

## LECTURE FIVE

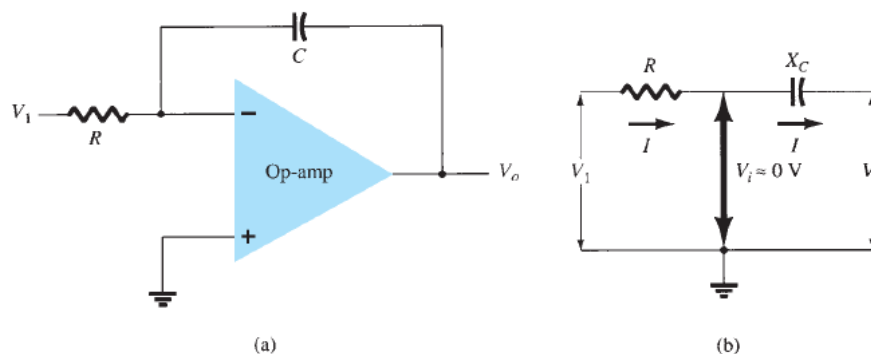
### Op-Amp's circuits (Integrator, Differentiator)

#### 4.1 INTEGRATOR

As mentioned in the previous lecture, the input and feedback components have been resistors. If the feedback component used is a capacitor, as shown in Fig. 5.1 a, the resulting connection is called an *integrator*. The virtual-ground equivalent circuit (Fig. 5.1b) shows that an expression for the voltage between the input and output can be derived in terms of the current  $I$  from the input to the output.

Recall that virtual ground means that we can consider the voltage at the junction of  $R$  and  $X_C$  to be ground (since  $V_i = 0$  V), but that no current goes into ground at that point. The capacitive impedance can be expressed as

$$X_C = \frac{1}{j\omega C} = \frac{1}{sC}$$



**Fig 5. 1: Integrator.**

where  $s = j\omega$  is in the Laplace notation. Solving for  $V_o/V_1$  yields

$$\frac{V_o}{V_1} = \frac{-1}{sCR}$$

This expression can be rewritten in the *time domain* as

$$v_o(t) = -\frac{1}{RC} \int v_1(t) dt$$

The integration operation is one of summation, summing the area under a waveform or a curve over a period of time.

**How a Capacitor Charges** To understand how an integrator works, it is important to review how a capacitor charges. Recall that the charge  $Q$  on a capacitor is proportional to the charging current ( $I_C$ ) and the time ( $t$ ).

$$Q = I_C t$$

Also, in terms of the voltage, the charge on a capacitor is

$$Q = C V_C$$

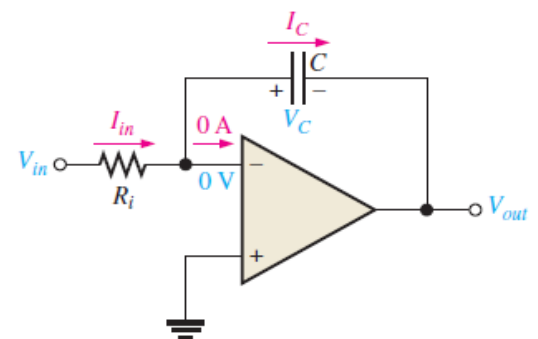
From these two relationships, the *capacitor voltage* can be expressed as

$$V_C = \left(\frac{I_C}{C}\right)t$$

In Figure 5–2, the inverting input of the op-amp is at virtual ground (0 V), so the voltage across  $R_i$  equals  $V_{in}$ . Therefore, the input current is

