x}I_T$$

Example 3.9 For the parallel network shown, determine current I_1.



$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$= \frac{1}{\frac{1}{1\text{ k}\Omega} + \frac{1}{10\text{ k}\Omega} + \frac{1}{22\text{ k}\Omega}}$$

$$= \frac{1}{1 \times 10^{-3} + 100 \times 10^{-6} + 45.46 \times 10^{-6}}$$

$$= \frac{1}{1.145 \times 10^{-3}} = \mathbf{873.01\ \Omega}$$

$$I_1 = \frac{R_T}{R_1} I_T$$

$$= \frac{(873.01\ \Omega)}{1\text{ k}\Omega}(12\text{ mA}) = (0.873)(12\text{ mA}) = \mathbf{10.48\ mA}$$

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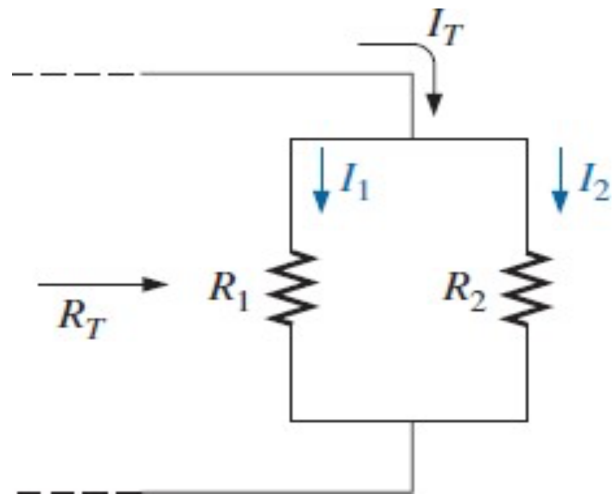
Special Case: Two Parallel Resistors

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$



$$I_1 = \frac{R_T}{R_1} I_T = \frac{\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)}{R_1} I_T$$

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

For two parallel resistors, the current through one is equal to the other resistor times the total entering current divided by the sum of the two resistors

VOLTAGE SOURCES IN PARALLEL

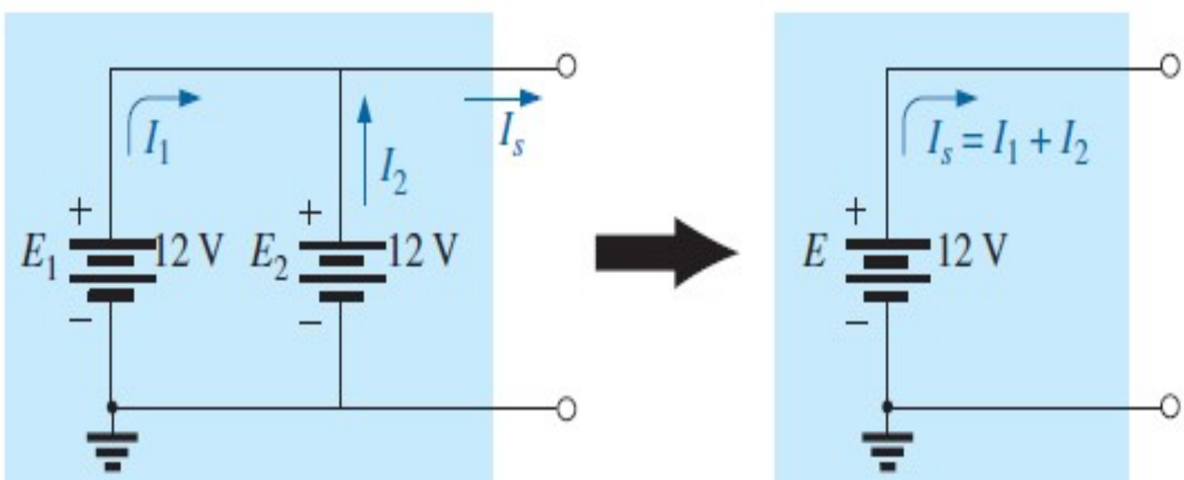
Voltage sources can be placed in parallel only if they have the same voltage.

The primary reason for placing two or more batteries or supplies in parallel is to increase the current rating above that of a single supply.

For example, in Fig shown, two ideal batteries of 12 V have been placed in parallel. The total source current using Kirchhoff's current law is now the sum of the rated currents of each supply. The resulting power available will be twice that of a single supply if the rated supply current of each is the same. That is,

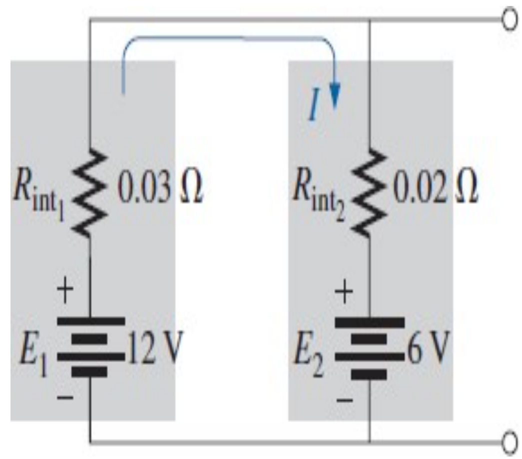
with $I_1 = I_2 = I$

then $P_T = E(I_1 + I_2) = E(I + I) = E(2I) = 2(EI) = 2P_{(\text{one supply})}$



If for some reason two batteries of different voltages are placed in parallel, both will become ineffective or damaged because the battery with the larger voltage rapidly discharges through the battery with the smaller terminal voltage.

