Grammar's rule for solving a set of linear equations:

Consider a set of linear equations in three unknowns x, y, z

$$a_{11}x + a_{12}y + a_{13}z = b_1 \qquad (1)$$

$$a_{21}x + a_{22}y + a_{23}z = b_2 \qquad (2)$$

$$a_{31}x + a_{32}y + a_{33}z = b_3 \qquad (3)$$

In matrices notation the system of linear equations may be written as:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

The above theorem called Grammar's rule to solve it we put:

$$D = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, D_1 = \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}, D_3 = \begin{bmatrix} a_{11} & a_{12} & b1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}$$

Where D = det. of the matrix

$$\therefore x = \frac{D_1}{D} \;, \; y = \frac{D_2}{D} \;, z = \frac{D_3}{D}$$

Example: Use Grammar's rule to solve the system

$5x - 2y = -1$

$2x + 3y = 3$

Sol:

$$\begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix} = 15 + 4 = 19$$

$$D_1 = \begin{bmatrix} -1 & -2 \\ 3 & 3 \end{bmatrix} = -3 + 6 = 3$$

$$D_2 = \begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix} = 15 + 2 = 17$$

$$\therefore x = \frac{D_1}{D} = \frac{3}{19} \qquad , \quad y = \frac{D_2}{D} = \frac{17}{19}$$

Example: Use Grammar's rule to solve the system

$x - 2z = 6$

$-3x + 4y + 6z = 30$

$-x - 2y + 3z = 8$

Sol:

$$\begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 30 \\ 8 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix} = 1(12 + 12) - 0 + 2(6 + 4) = 24 + 20 = 44$$

$$D_1 = \begin{bmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{bmatrix} = 6(12+12) + 2(-60-32) = 144 - 184 = -40$$

$$D_2 = \begin{bmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{bmatrix} = 1(90-48) - 6(-9+6) + 2(-24+30)$$

$$= 42 + 18 + 12 = 72$$

$$D_3 = \begin{bmatrix} 1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{bmatrix} = 1(32+60) - 6(6+4) = 92 + 60 = 152$$

$$\therefore x = \frac{D_1}{D} = -\frac{40}{44} \quad , \qquad y = \frac{D_2}{D} = \frac{72}{44} \quad , \qquad z = \frac{D_3}{D} = \frac{152}{44}$$

H.W:

1- Solve the following equations using Grammar's rule:

a) $2x - 5y = -2$
$4x + 6y = 1$

b) $x + 2y + z = 3$

$2x + y - z = 0$
$x - y + z = 0$

1- Find $A.A^{-1}$ *if the matrix A is given by*:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 1 & 4 \\ 6 & 0 & 3 \end{bmatrix}$$