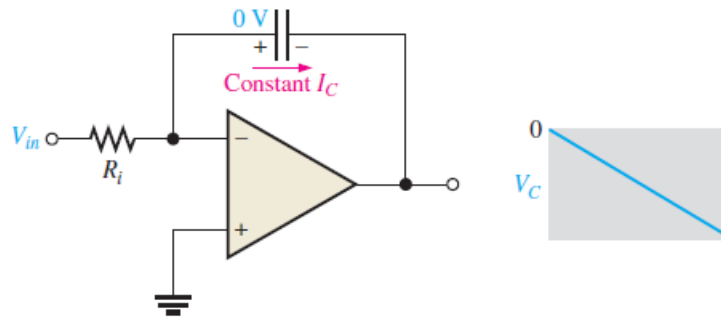
$$I_{in} = \frac{V_{in}}{R_i}$$

If  $V_{in}$  is a constant voltage, then  $I_{in}$  is also a constant because the inverting input always remains at 0 V, keeping a constant voltage across  $R_i$ . Because of the very high input impedance of the op-amp, there is negligible current at the inverting input. This makes all of the input current go through the capacitor,  $I_C = I_{in}$



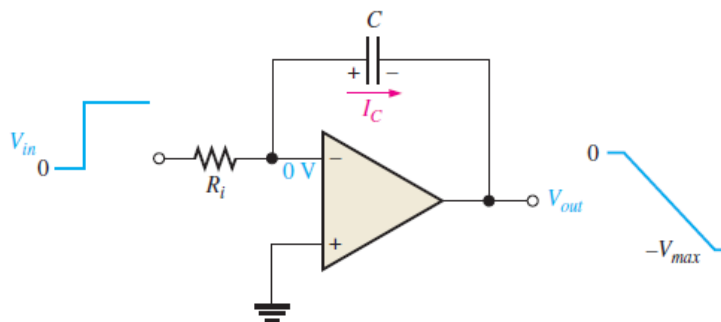
**Fig 5. 2:** Currents in an integrator.

**The Capacitor Voltage** Since  $I_{in}$  is constant, so is  $I_C$ . The constant  $I_C$  charges the capacitor linearly and produces a linear voltage across  $C$ . The positive side of the capacitor is held at 0 V by the virtual ground of the op-amp. The voltage on the negative side of the capacitor, which is the op-amp output voltage, decreases linearly from zero as the capacitor charges, as shown in Figure 5–3. This voltage,  $V_C$ , is called a *negative ramp* and is the consequence of a constant positive input



*Fig 5. 3: A linear ramp voltage is produced across the capacitor by the constant charging current*

**The Output Voltage**  $V_{out}$  is the same as the voltage on the negative side of the capacitor. When a constant positive input voltage in the form of a step or pulse (a pulse has a constant amplitude when high) is applied, the output ramp decreases negatively until the op-amp saturates at its maximum negative level. This is indicated in Figure 5–4.



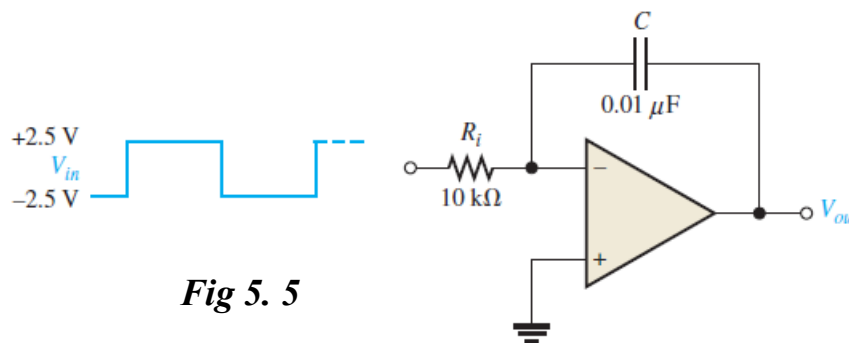
*Fig 5. 4: A constant input voltage produces a ramp on the output of the integrator.*

**Rate of Change of the Output Voltage** The rate at which the capacitor charges, and therefore, the slope of the output ramp, is set by the ratio  $I_C/C$ , as you have seen. Since  $I_C = V_{in}/R_i$ , the rate of change or slope of the integrator's output voltage is  $\Delta V_{out} / \Delta t$ .

$$\frac{\Delta V_{out}}{\Delta t} = -\frac{V_{in}}{R_i C}$$

**EXAMPLE 5–1:**

- a) Determine the rate of change of the output voltage in response to the input square wave, as shown for the ideal integrator in Figure 5–5. The output voltage is initially zero. The pulse width is  $100 \mu s$ .
- (b) Describe the output and draw the waveform.



