The Slope of a Line

Mathematicians have developed a useful measure of the steepness of a line, called the slope of the line. Slope compares the vertical change (the rise) to the horizontal change (the run) when moving from one fixed point to another along the line. A ratio comparing the change in y (the rise) with the change in x (the run) is used calculate the slope of a line.

Definition of Slope

The slope of the line through the distinct points (x_1, y_1) and (x_2, y_2)

$$slope = \frac{change\ in\ y}{change\ in\ x} = \frac{Rise}{Run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the slope of the line thru the points given:

- (-3, -1) and (-2,4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

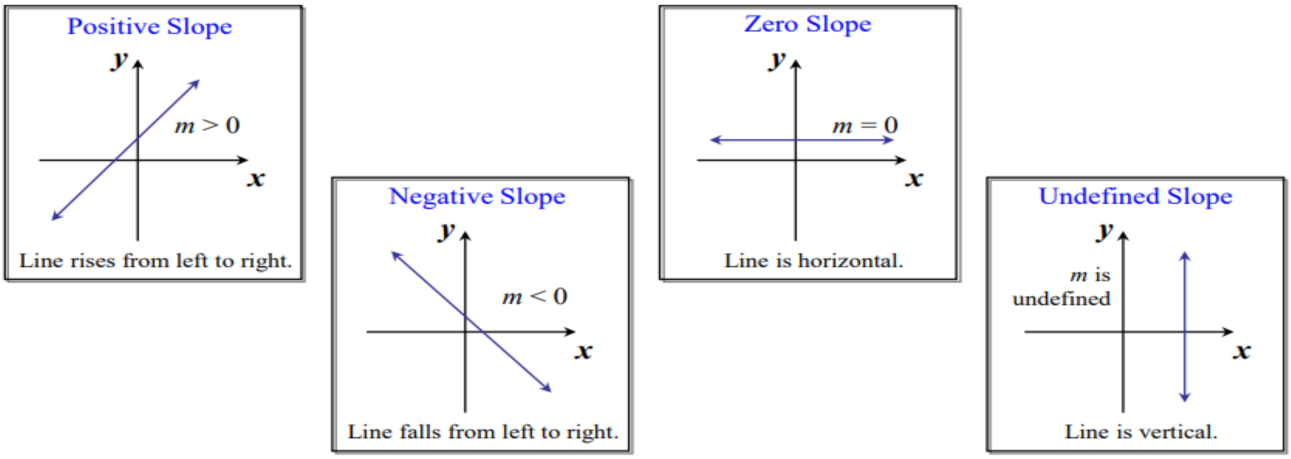
$$= \frac{4 - (-1)}{-2 - (-3)} = \frac{5}{1} = 5$$

- (-3,4) and (2, -2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - 4}{2 - (-3)} = \frac{-6}{5}$$

The Possibilities for a Line's Slope



Point-Slope Form of the Equation of a Line

The point-slope equation of a non-vertical line of slope m that passes through the point (x_1, y_1) is:

$y - y_1 = m(x - x_1)$.

Example: Writing the Point-Slope Equation of a Line

Write the point-slope form of the equation of the line passing through (-1,3) with a slope of 4. Then solve the equation for y.

Solution:

We use the point-slope equation of a line with $m = 4$, $x_1 = -1$, and $y_1 = 3$

y – y This is the point-slope form of the equation.

$y - 3 = 4[x - (-1)]$ Substitute the given values. Simply.

$y - 3 = 4[x + 1]$ We now have the point-slope form of the equation for the given line.
We can solve the equation for y by applying the distributive property.
$y - 3 = 4x + 4$
$y = 4x + 7$

Slope-Intercept Form of the Equation

Line The slope-intercept equation of a non-vertical line with slope m and yintercept b is:

$$y = mx + b.$$

Equations of Horizontal and Vertical Lines

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$y = b$$

where b is the y-intercept.

Note: $m = 0$.