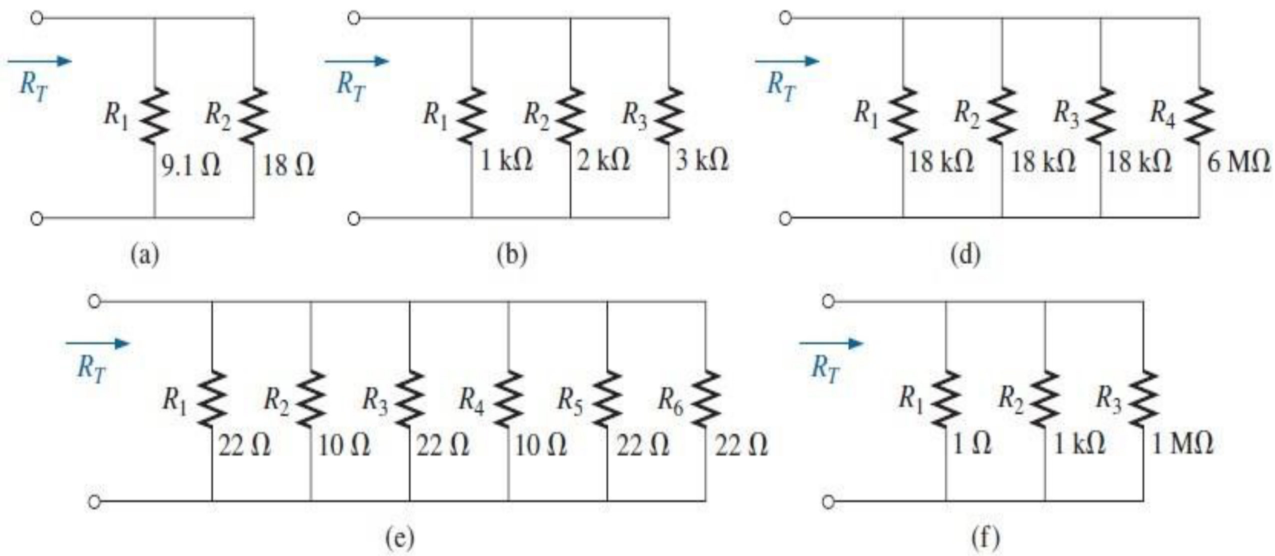
The only current-limiting resistors in the network are
the internal resistances, resulting in a very high discharge current for the battery with the larger supply voltage.



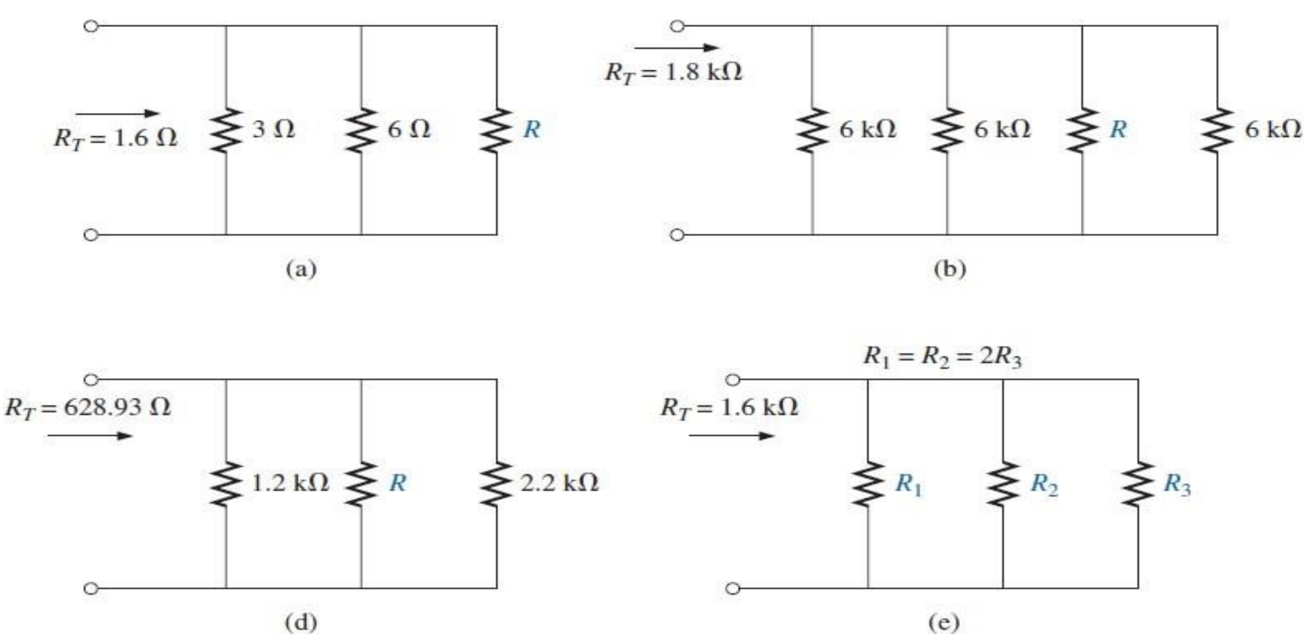
$$I = \frac{E_1 - E_2}{R_{int_1} + R_{int_2}} = \frac{12\text{ V} - 6\text{ V}}{0.03\ \Omega + 0.02\ \Omega} = \frac{6\text{ V}}{0.05\ \Omega} = 120\text{ A}$$