**Fig 5. 5**

**Solution**

(a) The rate of change of the output voltage during the time that the input is at  $+2.5 \text{ V}$  (capacitor charging) is

$$\frac{\Delta V_{out}}{\Delta t} = -\frac{V_{in}}{R_i C} = -\frac{2.5V}{(10k\Omega)(0.01\mu F)} = -25kV/s = -25mV/\mu s$$

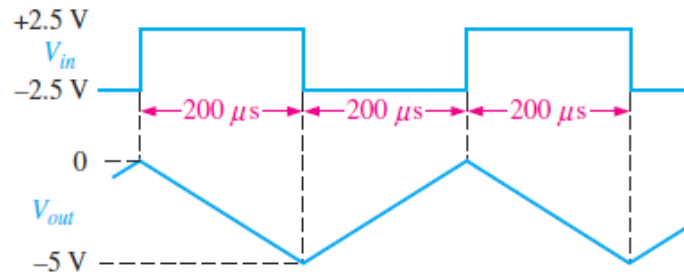
The rate of change of the output during the time that the input is negative (capacitor discharging) is the same as during charging, except it is positive.

$$\frac{\Delta V_{out}}{\Delta t} = +\frac{V_{in}}{R_i C} = +\frac{2.5V}{(10k\Omega)(0.01\mu F)} = +\frac{25kV}{s} = +25mV/\mu s$$

(b) When the input is at +2.5 V, the output is a *negative-going ramp*. When the input is at -2.5 V, the output is a *positive-going ramp*.

$$\Delta V_{out} = (25 \text{ mV}/\mu\text{s})(200 \mu\text{s}) = 5\text{V}$$

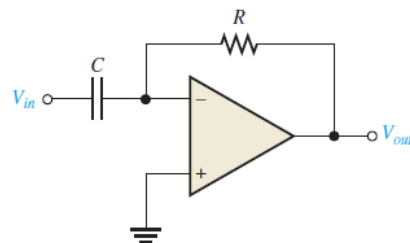
During the time the input is at +2.5 V, the output will go from 0 to -5V. During the time the input is at -2.5 V, the output will go from -5 V to 0. Therefore, the output is a triangular wave with peaks at 0V and -5V, as shown in Figure 5-6.



**Fig 5. 6**

### The Op-Amp Differentiator

The placement of the capacitor and resistor differs from the integrator. The capacitor is now the input element, and the resistor is the feedback element. A differentiator produces an output that is proportional to the rate of change of the input voltage.

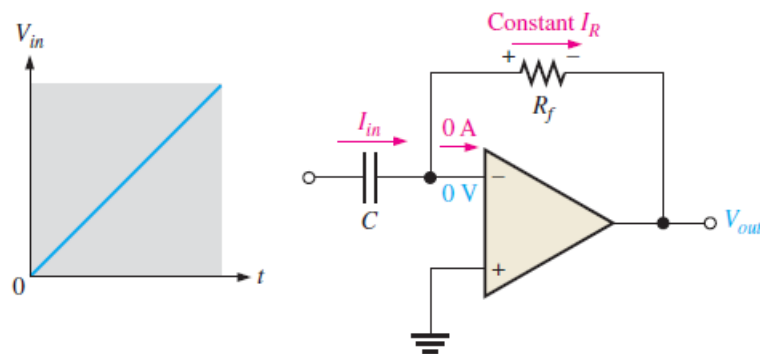


**Fig 5. 7: An op-amp differentiator.**

To see how the differentiator works, apply a positive-going *ramp voltage* to the input as indicated in Figure 5–8. In this case,  $I_C = I_{in}$ , and the voltage across the capacitor is equal to  $V_{in}$  at all times ( $V_C = V_{in}$ ) because of a virtual ground on the inverting input.