Equation of a Vertical Line

A vertical line is given by an equation of the form

$$x = a$$

where a is the x-intercept.

Note: m is undefined.

General Form of the Equation of a Line

Every line has an equation that can be written in the general form

$$Ax + By + C = 0$$

Where A, B, and C are three integers, and A and B are not both zero.

A must be positive.

Standard Form of the Equation of a Line

Every line has an equation that can be written in the standard form

$$Ax + By = C$$

Where A, B, and C are three integers, and A and B are not both zero.

A must be positive.

In this form, $m = -A/B$ and the intercepts are $(0, C/B)$ and $(C/A, 0)$.

<u>Equations of Lines</u>

- Point-slope form: $y - y_1 = m(x - x_1)$
- Slope-intercept form: y = m x + b
- Horizontal line: y = b
- Vertical line: x = a
- General form: Ax + By + C = 0
- Standard form: Ax + By = C

EX:

Find the slope and the y-intercept of the line whose equation is 2x – 3y + 6 = 0. Solution:

The equation is given in general form, Ax + By + C = 0.

One method is to rewrite it in the form y = mx + b. We need to solve for y.

2x – 3y + 6 = 0 This is the given equation.

2x + 6 = 3y To isolate the y-term, add 3 y on both sides.

3y = 2x + 6 Reverse the two sides. (This step is optional.)

$y = \frac{2}{3}x + 2$ Divide both sides by 3

The coefficient of x, 2/3, is the slope and the constant term, 2, is the y-intercept. Divide both sides by 3

EX:

standard form using integers Find the equation of the line through (0,1/3) with slope 1/2. Write the equation in standard form using only integers.

Solution:

Since we know the slope and y- intercept, start with slope-intercept form:

$$y = \frac{1}{2}x + \frac{1}{3}$$

$$-\frac{1}{2}x + y = \frac{1}{3}$$

$$-6 * \left[-\frac{1}{2}x + y = \frac{1}{3}\right]$$

$3x - 6y = -2$ Standard form with integer

Any integral multiple of 3x - 6y = -2 would also be standard, but we usually use the smallest possible positive coefficient for x.

Parallel Lines

Two lines in a plane are said to be parallel if they have no points in common. Any two vertical lines are parallel, and slope can be used to determine whether non vertical lines are parallel. For example, the lines y = 3x - 4 and y = 3x +1 are parallel because their slopes are equal and their y intercepts are different.

Theorem: Parallel Lines Two non-vertical lines in the coordinate plane are parallel if and only if their slopes are equal.

Writing equations of parallel tines

Ex:

Find the equation in slope-intercept form of the line through (1, -4) that is parallel to y=3x+2

Solution:

Since y =3x + 2 has slope 3, any line parallel to it also has slope 3. Write the equation of the line through (1, -4) with slope 3 in point-slope form:

Y-(-4) = 3(x-1)

Y+4 = 3x-3

Y= 3x-7 Slope-intercept form

Perpendicular Lines

Two lines are perpendicular if they intersect at a right angle. Slope can be used to determine whether lines are perpendicular. For example, lines with slopes such as 2/3 and -3/2 are perpendicular. The slope -3/2 is the opposite of the reciprocal of 2/3. In the following theorem we use the equivalent condition that the product of the slopes of two perpendicular lines is -1, provided they both have slopes.

Theorem: Perpendicular Lines

Two lines with slopes m_1 and m_2 are Perpendicular if and only if $m_1m_2=-1$

EX:

Find the equation of the line perpendicular to the line 3x - 4y =8 and containing the point (-2, 1). Write the answer in slope-intercept form.

Solution:

Rewrite 3x - 4y = 8 in slope-intercept form;

-4y= -3x + 8

Y = (3/4) x -2

Slope of this line is 3/4.