PROBLEMS

1. Find the total resistance for each configuration in Fig. Note that only standard value resistors were used.



2. The total resistance of each of the configurations in Fig. is specified. Find the unknown resistance.



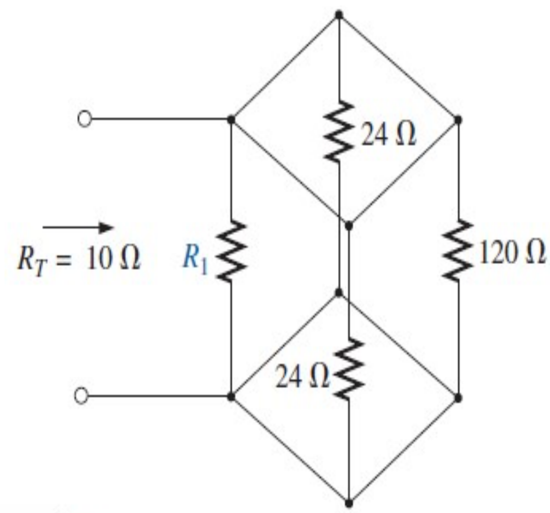
3. For the parallel network in Fig.:

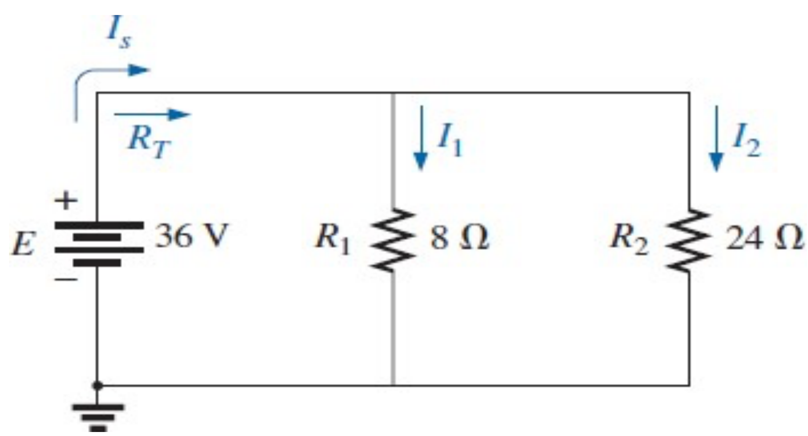
a. Find the total resistance.

b. What is the voltage across each branch?

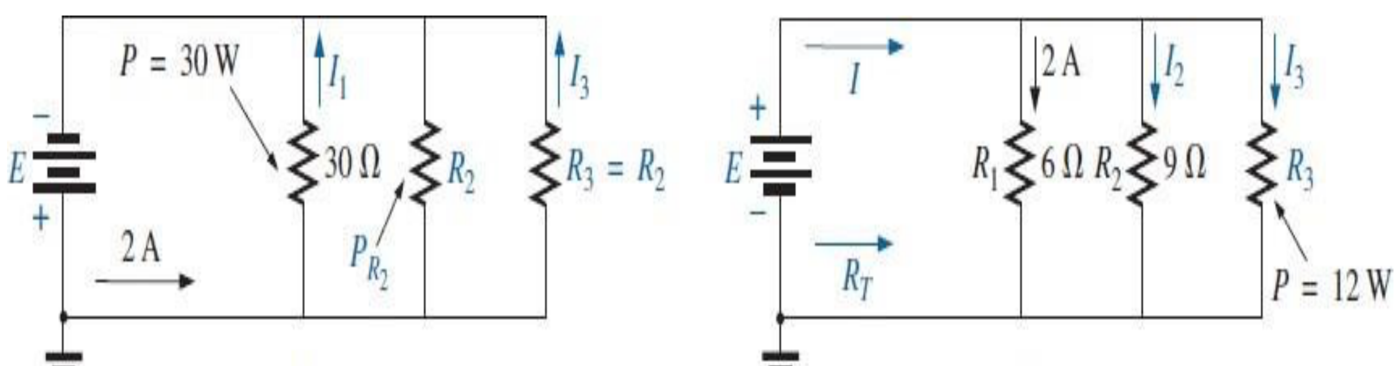
c. Determine the source current and the current through each branch.

d. Verify that the source current equals the sum of the branch currents.





4. Find the unknown quantities for the networks in Fig. using th information provided.



(d) (b)

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