From the basic formula,  $V_C = (I_C/C)t$ , the capacitor current is

$$I_C = \left(\frac{V_C}{t}\right)C$$



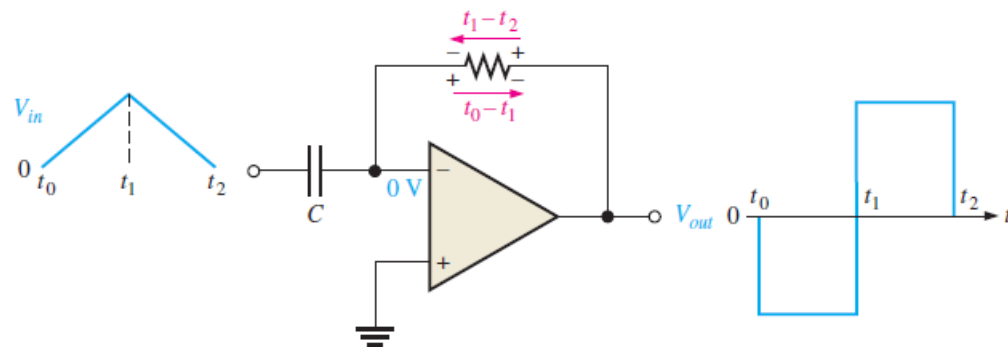
**Fig 5. 8:** differentiator with a ramp

Since the current at the inverting input is negligible,  $I_R = I_C$ . Both currents are constant because the slope of the *capacitor voltage* ( $V_C/t$ ) is constant. The output voltage is also constant and equal to the voltage across  $R_f$  because one side of the feedback resistor is always 0 V (virtual ground).

$$V_{out} = I_R R_f = I_C R_f$$

$$V_{out} = -\left(\frac{V_C}{t}\right)R_f C$$

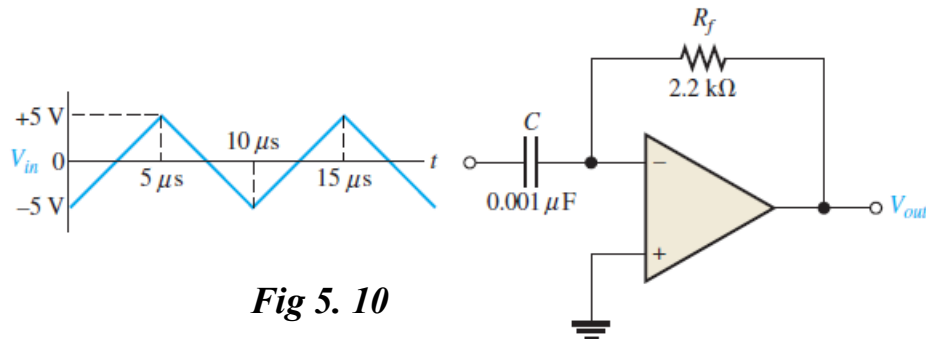
The output is negative when the input is a *positive-going ramp* and *positive when the input is a negative-going ramp*, as illustrated in Figure 5–9. During the positive slope of the input, the capacitor is charging from the input source, and the constant current through the feedback resistor is in the direction shown. During the negative slope of the input, the current is in the opposite direction because the capacitor is discharging.



**Fig 5. 9:** Output of a differentiator with a series of positive and negative ramps (triangle wave) on the input.

**EXAMPLE 5–2:**

Determine the output voltage of the ideal op-amp differentiator in Figure 5–10 for the triangular-wave input shown.



**Fig 5. 10**

### Solution

Starting at  $t = 0$ , the input voltage is a positive-going ramp ranging from -5 V to +5V (a +10 V change) in  $5\mu\text{s}$ . Then it changes to a negative-going ramp ranging from +5 V to -5 (a -10 V change) in  $5\mu\text{s}$ .