Since the product of the slopes of perpendicular lines is - 1, the slope of the line that we seek is -4/3. Use the slope -4/3 and the point (-2,1) in the point-slope form:

$$y-1=-\frac{4}{3}\left(x-(-2)\right)$$

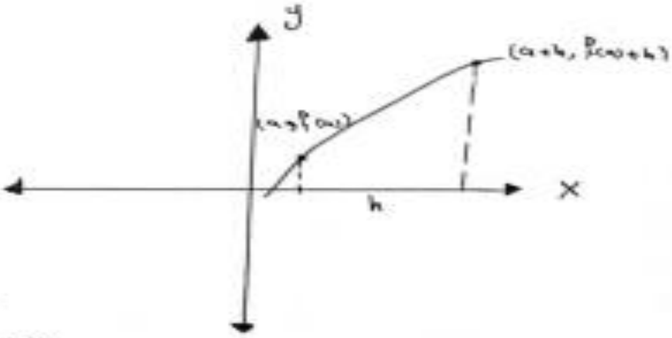
$$y - 1 = -\frac{4}{3}x - \frac{8}{3}$$

$$y = -\frac{4}{3}x - \frac{5}{3}$$

Slope of the Curve:

The slope of a curve $y = f(x)$ at the point $(a, f(a))$ on it is defined to be the number:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



Ex: let's use this limit to check our earlier experimental result that the slope of the curve $y = x^2$ at the point $(1,1)$ is 2. Here $a = 1$ and $f(x) = x^2$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{(1+h)^2 - 1^2}{h}$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h}$$

$$= \lim_{h \to 0} (2+h)$$

$$= 2$$

Ex: we find the slope of the curve $y=\sqrt{x}$ at the point $(4,2)$. Here $a=4$ and $f(x)=\sqrt{x}$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\sqrt{4+h}-\sqrt{4}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}*\frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$$

$$=\lim_{h\to 0}\frac{(4+h)-4}{h(\sqrt{4+h}+2)}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{4+h}+2}=\frac{1}{\sqrt{4}+2}$$

$$=\frac{1}{4}$$

Ex: let's use the definition to calculate the derivative of $f(x)=(x+3)^2$ at $x=0$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{(0+h+3)^2-(0+3)^2}{h}=\lim_{h\to 0}\frac{(h^2+6h+9)-9}{h}$$

$$=\lim_{h\to 0}(h+6)=6$$

H.W

Find the slope of the line containing each pair of points. (Example 1)

1. $(-2, 3), (4, 5)$ $\frac{1}{3}$

2. $(-1, 2), (3, 6)$ 1

3. $(1, 3), (3, -5)$ -4

4. $(2, -1), (5, -3)$ $-\frac{2}{3}$

5. $(5, 2), (-3, 2)$ 0

6. $(0, 0), (5, 0)$ 0

7. $\left(\frac{1}{8}, \frac{1}{4}\right), \left(\frac{1}{4}, \frac{1}{2}\right)$ 2

8. $\left(-\frac{1}{3}, \frac{1}{2}\right), \left(\frac{1}{6}, \frac{1}{3}\right)$ $-\frac{1}{3}$

9. $(5, -1), (5, 3)$ No slope

10. $(-7, 2), (-7, -6)$ No slope

Find the equation of the line through the given pair of points. Solve it for y if possible. (Example 2)

11. $(-1, -1), (3, 4)$ $y = \frac{5}{4}x + \frac{1}{4}$

12. $(-2, 1), (3, 5)$ $y = \frac{4}{5}x + \frac{13}{5}$

13. $(-2, 6), (4, -1)$ $y = -\frac{7}{6}x + \frac{11}{3}$

14. $(-3, 5), (2, 1)$ $y = -\frac{4}{5}x + \frac{13}{5}$

15. $(3, 5), (-3, 5)$ $y = 5$

16. $(-6, 4), (2, 4)$ $y = 4$

17. $(4, -3), (4, 12)$ $x = 4$

18. $(-5, 6), (-5, 4)$ $x = -5$