The time constant is

$$R_f C = (2.2 \text{ k}\Omega)(0.001 \mu\text{F}) = 2.2 \mu\text{s}$$

Determine the *slope* or rate of change ( $V_C/t$ ) of the *positive-going ramp* and calculate the *output voltage* as follows:

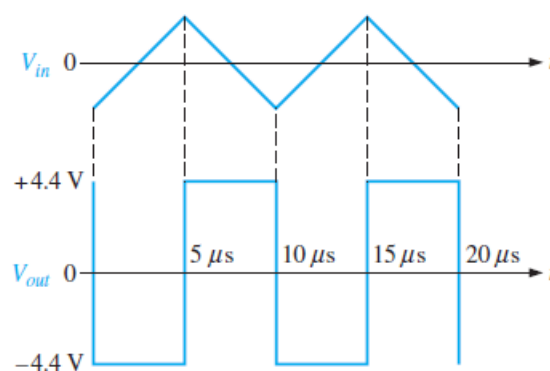
$$\frac{V_C}{t} = \frac{10V}{5\mu\text{s}} = 2V/\mu\text{s}$$

$$V_{out} = -\left(\frac{V_C}{t}\right)R_f C = -(2V/\mu\text{s})2.2\mu\text{s} = -4.4V$$

Likewise, the slope of the negative-going ramp is  $-2V/\mu\text{s}$ , and the output voltage is

$$V_{out} = -\left(-\frac{V_C}{t}\right)R_f C = -(-2V/\mu\text{s})2.2\mu\text{s} = +4.4V$$

Figure 5–11 shows a graph of the output voltage waveform relative to the input



**Fig 5. 11**

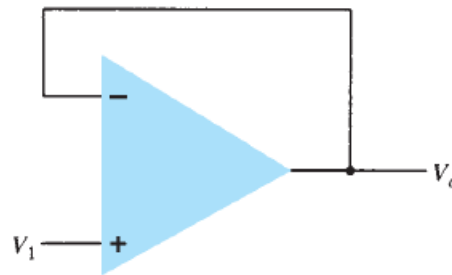
### 3. VOLTAGE BUFFER

A voltage buffer circuit provides a means of isolating an input signal from a load by using a stage having unity voltage gain, with no phase or polarity inversion, and acting as an ideal circuit with very high input impedance and low output impedance.

Figure 5.12 shows an op-amp connected to provide this buffer amplifier operation.

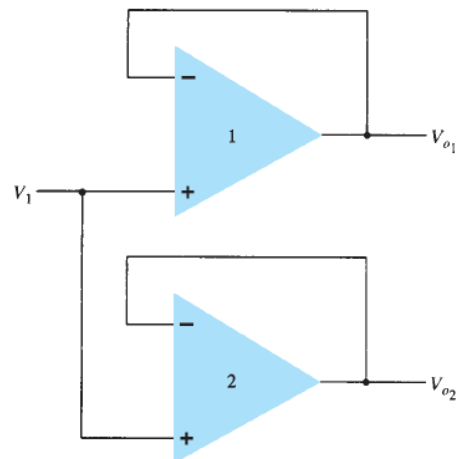
The output voltage is determined by

$$V_o = V_i$$



*Fig 5. 12: Unity-gain (buffer) amplifier*

Figure 5.13 shows how an input signal can be provided to two separate outputs. The advantage of this connection is that the load connected across one output has no (or little) effect on the other output. In effect, the outputs are buffered or isolated from each other.



*Fig 5. 13: Use of a buffer amplifier to provide output signals.*



**Department of Biomedical Engineering**  
**Bio-Medical circuits and electronics / Third stage**  
**Lecturer: Dr. Hussam Jawad Kadhim AL\_Janabi**  
**Email: [hussam.jawad@uomus.edu.iq](mailto:hussam.jawad@uomus.edu.iq)**

---

